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Equilibrium Analysis of Cournot Competition Model

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ABSTRACT

The paper discusses equilibrium solutions of Cournot competition strategy, which is a quantity based competition strategy. The paper derives equilibrium solutions of the competition strategy and analyses differences and influences of various scenarios on upstream and downstream enterprises. The paper later focuses on the comparison of the equilibrium points of Cournot and Collusion strategies when considering similar and dissimilar market demands. The difference in degrees of substitutability between duopoly retailers also has an effect on the equilibrium solution. The paper takes a numerical example to make a relevant analysis. Finally, empirical research explains pricing strategies of the duopoly in real life.

Keywords: Competition strategies; Equilibrium solutions; Cournot competition; Cartel (collusive) competition; Monopoly; Duopoly; Oligopoly.

1.0 INTRODUCTION

1.1 Background

Economics is the social science that studies the production, distribution, and consumption of goods and services. Economics is the study of how economies function and the activities and interactions of economic agents. Microeconomics is a branch of economics that studies individual actors and markets, as well as how they interact and what happens as a result of those interactions. Examples of individual agents include homes, businesses, buyers, and sellers. Macroeconomics examines the economy as a system in which production, consumption, saving, and investment coexist, as well as factors influencing it such as the employment of labor, capital, and land resources, currency inflation, economic growth, and public policies that have an effect on these components. Competition in economics refers to a situation in which various economic enterprises compete for restricted commodities by altering the four marketing mix's components: pricing, product, promotion, and site. According to traditional economic theory, competition drives businesses to create new goods, services, and technologies, giving customers more options and higher-quality goods. Prices for products are often cheaper the more options there are on the market, compared to what they would be if there was little (oligopoly) or no competition (Monopoly) (Tepper, 2018).

A multitude of factors, including those on the firm/seller side, such as the number of firms, entry hurdles, information, and the availability/accessibility of resources, affect how competitive a market is. With each customer having a willingness to pay, the quantity of customers in the market also affects competitiveness and affects the market's overall demand for the product.

A firm's sub-sector, or country's ability and performance to sell and supply goods and services in a given market is referred to as its competitiveness when compared to the ability and performance of other firms, sub- sectors, or countries in the same market. One firm is attempting to determine ways to deprive another company of market share. The Latin word "competere", which describes the competition that exists between entities in markets and industries, is where the word "competitiveness" originates. It is frequently used in management discourse when discussing comparisons of regional and global economic performance (Zeng, et al, 2021).

The number of competitors, how closely they resemble one another in size, and especially how much of the industry's output is controlled by the largest firm can all be used to gauge the level of competition in a given market.

1.2 Economic history of competition theory

Modern economic theory has focused on the many-seller limit of general equilibrium while earlier economic research concentrated on the distinction between price and non-price based competition.

Antoine Augustin Cournot, a 19th-century economist, defined competition as an environment where pricing does not change with quantity or when the demand curve facing the firm is horizontal (Menard, 2024).

Resources (capital, labor, technology, and talent) tend to congregate geographically, according to empirical observation (Florida and Mellander 2020). This outcome reflects the fact that businesses are intertwined with networks of suppliers, customers, and even rival businesses, giving them a competitive edge when selling their goods and services. Although arms-length market links do offer these advantages, there are occasionally externalities that result from connections

among businesses in a particular region or sector (textiles, leather goods, silicon chips) that cannot be captured or promoted by markets alone. Models that highlight the benefits of networks include the "clusterization" process, "value chains," and "industrial districts".

The motivation of businesses under capitalist economic systems is to uphold and enhance their own competitiveness; this essentially applies to all business sectors.

1.3 Different Economics market structures

1.1.1 Monopoly

Monopoly is a situation, by legal privilege or other agreement, which solely one party (company, cartel, etc) exclusively provides a particular product or service, dominating that market or generally exerting powerful control over it. That is an exclusive control over the trade or production of a company or service through exclusive possession (or a commodity controlled by one party) (Hagiu and Lee, 2011).

Monopolies occur when a single company controls the whole market. Monopolies are the one firm that control and define the whole market, as opposed to industries or markets determining the firms (Davis and Orhangazi, 2021). Where one or more of the requirements are not met, monopolies exist and make it difficult for new businesses to enter the market at low cost. High entry barriers are used by monopoly businesses to deter and stop competitors from entering the market, ensuring that they remain the only supplier there. A natural monopoly is a sort of monopoly that develops as a result of a particular industry's high start-up costs or significant economies of scale. These monopolies develop in sectors of the economy that depend on specialized technology, raw materials, or other elements. Monopolies can develop through both ethical and unethical corporate practices. Collusion, mergers, acquisitions, and hostile takeovers are some of these strategies. When two competitors that are at odds with one another work together to fix prices or raise prices in a coordinated manner, this is called collusion. Natural monopolies arise from ethical business activities where a company exploits the high barriers of an industry. The substantial capital or cash required to buy fixed assets, which are tangible assets a company requires to run, is frequently the cause of the high entry barriers. Because they produce and sell to consumers at lower costs than if there were competition in the market, natural monopolies are able to continue operating as they normally can. Monopolies in this situation make efficient use of the resources to sell the goods for less. Monopolists create a quantity at which marginal revenue equals marginal cost, just like competitive enterprises. The distinction here is that in a monopoly, marginal revenue does not equate to price since monopolists have the option to choose the price at which customers are prepared to pay in order to provide a quantity that maximizes profits as the only supplier in the market.

