

Local Stability Analysis of Modeling the Effect of Double Dose Vaccination of Measles Disease Dynamics in Zamfara State and Nigeria

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ABSTRACT

Measles is one of communicable diseases caused by the measles virus. In this research paper, we proposed and established a model for Measles disease dynamics, we incorporated the Treatment rate of both Exposed and Infected individuals, Enlightenment Rate and Effectiveness of Drug usage. We analyzed and shown the dynamics of the model system is in the region Σ and the model equations is considered epidemiologically and mathematically well posed with the invariant region method. The non-negativity solutions was presented and we found that the system is positive and remain in \mathcal{R}_+^6 . The equilibrium states were obtained and we present the effective reproduction number of the model system. Analysis of the reproduction number was obtained. The Local stability analysis of the model equation was obtained and we found that the model is locally asymptotically stable if $R_e < 1$.

Keywords: Double dose; Dynamic, Effectiveness of Treatment; Measles disease modeling and Public Enlightenment

1.0 INTRODUCTION

Measles is a highly spreadable viral infection caused by the virus of the measles (AbdulKuddus, Mohiuddin & Rahman, 2021). The disease belongs to the family of paramyxovirus and found in the human system (Ibrahim and Dénes, 2023). This disease spread from individual to individual through cough and sneeze of the infected individual (AbdulKuddus, Mohiuddin and Rahman, 2021). The clinical symptoms of the disease are fever, cough, runny nose, red eyes, and a distinctive rash (WHOrganization. 2021; CDC, nd; Liu, Wang and Zhao, 2020). This measles disease is extra hazardous, particularly for children under five years and adults of 20 years of age above. The difficulties including mouth ulcer, diarrhea, malnutrition, blindness and brain damage may arise due to measles (Goodson, Masresha, Wannemuehler, Uzicanin and Cochi, 2011). The vaccination for the measles is much available, not expensive and effective. It has curiously reduced the figure of people vanishing from measles (AbdulKuddus, Mohiuddin and Rahman, 2021; Ibrahim and Dénes, 2023). Several countries and numerous public health organizations have conducted wonderful efforts globally to combat against measles (AbdulKuddus, Mohiuddin and Rahman, 2021; Garba *et al*, 2022; Integrated Post Campaign Coverage Survey, 2024). However, in spite of being vaccine preventable, the disease stills a community health problem in several Africa countries globally.

Nigeria is the highest populated country in Africa and among the most crowded in the world rated 7th position (Ibrahim, Usman, Mohammed, Datti, Okunromade,

Abubakar, Nguku, 2019; Saleh, 2016; Olaniyan, Yahaya, 2016). The nation occupied a space of 923,768 square kilometers (Garba *et al*, 2022). The measles transmission peak season in Nigeria started in January to May. Nigeria announced measles vaccination into the routine immunization program in 1978 for children aged 9 months implemented series of protective immunization campaigns to fight measles disease from October to November 2024 (Faruk, Adebowale, Balogun, Taiwo, Adeoye, Mamuda and Waziri, 2020). This positive approach is very crucial, with the Nigeria history of measles disease outbreaks and continuing tasks in achieving peak vaccination coverage (Integrated Post Campaign Coverage Survey, 2024). Nigeria measles disease vaccination coverage reached 84.2% countrywide, and then fell short of the 95% campaign goal threshold which set for measles disease eradication during Extra Immunization Activity. Vaccination coverage by state ranged between 59% in Federal Capital Territory and 97% in Ekiti State. Children aged 48 to 59 Months had the maximum coverage at 85.9% while those Aged 9 to 11 Months had the bottommost coverage at 78.2%. Also, urban areas had measles higher coverage vaccination rate of 86.0% compared to rural areas at 81.8% (Integrated Post Campaign Coverage Survey, 2024). North Central had the highest percentage of first time vaccinations at 16.1%, followed by South South at 12.9%, although North East had the lowest at 8.2% (Integrated Post Campaign Coverage Survey, 2024). Measles vaccination first dose official and administrative data, from 2019 to 2024 shown that in 2019 we got 80.52% admin: 54% official, in 2020 we got 76.16%



admin: 54% official, in 2021 we got 72.69% admin: 59% official, in 2022 we got 79.73% admin: 60% official, in 2023 we got 82.73% admin: 60% official, in 2024 we got 85.92% admin: 51% official. While, for second dose, it was reported that, in 2020 we have 16.26% admin: 35% official, in 2021 we have 17.26% admin: 36%, in 2023 we got 24.34% admin: 37% official, in 2024 we got 26.21% admin: 30% official in Nigeria (Integrated Post Campaign Coverage Survey, 2024).

