



## Two-State Stochastic Model with Application to the Nigeria's Labour Market

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### ABSTRACT

This research seeks to address the critical issue of unemployment in Nigeria by developing a two-state stochastic model to analyse transitions between employment and unemployment states in the labour market. Traditional models, such as ARIMA, rely heavily on historical data and are limited to short-term predictions. To overcome these limitations, a Modified Exponential Distribution (MED) is introduced in this work that can model long-term transition probabilities, contrasting it with the conventional exponential distribution (ED). The model is applied to Nigeria's labour market, segmented into public and private sectors, with the private sector further divided into formal and informal sectors. Key findings reveal that the MED is more suitable for long-term analysis, showing gradual transitions, while the ED predicts rapid changes, ideal for short-term forecasts. Regional analysis highlights disparities across Nigeria's geopolitical zones, with the South-East and South-South exhibiting the highest unemployment and employment probabilities, indicating dynamic but unstable labour markets. The private sector dominates employment absorption, offering an 82% probability by year 10, compared to 18% in the public sector. Similarly, the informal sector provides immediate employment but plateaus early, while the formal sector shows steady growth, suggesting long-term stability. The study underscores the need for targeted policies to address regional imbalances and promote formal sector growth. It recommends prioritizing private sector opportunities for job seekers while advocating for structural reforms to enhance formal employment. By providing a robust framework for long-term labour market analysis, this research contributes to informed policy-making and economic planning in Nigeria.

**Keywords:** Unemployment, Stochastic Model, Modified Exponential Distribution, Labour Market, Nigeria.

### 1.0 INTRODUCTION

The labour market, which consists of the demand and supply for labour, is the main component of every economy. Employers provide the demand, while employees provide the supply (Kwaghkor *et al.*, 2019a; Kunawotor *et al.*, 2020; ILO, 2020; OECD, 2023). Unemployment represents a significant threat to societal stability in many countries, including Nigeria. Consequently, understanding its magnitude and identifying effective solutions is imperative (Chen *et al.*, 2020; Oluwaleye, 2021). The two states of the labour market, according to Von (2009), are unemployment and employment, and the change between them may be predicted using probability theory. The term "labour force" refers to the total population of the two labour market states. All individuals between the ages of fifteen and sixty-four who are prepared and capable of working comprise the labour force (NBS, 2015; NBS, 2016; Kwaghkor *et al.*, 2019b; World Bank, 2022). Some of the reasons for unemployment in Nigeria among others include; speedy population growth, the rapid increase of the educational system, rural-urban migration, the near-to-the-ground standard of education, corruption, unsteady and unsustainable power, poor practice of management, negligence of the agricultural science sector (Uddin & Uddin, 2013; Asaju *et al.*, 2014; Biswas, 2020; Sani, 2022; OECD, 2022; Agwu, 2022; Nnachi & Ebubechima, 2023). The effects of unemployment in Nigeria include income disparity, insecurity,

deterioration in the quality of life, and job insecurity (NLC, 2012; Uddin & Uddin, 2013; Agwu, 2022; Tukur & Aguiyi, 2022; Gado, 2025). The effects are grouped into economic, political, and social implications by Asaju *et al.* (2014).

Mathematics plays a vital role in describing and studying real word situations (see Kwaghkor *et al.*, 2021; Kwaghkor *et al.*, 2022; Danjuma & Kwaghkor 2025; Kwaghkor *et al.*, 2025). There are several models to predict the unemployment rate. From these models, the ARIMA model is the most flexible in predicting the unemployment rate with no government intervention (Dritsaki, 2016; Carista *et al.*, 2025). Most models used to predict the unemployment probabilities give short-term predictions (Kwaghkor *et al.*, 2019a; Ridwan *et al.*, 2020). These existing unemployment prediction models heavily rely on historical data (Kwaghkor *et al.*, 2019a; Kwaghkor *et al.*, 2019b). This dependence presents a significant limitation, as the absence of such data makes accurate forecasting difficult. Our research aims to fill this gap by formulating and deriving a two-state stochastic model. This model will predict long-term unemployment probabilities and employment generation, and we will apply it specifically to analyse the movement between Nigeria's two labour states in both the public and private labour market sectors.