1.1.2 Duopoly

Duopoly is a market situation in which two companies exclusively provide a particular product or service. That is the domination of a field endeavor by two people or entities (or an oligopoly limited to two sellers) (Patel, 2021).

When two businesses jointly control all or almost all of the market for a certain good or service, this is known as a duopoly. The simplest type of oligopoly, or a market dominated by a limited number of businesses, is a duopoly. If two players agree on prices or output (i.e; they form a Cartel), a duopoly can have the same effect on the market as a monopoly.

Note the following:

i. A duopoly is a form of oligopoly, where only two companies dominate the market.
ii. The companies in a duopoly tend to compete against one another, reducing the chance of monopolistic market power.
iii. Visa and MasterCard are examples of a duopoly that dominates the payments industry in Europe and the United States.
iv. One disadvantage of duopolies is that consumers have little choice in products.
v. Another disadvantage of duopolies is that the two players may collude and increase prices for the consumer.

1.1.3 Oligopoly

Oligopoly is an economic condition in which a small number of sellers exert control over the market of a commodity, each of the producers affects but does not control the market (Sevorova, et al, 2011).

Another type of market structures with imperfect competition is oligopolies. An oligopoly occurs when a small number of businesses conspire, either overtly or covertly, to limit output and/or control prices in order to generate profits above those of the market. Oligopolies can consist of two or more businesses. A highly concentrated market structure is an oligopoly. The Cournot model effectively defines competition because, when there are an unlimited number of businesses in the market, the excess of price over marginal cost will eventually approach zero. When there are only two companies competing on the market, it is known as a duopoly. Only a small number of enterprises control the market; for instance, big airlines like American Airlines and Delta operate with a small number of direct rivals, but there are many other smaller airlines that compete in this market. Oligopolies can emerge due to the same conditions that permit monopolies to exist. High entry hurdles, legislative protections, government outsourcing to a select few firms to construct public infrastructure (such as railroads), and access to scarce resources—often natural resources found within a country—are a few of

these. Instead of depending on the free market to determine pricing, companies in an oligopoly gain from price-fixing, which involves setting prices jointly or under the direction of one company in the group. To prevent the entry of new businesses into the market and guarantee that they maintain market share, oligopolies might form cartels. Markets that are prone to oligopolies are typically subject to strict government regulation to protect consumers from unfair pricing and maintain fair competition.

2.0 BACKGROUND OF COURNOT MODEL

Adam Smith defined competition in his 1776 book The Wealth of Nations as the process of allocating productive resources to their most highly valued uses and promoting efficiency. This definition quickly gained attraction among liberal economists who opposed the monopolistic practices of mercantilism, the most prominent economic theory of the time (Magnusson, 2019). Prior to Cournot, Smith and other classical economists discussed price and non-price rivalry among producers in order to offer their goods on the most favorable terms to the highest-bidding purchasers, not necessarily to a sizable number of sellers or to a market that had reached final equilibrium.

Later, microeconomic theory distinguished between perfect and imperfect competition, concluding that perfect competition is Pareto efficient, while imperfect competition is not. Conversely, according to Edgeworth's limit theorem, the addition of more firms to an imperfect market will cause the market to tend towards Pareto efficiency (Petri, 2021). Pareto efficiency, named after the Italian economist and political scientist Vilfredo Pareto (1848- 1923), is an economic state where resources cannot be reallocated to make one individual better off without making at least one individual worse off (Eisenbach and Phelan, 2022). It implies that resources are allocated in the most economically efficient manner; however, it does not imply equality or fairness.

2.1 Cournot competition

Antoine Augustin Cournot (1801–1877) initially described his theory of competition in his book in 1838, Recherches sur les Principes Mathèmatiques de la Thèorie des Richesses as a way to explain the competition with a market for spring water that was dominated by two suppliers (a duopoly). It was one of a number of models that Cournot "explicitly and with mathematical accuracy" laid forth in the volume. To be more specific, Cournot created profit functions for every firm before using partial differentiation to create a function that represented each firm's optimal response to a set of (exogenous) output levels from the other firm(s) in the market. He subsequently demonstrated that the point at which these functions intersect is a stable equilibrium (i.e. the simultaneous solution of the best response functions of each firm) (Von Stackelberg, 2010).