In addition, disease has been a significant public health concern in Zamfara State, Nigeria (Garba et al, 2022). There was a suspected measles outbreak which was reported in Bakura LGA, Zamfara State (Garba et al, 2022; Usman, Umar, Gidado, Gobir, Obi, Ajayi and Ajumobi, 2018). In May 2008, with 21 children presenting with rash and febrile illness (Garba et al, 2022), year 2022 outbreak according to the NCDC reported outbreak of suspected measles disease cases in Zamfara State (Garba et al, 2022). Between January and August 2021, Nigeria recorded 67,000 cases of measles disease, with Zamfara State being one of the affected areas (Garba et al, 2022). The NCDC affirmed that 58.7% of confirmed measles disease cases in affected states, including Zamfara, were individuals who did not received any dose of the vaccination (Olaniyan, Yahaya, 2016; Gunnala, Ogbuanu, Adegoke, Scobie, Uba, Wannemuehler, Ruiz, Elmoussaad, Ohuabunwo, Mustafa, Nguku, Waziri and Vertefeuille, 2016; Ameh, Sufiyan, Jacob, Waziri and Olayinka, 2016; Faruk, Adebawale, Balogun, Taiwo, Adeoye, Mamuda and Waziri, 2020). Meanwhile, Measles remains a major public health concern in Zamfara State due to frequent outbreaks, despite the availability of vaccines for over four decades (Garba et al, 2022). The state has experienced outbreaks of vaccine preventable diseases, including measles, Efforts to improve vaccination coverage and healthcare infrastructures are crucial to controlling measles outbreaks in Zamfara State (Garba et al, 2022).

There are many research related to this disease in epidemiology field, which is one of the very significant research area to the scholars. Several researchers have offered their thoughts and accomplished their investigation mathematically. Mathematical models have

been helpful and important in understanding and predicting the dynamics of both transmission and infectious of individuals (Lasisi, Akinwande, Olayiwola et al., 2018). Among the common are Ebola virus (Lasisi, Akinwande, Olayiwola et al., 2018). Hepatitis B virus (Bertoletti, Maini, & Williams, 2003). The Human Immunodeficiency virus (HIV) (Abdulrahman, Akinwande, Awojoyogbe and Abubakar, 2013; Akinwande, 2006) while Abdullahi, Lasisi and Suleiman (2024); Abdullahi and Lasisi (2024); Mohammed, Lasisi and Suleiman, (2024); Lasisi and Suleiman (2024); Adeyemo, Lawal, Azeez, Philemon, Falade, Nwachukwu, Omame, Lasisi and Aye (2024); Abdullahi and Lasisi, (2024); Lasisi and Suleiman, (2024); Lasisi, Ejiwole, Azuaba and Ibrahim (2024); Codecco (2001); Pascual, Bouma and Dobson (2002); Lasisi, Akinwande, and Oguntolu, (2020) have proposed model system to explored the communication dynamics and control of the measles disease. In this study, we incorporated the following into the existing work on epidemiology modeling; Treatment rate of Exposed individuals, Enlightenment Rate and Effectiveness of Drug usage.

2.0 FORMULATION OF THE MODEL AND ANALYSIS

We formulate a model system for spread of Measles disease infection. We classified the populations to compartmental classes as displayed in Figure 1. The model is divided into six sub-groups that is the susceptible, $X(t)$, first dose of vaccination class as $V_s(t)$, second dose of vaccination as $V_D(t)$, exposed class as $E(t)$, infected class as $I(t)$, and recovery class as $R(t)$. As indicated in the diagram in Figure 1, individuals entered susceptible class from the birth and immigration of persons Λ where the susceptible individual 'X' are visible to Measles disease at α , and infected persons at β , with effectiveness of drug usage of ϕ , the natural death of individuals is k and death due to the disease is τ , the recovery persons is γ , the recovery individuals due to the treatment of the infected person is δ and recovery due to the treatment of exposed person is θ . Also, σ_1 is the rate of vaccination of the first dose, and σ_2 is the rate of losing vaccination of first dose, φ_1 is the rate of second vaccination dose and φ_2 is the rate of recovery from second vaccination dose.