The public sector is the sector of the economy that is owned, administered, and financed by governmental

agencies. Its principal objective is to offer needed services and goods to the public, promote social welfare, and sustain general societal well-being, rather than making a profit. These cover government agencies, public schools, state-owned hospitals, and public transit systems. Funding often stems from tax revenues (Poister & Henry, 1994; Andrews *et al.*, 2020; Karagiannidou & Wittenberg, 2022). The private sector encompasses the portion of the economy that is owned and managed by individuals, private enterprises, and organisations, with the primary purpose being profit generation. All privately owned for-profit firms, not under government ownership or control, fall under this category. Innovation, competition, and market demand serve as essential drivers within the private sector. The Private Sector is further subdivided into the Formal and Informal Sectors (Poister & Henry, 1994; Gielens *et al.*, 2021). The Formal Sector contains enterprises and jobs that are officially recognised and regulated by the government. Companies in the formal sector often pay taxes, follow to labour regulations, and offer legal protections and benefits to their employees, such as written contracts, set salary, health insurance, and retirement plans. It includes large private enterprises, commercial banks, and educational or healthcare institutions (Haanwinckel & Soares, 2021). Informal Sector also known as the informal economy or grey economy, on the other hand, consists of economic activity and employment that are not taxed, monitored, or regulated by the government. It generally involves firms that are not formally recognised and people who lack basic social or legal protection and job benefits (Haanwinckel & Soares, 2021; Dell'Anno, 2021). Examples include street sellers, subsistence farmers, small-scale manufacturers, and some independent service providers like unlicensed taxi drivers or hairdressers operating unofficially. While it contributes significantly to employment and income in many developing countries, such as Nigeria, it typically comes with low work security and inadequate social protection (Poister & Henry, 1994; Etim & Daramola, 2020; Haanwinckel & Soares, 2021; Dell'Anno, 2021). The new two-state stochastic model is helpful in studying the transition between the two labour market states (unemployment and employment states) in both sectors of the Nigerian economy. The existence and uniqueness of the model's solutions are established. Using empirical data from the National Bureau of Statistics (NBS), we derive a best-fit model to predict the long-term movement of individuals between the labour states. This work will provide crucial insights into Nigeria's long-term unemployment rate probabilities and employment generation. This understanding will help formulate appropriate measures to reduce unemployment and improve employment generation.

## 2.0 THE TWO-STATE STOCHASTIC MODEL

The process by which the two labour market states (unemployment and employment) change is stochastic. The rationale behind this is that an individual may be unemployed or employed at a later date. In this section, we shall derive and formulate a two-state stochastic model with application to the analysis of the labour market. Before the formulation, we shall define some important concepts useful for the formulation as follows.

### 2.1 Stochastic Process

A stochastic process is an assemblage of random variables  $\{X_t\}_{t \in T}$  defined on a shared probability space  $(\Omega, F, P)$ , where:  $\Omega$  is the sample space,  $F$  is a  $\sigma$ -algebra (event space),  $P$  is a probability measure, and  $T$  is an index set (which often represents time).

### 2.2 Markov Chain

A Markov chain is a stochastic process that the future state depends only on the present state and not on the past. Formally, it is defined as a discrete-time or continuous-time stochastic process  $\{X_t\}_{t \in T}$  (where  $T$  is an index set) is a Markov chain if for all times  $t_0 < t_1 < \dots < t_n < t_{n+1}$  and all states  $X_0, X_1, \dots, X_n, X_{n+1}$  in the state space  $S$ , the following holds:

$$P\{X_{n+1} = j | X_0 = x_0, X_1 = x_1, \dots, X_n = i\} = P\{X_{n+1} = j | X_n = i\} = P_{ij} \quad (1)$$

where  $P_{ij}$  satisfies the probabilities conditions below,

$$P_{ij} \geq 0, \text{ and } \sum_{j=1}^N P_{ij} = 1, \quad i, j = 1, 2, \dots, N \quad (2)$$

$N$  is the states in the system. Equations (1) and (2) are called the Markovian properties.