As a result, in equilibrium, each firm's predictions of how other firms will behave are proven to be accurate; after everything is made clear, no firm wants to reverse its output decision (Agrawal, et al, 2022).

Before L´eon Walras acknowledged Cournot as a pioneer, his economic theory received little attention. As a result, Joseph Bertrand wrote an unkind review of Cournot's book, which in turn drew harsh criticism (Puu, 2018). Cournot's analysis of oligopoly was deemed "clever and intriguing, but not free from fundamental difficulties" by Irving Fisher. In 1897, he made arrangements for Nathaniel Bacon to translate the text (Friedman, 2000).

An industry structure where businesses compete on the volume of output they will generate, which they decide on simultaneously and independently, is described by the economic theory of Cournot competition. It bears the name of Antoine Augustin Cournot (1801–1877), whose inspiration came from seeing rivalry in a spring water duopoly (Hossain and Chowdhury, 2018). It has the following properties: there are multiple firms and no discernible differences between the products that each firm produces; there is no conspiracy between businesses, or cooperation; Companies have market power, which means that each firm's output decision affects the price of the good; the number of firms is fixed; rather than pricing, firms compete on quantity; the businesses behave strategically and economically, usually trying to maximize profit in light of the choices made by their rivals.

The "not hypothesis" that each firm seeks to maximize profits is a key tenet of this model. It is predicated on the belief that a firm's output decision won't have an impact on that of its competitors. The price has a well-known diminishing relationship with overall output. All businesses assume that the production of the others is given and are aware of the market's total number of businesses, N. The market price is determined at a point where demand equals the entire amount of goods produced by all businesses. Each company considers the quantity established by its rivals as a given, assesses the residual demand, and then conducts business as usual.

2.2 Nash Equilibrium

The most popular technique to determine the outcome of a non-cooperative game involving two or more participants in game theory is the Nash equilibrium, which bears the name of the American mathematician John Forbes Nash Jr. In Nash equilibrium, each player is supposed to be aware of the equilibrium strategies of the other players, and changing one's own strategy offers no advantages (Kim, 2021). The concept of Nash equilibrium has been around since Cournot used it to explain how rival enterprises choose their outputs

in 1838 (Nomidis, 2023).

The same concept was applied in a specific way in Antoine Augustin Cournot's oligopoly theory in 1838 (Beraud and Numa, 2023). According to Cournot's theory, various enterprises each decide how much production to generate in order to maximize their profits. The best output for one company is reliant upon the best outputs for the others. A pure-strategy Nash equilibrium, or Cournot equilibrium, arises when each firm's output optimizes its profits given the output of the other firms. In his examination of the stability of equilibrium, Cournot also proposed the idea of best response dynamics. However, Cournot did not define the concept broadly or apply it to any other applications.

The current set of strategy options represents a Nash equilibrium if each player has chosen a strategy—an action plan based on what has transpired so far in the game—and no one can raise their personal expected payoff by changing their strategy while the other players keep theirs the same.

If two players, Ali and Bilya, adopt strategies A and B, then (A, B) is a Nash equilibrium if neither Ali nor Bilya has any other viable strategies that perform better than A at maximizing their payoffs in response to the other player's choice of strategy. (A, B, C, D) in a game in which Chika and Daniel are also participants is a Nash equilibrium if A is Ali's best response to (B, C, D), Bilya's best reaction to (A, C, D), and so on.
Note: Nash demonstrated that every finite game has a Nash equilibrium.

3.0 MODELING

In this chapter we will show various models in competition modeling and also use the models to solve some problems specifically using duopoly model. We will also apply oligopoly and monopoly on some problems.

3.1 Cournot competition

Cournot competition is an economic model used to describe an industry structure in which companies compete on the amount of output they will produce, which they decide independently of each other and at the same time (Tremblay, C. H. and Tremblay, V. J., 2019).

An example of this in the actual world is the production of oil, as the market price that customers pay to the companies is based on the total amount of oil produced, which is something that all companies do independently and virtually simultaneously.
Note, in Cournot competition:

i. Firms move simultaneously, no leader or follower for firm 1 as: (as in the Stackelberg competition which will be discussed later).
ii. The firms compete in quantities rather than prices (as in the Bertrand competition which was discussed earlier).
iii. The firms here are referred to as rival firms

3.2 Cournot Duopoly Model

Here the firms are rival to each other. In a duopoly model, the Cournot competition will be a competition between two firms: firm 1 and firm 2
Let the price function $P(Q)$ be:

$$P(Q) = a - bQ$$

Where $P(Q)$ is the inverse demand function (i.e; P is the price)
Q is the total quantity demand and $Q = \sum_i^n \quad q_i, \ i = 1, \dots, n,$
but here $Q = q_1 + q_2$
a and b are constants