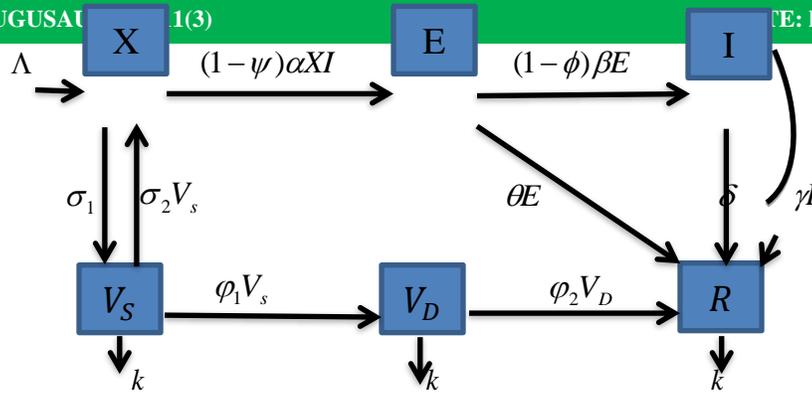


Figure 1: Schematic representation of the Systems

From the above figure 1, the model for dynamics of Measles is described by a system of ordinary differential equations below:

$$\frac{dX}{dt} = \Lambda + \sigma_2 V_s - \sigma_1 X - (1 - \psi)\alpha XI - kX \quad (1)$$

$$\frac{dE}{dt} = (1 - \psi)\alpha XI - (1 - \phi)\beta E - (k + \theta)E \quad (2)$$

$$\frac{dI}{dt} = (1 - \phi)\beta E - \delta I - \gamma I - (k + \tau)I \quad (3)$$

$$\frac{dV_s}{dt} = \sigma_1 X - \sigma_2 V_s - \phi_1 V_s - kV_s \quad (4)$$

$$\frac{dV_D}{dt} = \phi_1 V_s - \phi_2 V_D - kV_D \quad (5)$$

$$\frac{dR}{dt} = \gamma I + \delta I + \theta E + \phi_2 V_D - kR \quad (6)$$

Definition of the Parameters

Table 1: Symbols, and Parameters definition

Symbols	Definition of Parameters of the Model
Λ	Human Population Rate
ϕ_1	Vaccination rate of individual with first dose
ϕ_2	Recovery from vaccination with dose two
α	Exposure individuals to Measles infection
ϕ	Effectiveness of Drug Usage
γ	Recovery rate of infected individuals
σ_2	Wine out of vaccination with dose one
σ_1	Vaccination with dose one
k	Natural human mortality
δ	Rate of Treatment Infected person
θ	Treatment rate of Exposed individuals
τ	Induced death rate
ψ	Enlightenment Rate

2.1 Analysis of Model Equations

Invariant Region

From this analysis, we considered the total populace (N), which implies, $N(t) = X(t) + E(t) + I(t) + V_s(t) + V_D(t) + R(t)$. Then, the differentiation of N with respect to time leading to:

$$\frac{dN}{dt} = \frac{dX}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dV_s}{dt} + \frac{dV_D}{dt} + \frac{dR}{dt}$$

Theorem 1: Consider the region $\Sigma = \{X, E, I, V_s, V_D, R \in \mathbb{R}^6, 0 \leq N \leq \frac{\Lambda}{k}\}$, Σ is a positively invariant set and a global attractor of the model.

Proof: To obtain the invariant analysis, we have, $N(t) = X(t) + E(t) + I(t) + V_s(t) + V_D(t) + R(t)$. Then, the differentiation of N (t) with respect to time leading to:

$$\frac{dN}{dt} = \frac{dX}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dV_s}{dt} + \frac{dV_D}{dt} + \frac{dR}{dt} \tag{7}$$

Then we have:

$$\frac{dN}{dt} \leq \Lambda - k(N)$$

Applying integration factor to gives

$$N(t) \leq \frac{\Lambda}{k} + \{N(0) - \frac{\Lambda}{k}\}e^{-kt} \tag{8}$$

As t move closed to infinity in (8), the size $N \rightarrow \frac{\Lambda}{k}$ which implies, $0 \leq k \leq \frac{\Lambda}{k}$. Thus, the feasible solution set of

the system equations of the model enters and remains in the region:

$$\Sigma = \{X, E, I, V_s, V_D, R \in \mathbb{R}^6, 0 \leq N \leq \frac{\Lambda}{k}\}$$

So, the system is mathematically and epidemiologically well posed, and it is sufficient to study the model in region Σ .