### 2.3 Semi-Markov Chain

A Semi-Markov Chain (SMC) or Semi-Markov Process (SMP) generalizes a Markov chain by allowing the time spent in each state to follow an arbitrary probability distribution, rather than being restricted to exponential (in continuous time) or geometric (in discrete time) distributions. The next state depends only on the current state (like a Markov chain), but the holding time (time spent in a state before transitioning) can follow any distribution. Unlike continuous-time Markov chains (CTMCs), where holding times are exponentially distributed, SMCs allow for non-memoryless distributions. Mathematically, a Semi-Markov Chain is defined as  $\{X_t\}_{t > 0}$  with state space  $S$  is defined by a transition probability matrix  $P = [P_{ij}]$ , and a family of holding time distributions  $F_{ij}(t)$  for each  $i, j \in S$  (Howard, 1971; Gheorghe, 1990). The same equations (2) that apply to the transition probabilities for a Markov process also apply to  $P_{ij}$ .

### 2.4 The Holding time (Howard, 1971)

The holding time (or sojourn time) in a Semi-Markov Chain (SMC) is the random time the process spends in a given state before transitioning to another state.

Unlike in a continuous-time Markov chain (CTMC), where holding times must follow an exponential distribution (due to the memoryless property), an SMP allows the holding time to follow any arbitrary probability distribution. The holding times are positive integer-valued random variables, each defined by a distribution function for probability  $h_{ij}(\cdot)$  that represents the holding time distribution for moving from state  $i$  to state  $j$  and is defined as

$$P(X_{ij} = n) = h_{ij}(n), n = 1, 2, \dots; i, j = 1, 2, \dots, N \quad (3)$$

### 2.5 The Waiting Time

In a semi-Markov chain (SMC), the waiting time is the arbitrary amount of time the process remains in a particular state prior to transition while we are unsure about the successor state. Unlike in a typical Markov chain, where waiting times are restricted to follow an exponential (continuous-time) or geometric (discrete-time) distribution, an SMC allows for any distributions of waiting times. This type of chain is also characterised by a probability density function represented by  $w_i(\cdot)$ .

### 2.6 Cumulative probability distribution (CPD) for the semi-Markov process's holding and waiting times

The CPD of the positive (continuous) integer-valued random variable (holding time)  $X_{ij}$  can be represented by the symbol  $F_{ij}(\cdot)$  so that,

$$F_{ij}(t) = P(T_{ij} \leq t) = \int_{n=0}^t h_{ij}(n)dn \quad (4)$$

and the complementary CPD of  $T_{ij}$  denoted by  $\overline{F}_{ij}(\cdot)$  is given as,

$$\overline{F}_{ij}(t) = P(T_{ij} > t) = \int_{n=t+1}^{\infty} h_{ij}(n)dn = 1 - F_{ij}(t) \quad (5)$$

If  $Y_i$  is the amount of time the process remains in state  $i$  before exiting it, then  $Y_i$  is known as the state  $i$  waiting time.

If  $w_i(m)$  be the probability that the system will spend  $m$  time units in state  $i$ , called the probability distribution function of the waiting time  $Y_i$ , then

$$w_i(n) = P(Y_i = n) = \sum_{j=1}^N P_{ij}h_{ij}(n) \quad (6)$$

The CPD for waiting times is

$$w_i(x) = P(Y_i \leq x) = \int_{n=1}^x w_i(n)dn = \sum_{j=1}^N P_{ij}F_{ij}(x) \quad (7)$$

and the complementary CPD for waiting times is

$$\overline{w}_i(x) = P(Y_i > x) = 1 - w_i(x) = \int_{n=x+1}^{\infty} w_i(n)dn = \sum_{j=1}^N P_{ij}\overline{F}_{ij}(x) \quad (8)$$

### 2.7 The Exponential Distribution (ED)

The ED is a continuous probability distribution commonly used to model the time between events. The

ED is a model for durations and often concerns the amount of time until some specific event happens. The exponential distribution has several unique properties that make it particularly useful in real-world applications. One of which is the memoryless property. This is perhaps the most distinctive characteristic of the exponential distribution. It means that the future behaviour of the system does not depend on its history. It also has a constant hazard rate, which helps the exponential distribution maintain a constant failure rate over time (Dshalalow, 2023). The exponential density function of the exponential distribution is given by