Let $TC_i = C_i(q) = Cq_i$ i.e the total cost

$MC_i = C,$ i.e the marginal cost

then $MC_i = \dfrac{\partial TC_i}{\partial q_i} = c$

$AC_i = C,$ i.e; the Average cost

also $AC_i = \dfrac{TC_i}{q_i} = \dfrac{Cq_i}{q_i} = c$

Thus $MC_i = AC_i$

Therefore $P(Q) = a - bQ = a - bq_1 - bq_2$

Note: the profit of a given firm is equal to the revenue minus cost. Revenue is the product of price and quantity of the firm and cost is given by the firm's cost structure, so profit is:
Profit function for firm 1:

$$\begin{aligned}\pi_1 &= P(Q)q_1 - cq_1 \\ &= (a - bQ)q_1 - cq_1 \\ &= (a - bQ - c)q_1 \\ &= (a - bq_1 - bq_2 - c)q_1,\end{aligned}$$

(i.e; the profit of firm 1 in terms of q_1 and q_2)
Profit function for firm 2:

$$\begin{aligned}\pi_2 &= P(Q)q_2 - cq_2 \\ &= (a - bQ)q_2 - cq_2 \\ &= (a - bQ - c)q_2 \\ &= (a - bq_1 - bq_2 - c)q_2,\end{aligned}$$

(i.e; the profit of firm 2 in terms of q_1 and q_2)
For firm 1:

$$\begin{aligned}\text{since } \pi_1 &= (a - bq_1 - bq_2 - c)q_1 \\ &= aq_1 - bq_1^2 - bq_2q_1 - cq_1\end{aligned}$$

We will now find the first order condition (FOC)

$$\frac{\partial \pi_1}{\partial q_1} = 0$$

$$\Rightarrow \quad a - 2bq_1 - bq_2 - c = 0$$

$$\text{therefore } 2bq_1 = a - bq_2 - c$$

$$thus \;\; q_1 = \frac{a - bq_2 - c}{2b},$$

(i.e; the product quantity of firm 1 in terms of q_2)
For firm 2:

$$\pi_2 = (a - bq_1 - bq_2 - c)q_2$$

$$= aq_2 - bq_1q_2 - bq_2^2 - cq_2$$

We will now find the first order condition (FOC) for firm 2 as:

$$\frac{\partial \pi_2}{\partial q_2} = 0$$

$$\Rightarrow \quad a - bq_1 - 2bq_2 - c = 0$$

$$\text{therefore } 2bq_2 = a - bq_1 - c$$

$$thus \;\; q_2 = \frac{a - bq_1 - c}{2b},$$

(i.e. the product quantity of firm 2 in terms of q_1)
Then substituting q_2 in q_1, we have:

$$q_1 = \frac{a - bq_2 - c}{2b},$$

$$= \frac{a - \left(\frac{a - bq_2 - c}{2}\right) - c}{2b}$$

$$= \frac{2a - a + bq_1 + c - 2c}{4b}$$

Thus

$$4bq_1 = a + bq_1 - c$$

$$4bq_1 - bq_1 = a - c \;\; and$$

$$3bq_1 = a - c$$

$$then \; q_1^* = \frac{a - c}{3b}$$

(i.e; the product quantity of firm 1 in terms of *a*, *b* and *c*)

Also substituting $q_1^* = \frac{a - c}{3b}$ in q_2 we have:

$$q_2 = \frac{a - bq_1 - c}{2b},$$

$$= \frac{a - \left(\frac{a - c}{3}\right) - c}{2b}$$

$$= \frac{2a - 2c}{6b}$$

$$= \frac{a-c}{3b}$$

$$then\ q_2^* = \frac{a-c}{3b}$$

(i.e; the product quantity of firm 2 in terms of $a, b\ and\ c$)

$$\Rightarrow \quad q_1^* = q_2^* = \frac{a-c}{3b}$$

$$Then\ \ Q = q_1 + q_2$$

$$= \frac{a-c}{3b} + \frac{a-c}{3b}$$

$$= \frac{2(a-c)}{3b} \qquad Thus\ Q^* = \frac{2(a-c)}{3b},\ (i.e;\ the\ total\ \ quantity\ \ demand)$$

$$and\ P(Q) = a - bQ$$

$$= a - \frac{2(a-c)}{3}$$

$$\frac{3a-2a+2c}{3}$$

$$= \frac{a+2c}{3}$$

$$Thus\ P^* = \frac{a+2c}{3}, \quad (i.e;\ the\ price)$$

$$Then\ \ \pi_1 = (a - bQ - c)q_1 \qquad and\ \ \pi_2 = (a - bQ - c)q_2$$

since $q_1^* = q_2^*$, therefore $\pi_1 = \pi_2$

$$\Rightarrow \quad \pi_1 = (a - bQ - c)q_1$$

$$= \left(a - \frac{2(a-c)}{3} - c\right)\left(\frac{a-c}{3b}\right)$$

$$= \frac{3a-2a+2c-3c}{3}\left(\frac{a-c}{3b}\right)$$

$$= \left(\frac{a-c}{3}\right)\left(\frac{a-c}{3b}\right)$$

$$= \frac{(a-c)^2}{9b}$$

$$= \pi_2^*$$

Note: In Cournot, the profits of firm 1 and 2 are equal. Therefore the total profit for both firms is:

$$\pi^* = \pi_1^* + \pi_2^*$$

$$= \frac{(a-c)^2}{9b} + \frac{(a-c)^2}{9b}$$

$$= \frac{2(a-c)^2}{9b}$$

Hence,

$$\pi = 2\pi_1 = 2\pi_2, \quad since\ \ \pi_1 = \pi_2$$

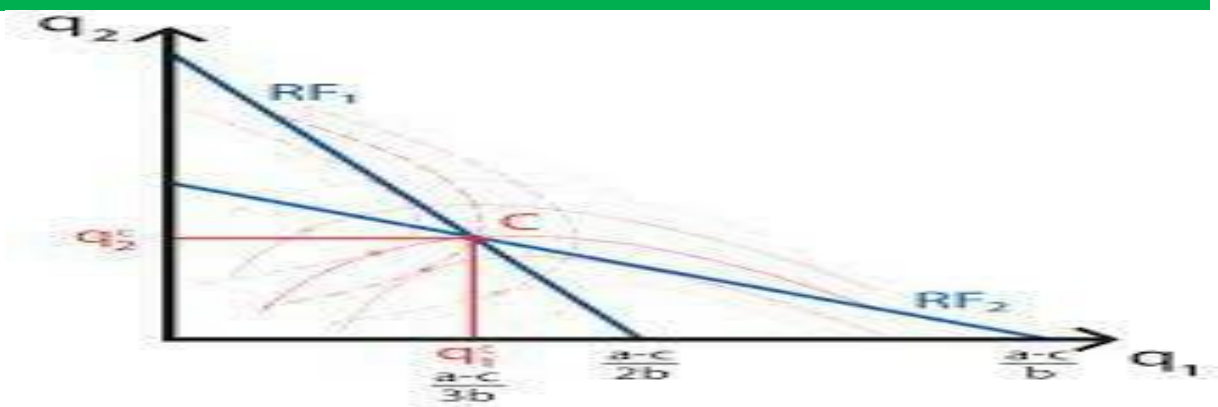

Figure 1: Cournot Duopoly (from policonomics.com)

3.2.1 Nash Equilibrium (NE) strategy for Cournot model

The Nash equilibrium for Cournot competition are $(q_1^*,\ q_2^*)$ i.e; when;

$$q_1 = \frac{a - bq_2 - c}{3b}, \quad \Rightarrow \qquad q_1 = \frac{a-c}{3b}$$

$$q_2 = \frac{a - bq_1 - c}{3b}, \quad \Rightarrow \qquad q2 = \frac{a-c}{3b}$$

Hence $q_1^* = q_2^*$

3.3 Cournot Oligopoly Model (n-firms model)

Here we will also focus on Cournot models involving more than two firms (note; duopoly model is also an oligopoly model involving only two firms).
For oligopoly model with n firms:
Let the price function be:

$$P(q) = a - bQ,$$

where is P (Q) is the price,
Q is the total quantity demand, $Q = \sum_i^n \quad q_{i,}\ i = 1, \dots, n,$
a and b are constants.
Let the total cost be:

$$TC_i = C_i(q) = Cq_i$$

$$\Rightarrow \quad MC_i = \frac{\partial \mathrm{TC_i}}{\partial q_i} = c$$

So $MCi = C$, i.e.; the marginal cost,
also $ACi = \frac{TC_i}{q_i} = \frac{Cq_i}{q_i} = c$
then $AC_i = C$, i.e.; the Average cost.
Thus $MCi = ACi = c$.

This implies the marginal cost and the corresponding average cost of a given firm are equal.

$$\begin{aligned} Then\ P(Q) &= a - bQ \\ &= a - b(q_1 + q_2 + \dots + q_n) \\ &= a - bq_1 - bq_2 - \dots - bq_n \end{aligned}$$

Note: the profit of a given firm is equal to the revenue minus cost. Revenue is the product of price and quantity of the firm and cost is given by the firm's cost structure, so profit of the firms is as follows:
Profit function for firm 1:

$$\begin{aligned} \pi_1 &= P(Q)q_1 - cq_1 \\ &= (a - bQ)q_1 - cq_1 \\ &= (a - bQ - c)q_1 \\ &= (a - bq_1 - bq_2 - \dots - q_n - c)q_1 \end{aligned}$$

Profit function for firm 2:

$$\pi_2 = P(Q)q_2 - cq_2$$
$$= (a - bQ)q_2 - cq_2$$
$$= (a - bQ - c)q_2$$
$$= (a - bq_1 - bq_2 - ... - q_n - c)q_2$$