2.2 Existence of Non-negativity Solution

Theorem 2: (Non-negativity solution of the system)

Let $t_0 > 0$, the initial conditions satisfied $X(0) > 0, E(0) > 0, I(0) > 0, V_s(0) > 0, V_D(0) > 0, R(0) > 0$, then the solutions X, E, I, V_s, V_D, R of the model equations (1) – (6), are all non-negativity for $t_0 > 0$.

Proof: To show that for all $t \in [0, t_0]$, $X(t), E(t), I(t), V_s(t), V_D(t), R(t)$ in nonnegative. We, have from model equations (1)-(6) that

$$\frac{dX}{dt} = \Lambda + \sigma_2 V_s - \sigma_1 X - (1 - \psi)\alpha XI - kX \geq -\sigma_1 X - (1 - \psi)\alpha XI - kX \tag{9}$$

So that, $\frac{dX}{dt} \geq -\sigma_1 X - (1 - \psi)\alpha XI - kX > 0$

Implies, $X(t) \geq X(0)e^{\int(\sigma_1 - (1 - \psi)\alpha I - k)dt} > 0$ (10)

Similarly,

$$\frac{dE}{dt} = (1 - \psi)\alpha XI - (1 - \phi)\beta E - (k + \theta)E \geq -(1 - \phi)\beta E - (k + \theta)E \tag{11}$$

$$E(t) \geq E(0)e^{-\int((1 - \phi)\beta + (k + \theta))dt} > 0 \tag{12}$$

Similarly, $I(t) > 0, V_s(t) > 0, V_D(t) > 0, R(t) > 0$. Hence, for all $t \in [0, t_0]$, $X(t), E(t), I(t), V_s(t), V_D(t), R(t)$ will be nonnegative and in \mathbb{R}_+^6

2.3 Disease Free Equilibrium (DFE)

We set the equations (1) - (6) to zero and solve, the Disease Free Point for Measles infection is obtained by setting

$$\frac{dX}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dV_s}{dt} = \frac{dV_D}{dt} = \frac{dR}{dt} = 0. \tag{13}$$

Thus, the DFE denoted by E_0 of the model equations (1)-(6) which is obtained as,

$$E_0 = \left(X^0, E^0, I^0, V_s^0, V_D^0, R^0, \right) \\ E_0 = \left(\begin{array}{c} \frac{\Lambda \cdot (\sigma_2 + \varphi_1 + k)}{(\sigma_1 + k) \cdot (\sigma_2 + \varphi_1 + k) - \sigma_1 \cdot \sigma_2}, 0, 0, \frac{\Lambda \cdot \sigma_1}{(\sigma_1 + k) \cdot (\sigma_2 + \varphi_1 + k) - \sigma_1 \cdot \sigma_2}, \\ \frac{\Lambda \cdot \varphi_1 \cdot \sigma_1}{(\varphi_2 + k) \cdot ((\sigma_1 + k) \cdot (\sigma_2 + \varphi_1 + k) - \sigma_1 \cdot \sigma_2)}, \frac{\Lambda \cdot \varphi_1 \cdot \varphi_2 \cdot \sigma_1}{k(\varphi_2 + k) \cdot ((\sigma_1 + k) \cdot (\sigma_2 + \varphi_1 + k) - \sigma_1 \cdot \sigma_2)} \end{array} \right) \tag{14}$$