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

The rate parameter is denoted by the value of  $\lambda$ . It is the opposite of the anticipated amount of time,  $\mu$ , in a state prior to the subsequent occurrence. In other words,  $\lambda = \frac{1}{\mu}$ . Integrating Equation (9) yields the Cumulative Distribution Function (CDF) of an exponential function in the time interval  $(0, t)$  as

$$F(t; \lambda) = \begin{cases} 1 - e^{-\lambda t}, & t > 0 \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

The CDF of an exponential function can be written as the probability of period being less than some value,  $t$ , as

$$P(T \leq t) = 1 - e^{-\lambda t} \quad (11)$$

When applied to the Nigerian labour market, the rate parameter of the exponential distribution—which is the inverse of the anticipated stay or duration in a state prior to transition—has limitations. This is due to the fact that the length of time people spend in a state prior to transition varies in a typical Nigerian labour market. Therefore, we used the potential movements between the two labour market situations to adjust the rate parameter of the exponential distribution.

### 2.8 The Modified Exponential Distribution (MED)

The modified exponential distribution was presented by Kwaghkor et al. (2019b) as:

$$f(t) = \left( 1 - \exp\left(-\frac{J}{n}\right) \right) \exp\left[-\left( 1 - \exp\left(-\frac{J}{n}\right) \right) t\right] \quad (12)$$

and the cumulative distribution function is given as

$$F(t; k) = 1 - \exp\left[-\left( 1 - \exp\left(-\frac{J}{n}\right) \right) t\right] \quad (13)$$

where  $J$  is the number of persons leaving the current state at present, and  $n$  is the total number of persons in the current state at present. The modified distribution's mean, variance, and validation are found in Kwaghkor et al. (2019b). Figure 1a is a graph showing the density of the exponential distribution (ED) and the modified exponential distribution (MED). Figure 1b is a graph showing the CDF of the ED and MED.

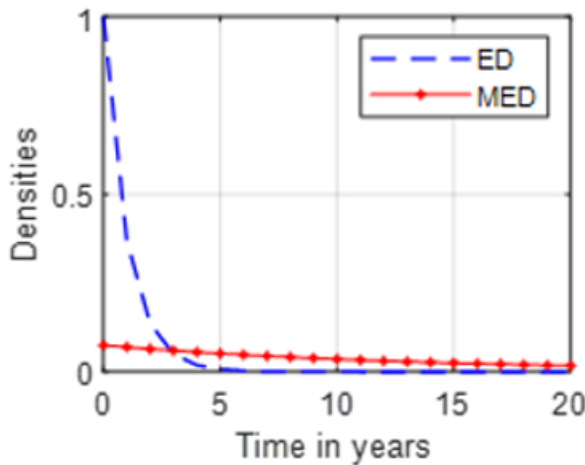


Figure 1a: ED vs MED

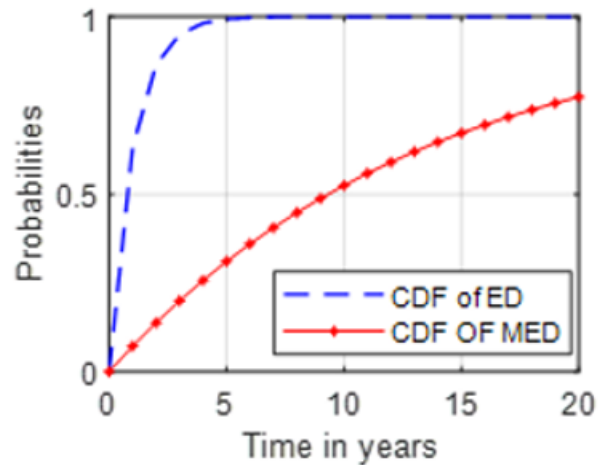


Figure 1b: CDF of ED vs CDF of MED

Figure 1a illustrates that the exponential distribution (ED) maintains a constant failure rate over a short time interval (within just a few years), whereas the modified exponential distribution (MED) maintains a constant failure rate over a much longer period. In Figure 1b, it is observed that by the fifth year, the cumulative distribution function (CDF) of the ED stabilizes, reflecting a constant probability rate, while the CDF of the MED continues to vary for over 20 years. This indicates that the time of stay in a state before transiting can be effectively measured in the short-term using the CDF of the ED and in the long-term using the CDF of the MED. These results suggest that the MED is well-suited for modelling long-term transition probabilities of processes, even when the full historical data of the process is not available.