Profit function for firm n:

$$\pi_n = P(Q)q_n - cq_n$$
$$= (a - bQ)q_n - cq_n$$
$$= (a - bQ - c)q_n$$
$$= (a - bq_1 - bq_2 - ... - q_n - c)q_n$$

To find the different quantities of the respective firms, we set their respective first order conditions (FOC) to zero

$$i.e; \quad \frac{\delta \pi_i}{\delta q_i} = 0$$

For firm 1:

$$\pi_1 = (a - bq_1 - bq_2 - ... - bq_n - c)q_1$$
$$= aq_1 - bq_1^2 - bq_1q_2 - ... - bq_1q_n - cq_1$$

We will now find the first order condition (FOC) for firm 1 as:

$$\frac{\partial \pi_1}{\partial q_1} = 0$$
$$\Rightarrow \quad a - c - 2bq_1 - bq_2 - ...bq_n = 0$$
$$\text{therefore } 2bq_1 = a - c - bq_2 - ... - bq_n$$
$$thus \ q_1^* = \frac{\mathrm{a} - c - bq_2 - ... - bq_n}{2b}$$

For firm 2:

$$\pi_2 = (a - bq_1 - bq_2 - ... - bq_n - c)q_2$$
$$= aq_1 - bq_1q_2 - bq_2^2 - ... - bq_2q_n - cq_2$$

We will now find the first order condition (FOC) for firm 2 as:

$$\frac{\partial \pi_2}{\partial q_2} = 0$$
$$\Rightarrow \quad a - c - bq_1 - 2bq_2 - ...bq_n = 0$$
$$\text{therefore } 2bq_2 = a - c - bq_1 - ... - bq_n$$
$$thus \, q_2^* = \frac{\mathrm{a} - c - bq_1 - ... - bq_n}{2b}$$

For firm n:

$$\pi_n = (a - bq_1 - bq_2 - ... - bq_n - c)q_n$$
$$= aq_1 - bq_1q_n - bq_1q_n - ... - bq^2 - cq_n$$

We will now find the first order condition (FOC) for firm n as:

$$\frac{\partial \pi_{\mathrm{n}}}{\partial q_n} = 0$$
$$\Rightarrow \quad a - c - bq_1 - bq_2 - ...2bq_n = 0$$
$$\text{therefore } 2bq_n = a - c - bq_1 - bq_2 - ... - bq_{n-1}$$

$$thus \quad q_n^* = \frac{a - c - q_1 - bq_2 - \ldots - bq_{n-1}}{2b}$$

So we have the following n-system of equations:

$$q_1 = \frac{a - c - bq_2 - \ldots - bq_n}{2b} \tag{1}$$

$$q_2 = \frac{a - c - bq_1 - \ldots - bq_n}{2b} \tag{2}$$

$$\vdots$$

$$q = \frac{a - c - q_1 - bq_2 - \ldots - bq_{n-1}}{2b} \tag{3}$$

Solving the system of equations, we have:
substituting (2) into (1):

$$q_1 = \frac{a - c - bq_2 - \ldots - bq_n}{2b}$$

$$= \frac{a - c - b\left[\frac{a - c - bq_1 - \ldots - bqn}{2b}\right] - \ldots - bqn}{2b}$$

$$= \frac{a - c - \frac{a}{2} + \frac{c}{2} + \frac{bq_1}{2} + \ldots + \frac{bq_n}{2} - \ldots - bq_n}{2b}$$

$$= \frac{a}{2b} - \frac{c}{2b} - \frac{a}{4b} + \frac{c}{4b} + \frac{bq_1}{4b} + \ldots + \frac{bq_n}{4b} - \ldots - \frac{bq_n}{2b}$$

$$= \frac{a}{4b} - \frac{c}{4b} + \frac{q_1}{4} + \ldots + \frac{q_n}{4} - \ldots - \frac{q_n}{2}$$

$$then \quad q_1 - \frac{q_1}{4} = \frac{a}{4b} - \frac{c}{4b} + \ldots + \frac{q_n}{4} - \ldots - \frac{q_n}{2}$$

$$\Rightarrow \quad q_1 = \frac{4}{3}\left[\frac{a}{4b} - \frac{c}{4b} + \ldots + \frac{q_n}{4} - \ldots - \frac{q_n}{2}\right]$$

$$= \frac{a}{3b} - \frac{c}{3b} + \ldots + \frac{q_n}{3} - \ldots - \frac{2q_n}{3}$$

$$= \frac{a}{3b} - \frac{c}{3b} - \frac{q_3}{3} - \ldots - \frac{q_n}{3},$$

(i.e.; the product quantity of firm 1 in terms of $q3, \ldots qn$),
also substituting (2) in (3):

$$q_n = \frac{a - c - q_1 - bq_2 - \ldots - bq_{n-1}}{2b}$$

$$= \frac{a - c - q_1 - b\left[\frac{a - c - bq_1 - \ldots - bqn}{2b}\right] - \ldots - bq_{n-1}}{2b}$$