2.4 Effective Reproduction Number

Effective reproduction number R_e for the Measles disease equations (1) - (6) is obtained with the method of next generation operator method as described in Lasisi, Akinwande and Oguntolu (2020); Lasisi (2020); Lasisi (2018); Lasisi (2021); Lasisi and Adeyemo (2021); Lasisi, Akinwande and Abdulrahman (2020). Both inflow and out flow of the infectious classes are $F(x)$ and $V(x)$ which are obtained from the model equations (2) and (3)

$$\frac{dE}{dt} = (1 - \psi) \cdot \alpha X \cdot I - (1 - \phi) \cdot \beta \cdot E - (k + \theta) \cdot E \\ \frac{dI}{dt} = (1 - \phi) \cdot \beta \cdot E - \delta \cdot I - \gamma \cdot I - (k + \tau) \cdot I$$

Consequently, we have the following Jacobian matrix

$$F = \begin{pmatrix} 0 & (1 - \psi) \alpha X^0 \\ (1 - \phi) \beta & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} (1 - \phi) \beta - (k + \theta) & 0 \\ 0 & (\delta + \gamma + \tau + k) \end{pmatrix} \tag{15}$$

Therefore, The characteristics system of $F \cdot V^{-1}$ is obtained with the inverse of V as:

$$|FV^{-1} - \lambda| = \begin{vmatrix} -\lambda & \frac{(1 - \psi) \alpha X^0}{(\delta + \gamma + k + \tau)} \\ \frac{(1 - \phi) \beta}{(1 - \phi) \beta - (k + \theta)} & -\lambda \end{vmatrix} = 0 \tag{16}$$

Therefore the dominant eigenvalue is given as follow,

$$\lambda = \pm \sqrt{\frac{(1 - \phi) \beta (1 - \psi) \alpha X^0}{(\delta + \gamma + k + \tau) ((1 - \phi) \beta - (k + \theta))}}$$

Implies, effective reproduction number (R_e) is achieved as following, which is the larger Eigen value

$$R_e = \sqrt{\frac{(1 - \phi) \beta (1 - \psi) \alpha X^0}{(\delta + \gamma + k + \tau) ((1 - \phi) \beta - (k + \theta))}} \tag{17}$$

Therefore, effective reproduction number (R_e) after substitution of X^0 is given as:

$$R_e^2 = \frac{(1 - \phi) \beta (1 - \psi) \alpha}{(\delta + \gamma + k + \tau) ((1 - \phi) \beta - (k + \theta))} * \frac{\Lambda (\sigma_2 + \varphi_1 + k)}{(\sigma_1 + k) (\sigma_2 + \varphi_1 + k) - \sigma_1 \sigma_2} \tag{18}$$

This is the average number of new infections generated by a single infectious of the measles.

2.5 Local Stability of DFE

The local stability of DFE is determined by linearize (1) – (6), we get

$$J(E_0) = \begin{pmatrix} -\sigma_1 - (1-\psi)\alpha.I^0 - k & 0 & -(1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ (1-\psi)\alpha.I^0 & -(1-\phi).\beta - (k+\theta) & (1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ 0 & (1-\phi).\beta & -(\delta+\gamma+k+\tau) & 0 & 0 & 0 \\ \sigma_1 & 0 & 0 & -(\sigma_2 + \varphi_1 + k) & 0 & 0 \\ 0 & 0 & 0 & \varphi_1 & -(\varphi_2 + k) & 0 \\ 0 & \theta & \delta + \gamma & 0 & \varphi_2 & -k. \end{pmatrix} \quad (19)$$

At DFE, we get

$$J(E_0) = \begin{pmatrix} -\sigma_1 - k & 0 & -(1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ 0 & -(1-\phi).\beta - (k+\theta) & (1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ 0 & (1-\phi).\beta & -(\delta+\gamma+k+\tau) & 0 & 0 & 0 \\ \sigma_1 & 0 & 0 & -(\sigma_2 + \varphi_1 + k) & 0 & 0 \\ 0 & 0 & 0 & \varphi_1 & -(\varphi_2 + k) & 0 \\ 0 & \theta & \delta + \gamma & 0 & \varphi_2 & -k. \end{pmatrix} \quad (20)$$

Solving the Equation (20), using Gaussian Elimination method, we get

$$J(E_0) = \begin{pmatrix} -\sigma_1 - k & 0 & -(1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ 0 & -(1-\phi).\beta - (k+\theta) & (1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ 0 & 0 & G & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\sigma_2 + \varphi_1 + k) & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\varphi_2 + k) & 0 \\ 0 & \theta & 0 & 0 & 0 & -k. \end{pmatrix} \quad (21)$$