**2.9 The Interval (0, t) transition probability**

When a process occupies state *i* at time zero, the probability that it will occupy state *j* at time *t* is known as  $\Phi_{ij}(t)$ . In the interval (0,t), this is known as the interval transition probability from state *i* to state *j*, and Howard (1971) provides it as:

$$\Phi_{ij}(t) = \delta_{ij}\bar{w}_i(t) + \sum_{k=0}^N p_{ik} \int_0^t h_{ik}(m)\Phi_{kj}(t-m)dm, h_{ij}(0) = 0 \quad (14)$$

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \quad i, j = 1, 2, 3, \dots$$

Transition Probability (14) from state *i* to state *j* in the interval(0, t) assume that the process could have made its first transition from state *i* to some state *k* at a time *m*,  $0 < m \leq t$ , and then by some succession of transitions, have made its way to state *j* at time *t*. The quantity  $\delta_{ij}$  ensures that the term in which it appears occurs only when  $i = j$  (Howard, 1971). In a practical sense,  $\Phi_{ij}(t)$  is referred to as the transition probabilities of moving from the employment to unemployment state and vice versa.

**2.10 The Nigeria’s Labour Market**

The Nigeria’s labour market, like any other country’s labour market, is an economic function with demand and supply of labour. Nigeria’s labour market population is segmented into two groups: the labour force and the non-labour force. The labour force comprises individuals aged 15 and above who are capable and willing to work, regardless of their current employment status. The non-labour force includes individuals under 15, as well as those 15 and above who are unable, unwilling, or unavailable to work. The labour force is further divided into the unemployed and employed populations. The unemployment, within this context, refers to those individuals in this age group who are actively seeking employment but are currently without a job. The employed are individuals who are currently with a job. Within the labour force, employment is further categorised as either employed or self-employed. Employees are those who receive wages or salaries, including paid apprentices. Self-employed individuals operate their own businesses, either independently or with partners, including family members (Sani, 2022; Albinowski & Lewandowski, 2022; NBS, 2024; Lisa *et al.*, 2024).

**2.11 The Transition matrix based on Nigeria’s labour market**

The transition path or matrix based on the Nigeria’s labour market is presented in Figure 2.



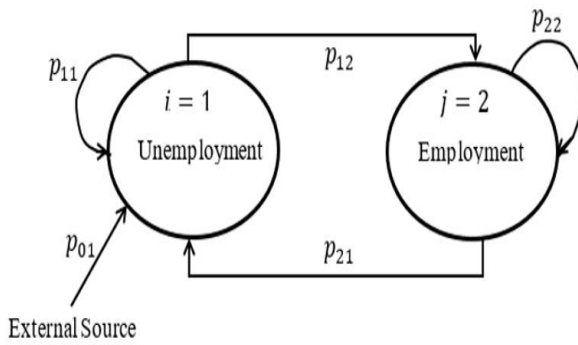


Figure 2: Flow Diagram of Nigeria's Labour Market

The possible transition matrix of the labour market model from the stage diagram in Figure 2 is given below as

$$A = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \quad (15)$$

- $p_{11}$  = the rate of remaining in state 1 after some time  $t$ ,
- $p_{12}$  = the rate of moving from state 1 to state 2 after some time  $t$ ,
- $p_{21}$  = the rate of moving from state 2 to state 1 after some time  $t$ ,
- $p_{22}$  = the rate of remaining in state 2 after some time  $t$ .