$$= \frac{a - c - bq_1 - \frac{a}{2} + \frac{c}{2} + \frac{bq_1}{2} + \ldots + bq_n - \ldots - bq_{n-1}}{2b}$$

$$= \frac{a}{2b} - \frac{c}{2b} - \frac{a}{4b} + \frac{c}{4b} + \frac{q_1}{2} + \frac{q_1}{4} + \ldots + \frac{q_n}{4} - \ldots - \frac{q_{n-1}}{2}$$

$$= \frac{a}{4b} - \frac{c}{4b} - \frac{q_1}{4} + \ldots + \frac{q_n}{4} - \ldots - \frac{q_{n-1}}{2}$$

$$then \;\; q_n - \frac{q_n}{4} = \frac{a}{4b} - \frac{c}{4b} - \frac{q_1}{4} + \frac{q_{n-1}}{4} - \ldots - \frac{q_{n-1}}{2}$$

$$\Rightarrow \;\; q_n = \frac{4}{3}\left[\frac{a}{4b} - \frac{c}{4b} - \frac{q_1}{4} - \ldots - \frac{q_{n-1}}{4}\right]$$

$$= \frac{a}{3b} - \frac{c}{3b} - \frac{q_1}{3} - \frac{q_3}{3} - \ldots - \frac{q_{n-1}}{3},$$

(i.e; the product quantity of firm n in terms of $q_1, q_3, \ldots, q_{n-1}$).
Here we can deduce:

$$q_2 = \frac{a}{3b} - \frac{c}{3b} - \frac{q_1}{3} - \frac{q_3}{3} - \ldots - \frac{q_n}{3},$$

(i.e; the product quantity of firm 1 in terms of $q_3, \ldots, q_n$)
and so on.

Table 1: comparing results from competition models.

4.0 RESULTS

In this chapter, we will show how the different models helps in solving problems on competition modelling in a simplified approach. We will then see the difference in the market prices and quantities of the various competition models discussed, we will also try to highlight on the similarities, differences and some advantages and disadvantages of the various models.

For instance, if we use Stackelberg model in the case where the price function $P(Q) = 100 - 2Q$ and the marginal cost $MC_1 = MC_2 = 20$, the total quantity will be $Q = 45$, market price will be $P = 55$ and the total profit will be $\pi = \pi_1 + \pi_2 = 1575$, while in the Cournot model with price function $P(Q) = 100 - 2Q$ and the marginal cost $MC_1 = MC_2 = 20$, the total quantity will be $Q = 40$, market price will be $P = 60$ and the total profit will be $\pi = \pi_1 + \pi_2 = 1600$. Showing that the Stackelberg price is lower than that of Cournot, so also the
profit.

Note: The Stackelberg price is lower than the Cournot price.but greater than the Bertrand price.

2.1.4

Let consider a given price function $P(Q) = 180-2Q$, with a marginal cost $MC = 20$ (i.e; $MC_1 = MC_2 = 20$) and relate between Stackelberg, Cournot, Bertrand and Cartel competition modelling. Note the following differences in the Table 1, below:

Market structure	market price	Market quantity
Stackelberg	60	60
Cournot	73.33	53.33
Bertrand	20	80
Cartel	100	40

Hence, we can see that from the table, the market price of Cartel is **different**
higher than that of all the other three models; we can also see that the market output of Cartel is lower than that of all the other three models.
Let also consider another price function given as $P(Q) = 500 - 2Q$, with marginal cost $MC_1 = MC_2 = 20$, we can also show the results in Table 2, as follows:
Table 2: comparing results from different competition models.

Market structure	market price	Market quantity
Stackelberg	140	180
Cournot	180	160
Bertrand	20	240
Cartel	260	120

Thus, considering the above tables and comparing with many other examples, we can summarize (deduce) the following facts:

i. The higher the output the lower the price.
ii. The lower the output the higher the price.
iii. Cartel (Collusion) always assumes the highest price.
iv. Bertrand always assumes the lowest price; among the four different models.

5.0 CONCLUSION

Competition causes commercial firms to develop new products, services and technologies, it also helps in enforcing the companies (industries) to improve the quality and quantity of their products, so that consumers can have a choice of which company will best suites their demand in the purchase of goods and services.

We have seen how the models of competition differs in terms of quantities, prices and profits despite having unique

prices functions and marginal costs. Competition is mainly categorized into two parts, the perfect and imperfect competition, where the perfect competition referred to a market situation when many small companies sell individual products. Because no company is large enough to control price, each simply accepts the market price (Nock, 2020). The price is determined by supply and demand. While the imperfect competition is referred to a situation where the characteristics of an economic market do not fulfill all the necessary conditions of a perfectly competitive market (Liverto, 2020).

Due the fact that perfect competition does not occur in real life, so we have considered the imperfect competition which consists of different models among which we discussed some of them.