Where,

$$G = \frac{-\psi.\phi.\beta.\alpha.X^0 - \psi.\beta.\alpha.X^0 - \phi.\beta.\alpha.X^0 + \beta.\alpha.X^0 - [(1-\phi).\beta - (k+\theta)].[-(\delta+\gamma+k+\tau)]}{-(1-\phi).\beta - (k+\theta)} \quad (22)$$

Taking the characteristics equation of (22), we have $|J(E_0) - \lambda I| = 0$.

$$|J(E_0) - \lambda I| = \begin{pmatrix} -\sigma_1 - k - \lambda & 0 & -(1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ 0 & -(1-\phi)\beta - (k+\theta) - \lambda & (1-\psi)\alpha.X^0 & 0 & 0 & 0 \\ 0 & 0 & G - \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\sigma_2 + \varphi_1 + k) - \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\varphi_2 + k) - \lambda & 0 \\ 0 & \theta & 0 & 0 & 0 & -k - \lambda. \end{pmatrix} \quad (23)$$

Implies,

$$\{[-\sigma_1 - k - \lambda].[-(1-\phi)\beta - (k+\theta) - \lambda].[G - \lambda].[-(\sigma_2 + \varphi_1 + k) - \lambda].[-(\varphi_2 + k) - \lambda].[-k - \lambda]\} = 0. \quad (24)$$

The Eigenvalues of (24) are:

$$\lambda_1 = -\sigma_1 - k < 0$$

$$\lambda_2 = -(1-\phi)\beta - (k+\theta) < 0$$

$$\lambda_4 = -(\sigma_2 + \varphi_1 + k)$$

$$\lambda_5 = -(\varphi_2 + k)$$

$$\lambda_6 = -k$$

$$\lambda_3 = G < 0 \text{ is this less than } 1?$$

$$\lambda_3 = \frac{\{-\psi\phi\beta\alpha X^0 - \psi\beta\alpha X^0\} + \{-\phi\beta\alpha X^0 + \beta\alpha X^0\} - [(1-\phi)\beta - (k + \theta)][-(\delta + \gamma + k + \tau)]}{-(1-\phi)\beta - (k + \theta)}$$

$$\lambda_3 = \frac{\frac{(1-\phi)\{(\psi+1)\beta\alpha X^0\}}{-(1-\phi)\beta - (k + \theta)} - [1]}{-(1-\phi)\beta - (k + \theta)}$$

$$\lambda_3 = \frac{\frac{(1-\phi)\{(\psi+1)\beta\alpha X^0\}}{-(1-\phi)\beta - (k + \theta)} - [1]}{1/-(\delta + \gamma + k + \tau)}$$

$$\lambda_3 = (\delta + \gamma + k + \tau)\{R_e - 1.\} \tag{25}$$

$$\lambda_3 = (\delta + \gamma + k + \tau)\{R_e - 1.\} < 0 \text{ if } R_e < 1 \tag{26}$$

Therefore, since all the Eigenvalues are zero(s) then, The Measles DFE is locally asymptotically stable if $R_e < 1$. Implies, the Measles infection will not persist in the populace. Hence, it shown that Measles disease can be eliminated from the population when $R_e < 1$.

Theorem 3: The Measles DFE ‘ E_0 ’ of the system (1)-(6) is Locally Asymptotically Stable (LAS) in Σ , if $R_e < 1$

3.0 CONCLUSION

The Measles disease dynamics model equation, by incorporating the Treatment rate of Exposed individuals, Enlightenment Rate and Effectiveness of Drug usage were presented. The DFE states and effective reproduction number of the system which is the average number of new infections caused by a single infectious of the measles were obtained. We obtained the invariant region of the model, we found that the system was mathematically and epidemiologically well posed and sufficient to study model dynamics in region Σ . The non-negativity solution of the model equations obtained and found that for all $t \in [0, t_0]$, $X(t)$, $E(t)$, $I(t)$, $V_s(t)$, $V_D(t)$, $R(t)$ remained nonnegative and in \mathfrak{R}_+^6 . Local stability of the measles disease was obtained and found that since all the Eigenvalues are less than zero(s) then the Measles DFE is locally asymptotically stable if $R_e < 1$. Hence, Measles disease can be control or eradicated from the population when $R_e < 1$.

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