From Figure 2, there are four (4) possible transition probabilities for the future between the two labour market states: an unemployed person can be employed after some time  $t$ , an unemployed person can remain unemployed after some time  $t$ , an employed person can become unemployed after some time  $t$  and, an employed person can remain employed after some time  $t$ . Since Nigeria's labour market has only two states without branching states,  $k = j, m = t$ , and Equation (14) now becomes

$$\Phi_{ij}(t) = \delta_{ij} \bar{w}_i(t) + p_{ij} \int_0^t h_{ij}(m) dm, \quad h_{ij}(0) = 0 \quad (16)$$

### 3 THE MODEL EQUATIONS

Based on the four possible transition probabilities for the future between the two labour market states, Equation (16) can be presented as follows:

$$\Phi_{12}(t) = p_{12} \int_0^t h_{12}(m) dm \quad (17)$$

$$\Phi_{11}(t) = \bar{w}_1(t) + p_{11} \int_0^t h_{11}(m) dm \quad (18)$$

$$\Phi_{21}(t) = p_{21} \int_0^t h_{21}(m) dm \quad (19)$$

$$\Phi_{22}(t) = \bar{w}_2(t) + p_{22} \int_0^t h_{22}(m) dm \quad (20)$$

With the initial conditions  $h_{12}(0) = h_{11}(0) = h_{21}(0) = h_{22}(0) = 0$ . (21)

Equations (17)–(20) with the initial conditions (21) represent the two-state stochastic model equations that describe the movement between the two labour market states. If the rate of unemployment is also increasing from an external source, as can be seen in Figure 2 above, then the model Equation (16) can be written as

$$\Phi_{01}(t) = p_{01} \int_0^t h_{01}(m) dm, \quad h_{01}(0) = 0 \quad (22)$$

### 3.1 Existence and uniqueness of the solution of the model equations

**Theorem 1:** Given an  $n$ -dimensional system of first-order equations

$$\left. \begin{aligned} \frac{dx_1}{dt} &= f_1(t, x_1, x_2, \dots, x_n) \\ \frac{dx_2}{dt} &= f_2(t, x_1, x_2, \dots, x_n) \\ &\vdots \\ \frac{dx_n}{dt} &= f_n(t, x_1, x_2, \dots, x_n) \end{aligned} \right\} \quad (23)$$

with the initial conditions  $x_1(t_0) = x_1^0, x_2(t_0) = x_2^0, \dots, x_n(t_0) = x_n^0$ .

If  $f_i$  and  $\frac{\partial f_i}{\partial x_j}, i, j = 1, \dots, n$  are continuous in a region

$D$  of the space  $(t, x_1, x_2, \dots, x_n)$  containing the point  $(t_0, x_1^0, x_2^0, \dots, x_n^0)$ , then there is an interval  $I$  containing  $t_0$  in which there exist a unique solution  $x_1 = y_1(t), x_2 = y_2(t), \dots, x_n = y_n(t)$  of initial value problem given above.

#### Proof

The right-hand side of the model, Equations (17) – (20), can be written as

$$f_1 = p_{12} \int_0^t h_{12}(m) dm \quad (24)$$

$$f_2 = \bar{w}_1(t) + p_{11} \int_0^t h_{11}(m) dm \quad (25)$$

$$f_3 = p_{21} \int_0^t h_{21}(m) dm \quad (26)$$

$$f_4 = \bar{w}_2(t) + p_{22} \int_0^t h_{22}(m) dm \quad (27)$$

Substituting Equations (5) and (8) in Equations (24) – (27), we have

$$f_1 = p_{12} \int_0^t h_{12}(m) dm \quad (28)$$

$$f_2 = p_{11} \int_{m=t+1}^{\infty} h_{11}(m) dm + p_{11} \int_0^t h_{11}(m) dm \quad (29)$$

$$f_3 = p_{21} \int_0^t h_{21}(m) dm \quad (30)$$

$$f_4 = p_{22} \int_{m=t+1}^{\infty} h_{22}(m) dm + p_{22} \int_0^t h_{22}(m) dm \quad (31)$$

Since  $h_{ij}(m)$  (the modified exponential distributions) considered in this work is a continuous probability distribution with initial condition  $h_{ij}(0) = 0$ , the functions  $f_1, f_2, f_3$ , and  $f_4$  satisfy the continuous conditions (Kwaghkor et al., 2019a), the functions are continuous functions within the interval  $(0, t)$ .