We explained the different forms of competition, where there may be only one competing as a monopolistic competition, in the case where two companies are competing, that will be referred to as duopolistic competition (it is also a form of oligopoly) and in the case where there are few (i.e; two and above) companies are competing is also known as oligopolistic competition.

This write up explains in details how companies compete in different forms and with different models, which depends on the market situation. For instance, in the case of Stackelberg competition, the companies must acts as a leader and a follower(s) and also move sequentially which violates the Cournot competition, where the companies must compete as rivals to one another and also move simultaneously. Note; a company using a Stackelberg model cannot change to Cournot model and so on.

In Bertrand competition, the companies compete in prices with differentiated products rather than in Stackelberg and Cournot competition, where the companies compete in quantities with homogeneous products. Note; in Bertrand competition if the price p_1 of firm 1 is greater than the price p_2 of firm 2, then firm 1 loses and firm 2 wins the market and vice-versa. If the price $p_1 = p_2$, then two firms will share the market.
Note also the firm with lower price will have zero profit and the firm with higher price will have a positive profit, i.e; after the lower price firm sold everything, then costumers has no any other option than to buy from from the higher price firm.

In Cartel (Collusion), there is no competition, the companies acts as in monopolistic markets, where they agreed upon some laid down conditions. In Cartel, the consumers have no any option than purchase at the said price.

The models solution in this write up specifically for Cournot and Stackelberg competition can be applied in finding the price, quantity and profit to a given problem in a simplified approach.

REFERENCES

Agrawal, A., Gans, J., & Goldfarb, A. (2022). *Prediction machines, updated and expanded: The simple economics of artificial intelligence*. Harvard Business Press.

Davis, L., & Orhangazi, Ö. (2021). Competition and monopoly in the US economy: What do the industrial concentration data show?. Competition & Change, 25(1), 3-30.

Easterly, W., & Levine, R. (2002). It's not factor accumulation: stylized facts and growth models. Banco Central de Chile.

Eisenbach, T. M., & Phelan, G. (2022). Cournot fire sales. American Economic Journal: Macroeconomics, 14(3), 508-542.

Florida, R., & Mellander, C. (2020). Technology, talent and economic segregation in cities. Applied Geography, 116, 102167.

Friedman, J. W. (2000). The legacy of Augustin Cournot. Cahiers d'économie politique, 37(1), 31-46.

Hagiu, A., & Lee, R. S. (2011). Exclusivity and control. Journal of Economics & Management Strategy, 20(3), 679-708.

Hossain, M. M., & Chowdhury, M. N. M. (2018). Antoine Augustin Cournot: The Pioneer of Modern Economic Ideas.

Kim, S. (2021). A Game Theoretic Power Allocation Strategy for Secrecy Capacity Optimization (Doctoral dissertation, DGIST).

Liberto, D. (2020). Imperfect competition definition. Investopedia. Retrieved.

Magnusson, L. (2019). Mercantilism. In The Elgar Companion to John Maynard Keynes (pp. 296-301). Edward Elgar Publishing.

Menard, C. (2024). Toward the Marginal Revolution: The Foundational Status of Antoine-Augustin Cournot. In Marginal Revolution in Economics: A Reappraisal (pp. 57-81). Singapore: Springer Nature Singapore.

Nock, D. (2020). "Let's Bid!"-A modular activity to promote interest in engineering economy. The Engineering Economist, 65(3), 195-212.

Nomidis, D. (2023). Revisiting Cournot and Neoclassical Economics. History of economics review, 84(1), 4-20.

Patel, P. (2021). Modelling cooperation, competition, and equilibrium: The enduring relevance of game theory in shaping economic realities. Social Science Chronicle, 1(1), 1-19.

Petri, F. (2021). Product Markets: Pricing, Capacity, Investment, Imperfect Competition. In Microeconomics for the Critical Mind: Mainstream and Heterodox Analyses (pp. 1021-1114). Cham: Springer International Publishing.

Puu, T. (2018). Bertrand. In Disequilibrium

Economics: Oligopoly, Trade, and Macrodynamics (pp. 97-123). Cham: Springer International Publishing.

Severová, L., Kopecká, L., Svoboda, R., & Brčák, J. (2011). Oligopoly competition on market with food products. Agricultural Economics, 57(12), 580-588.

Tepper, J. (2018). The myth of capitalism: Monopolies and the death of competition. John Wiley & Sons.

Tremblay, C. H., & Tremblay, V. J. (2019). Oligopoly games and the Cournot–Bertrand model: a survey. Journal of Economic Surveys, 33(5), 1555-1577.

Von Stackelberg, H. (2010). Market structure and equilibrium. Springer Science & Business Media.

Zeng, L., Li, R. Y. M., & Huang, X. (2021). Sustainable mountain-based health and wellness tourist destinations: The interrelationships between tourists' satisfaction, behavioral intentions, and competitiveness. Sustainability, 13(23), 13314.