Next,

$$\frac{\partial f_1}{\partial m} = p_{12} \frac{\partial}{\partial m} \int_0^t h_{12}(m) dm \quad (32)$$

$$\frac{\partial f_2}{\partial m} = p_{11} \frac{\partial}{\partial m} \left( \int_{m=t+1}^{\infty} h_{11}(m) dm \right) + p_{11} \frac{\partial}{\partial m} \int_0^t h_{11}(m) dm \quad (33)$$

$$\frac{\partial f_3}{\partial m} = p_{21} \frac{\partial}{\partial m} \int_0^t h_{21}(m) dm \quad (34)$$

$$\Phi_{12}(m) = p_{12} \int_0^t \left( 1 - \exp\left(-\frac{J}{n}\right) \right) \exp\left[-\left(1 - \exp\left(-\frac{J}{n}\right)\right)m\right] dm \quad (36)$$

$$\Phi_{11}(m) = \bar{w}_1(m) + p_{11} \int_0^t \left( 1 - \exp\left(-\frac{J}{n}\right) \right) \exp\left[-\left(1 - \exp\left(-\frac{J}{n}\right)\right)m\right] dm \quad (37)$$

$$\Phi_{21}(m) = p_{21} \int_0^t \left( 1 - \exp\left(-\frac{J}{n}\right) \right) \exp\left[-\left(1 - \exp\left(-\frac{J}{n}\right)\right)m\right] dm \quad (38)$$

$$\Phi_{22}(m) = \bar{w}_2(m) + p_{22} \int_0^t \left( 1 - \exp\left(-\frac{J}{n}\right) \right) \exp\left[-\left(1 - \exp\left(-\frac{J}{n}\right)\right)m\right] dm \quad (39)$$

Also, Equation (21) becomes

$$\Phi_{01}(m) = p_{01} \int_0^t \left( 1 - \exp\left(-\frac{J}{n}\right) \right) \exp\left[-\left(1 - \exp\left(-\frac{J}{n}\right)\right)m\right] dm \quad (40)$$

### 3.3 Analytical solution of the model equations with modified exponential distribution

The analytical solution of the model equations (36) – (40) using the integration by parts method is given as

$$\Phi_{12}(t) = p_{12} [1 - \exp(-(1 - \exp(-J/n))t)] \quad (41)$$

$$\Phi_{11}(t) = \bar{w}_1(t) + p_{11} [1 - \exp(-(1 - \exp(-J/n))t)] \quad (42)$$

$$\Phi_{21}(t) = p_{21} [1 - \exp(-(1 - \exp(-J/n))t)] \quad (43)$$

$$\Phi_{22}(t) = \bar{w}_2(t) + p_{22} [1 - \exp(-(1 - \exp(-J/n))t)] \quad (44)$$

and

$$\Phi_{01}(t) = p_{01} [1 - \exp(-(1 - \exp(-J/n))t)] \quad (45)$$

where  $\bar{w}_1(t) = p_{12} [\exp(-(1 - \exp(-J/n))t)]$  and  $\bar{w}_2(t) = p_{21} [\exp(-(1 - \exp(-J/n))t)]$ .

Equations (41) – (45) measure the transition probabilities between the labour market states if the modified exponential distribution governs the stay in any state.

## 4 RESULTS: Application of the Two-State Stochastic Model to the Nigeria's Labour Market

This section details the computation of transition probabilities between the two labour market states using MATLAB.

### 4.1 Comparing Transition Probabilities of the Labour States via Exponential and Modified Exponential Distributions

The duration of an individual's stay in a given state before transition from one state to the other is modelled using both exponential distribution (ED) and modified exponential distribution (MED). The resulting transition probabilities, illustrating movement between or staying within states over time (in years), are graphically presented below, using the data from Table 2.

$$\frac{\partial f_4}{\partial m} = p_{22} \frac{\partial}{\partial m} \left( \int_{m=t+1}^{\infty} h_{22}(m) dm \right) + p_{22} \frac{\partial}{\partial m} \int_0^t h_{22}(m) dm \quad (35)$$

Equations (32) – (35) also satisfy the conditions of Theorem (1) and are continuous within the interval  $(0, t)$ .

Hence, there exists a solution of the model Equations (17) – (20) with the initial condition (21), and the solution is unique.

### 3.2 The two-state stochastic model equations integrating the modified exponential distribution

Based on the modified exponential distribution (12), the two-state stochastic model Equations of the transition between the labour market states will be rewritten as follows;

**Table 2: Parameter Values for Figure 3a and 3b**

Figure 3a			Figure 3b		
Parameter	Value	Source	Parameter	Value	Source
$p_{12}$	0.957	ADSR, 2024	$p_{21}$	0.952	NBS 2023
$p_{11}$	0.043	ADSR, 2024	$p_{22}$	0.048	NBS 2023
$J$	187226	NBS, 2020	$J$	4070000	Amaefule et al. 2017, NBS 2023
$n$	4792295	NBS, 2023	$n$	84148567	NBS 2023
$\mu$	3	Assumed	$\mu$	5	Assumed

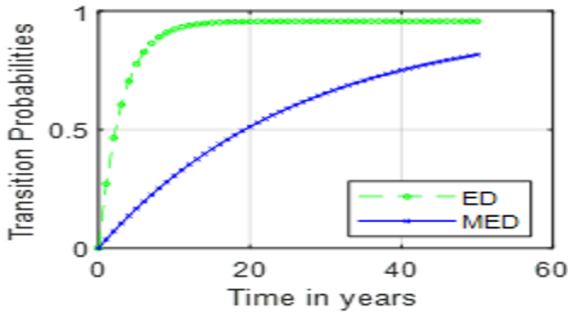


Figure 3a: Transition Probabilities of  $\Phi_{12}(t)$

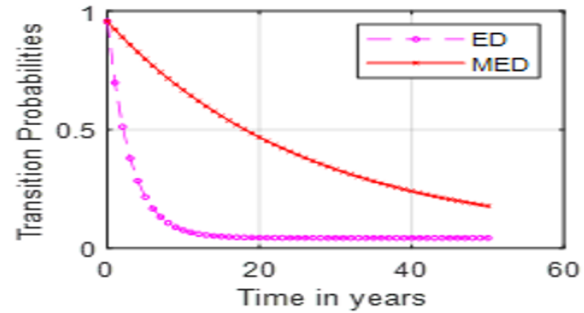


Figure 3b: Transition Probabilities of  $\Phi_{11}(t)$

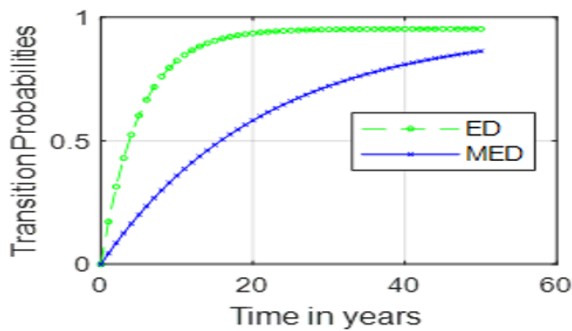


Figure 3c: Transition Probabilities of  $\Phi_{21}(t)$

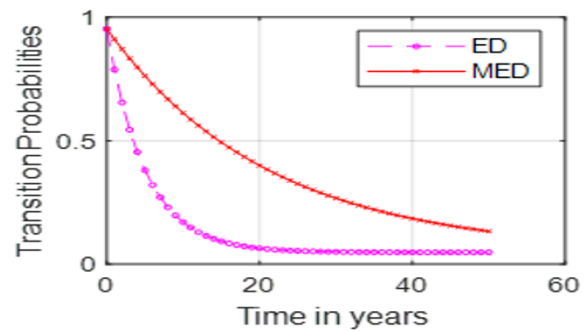


Figure 3d: Transition Probabilities of  $\Phi_{22}(t)$

Figures 3a to 3d present four subplots illustrating the transition probabilities between two labour market states over time (in years). These probabilities are defined for transitions from one state to another and for remaining in the same state, labelled as  $\Phi_{12}$ ,  $\Phi_{11}$ ,  $\Phi_{21}$  and  $\Phi_{22}$ , respectively. Each subplot compares the results derived from the Exponential Distribution (ED) and the Modified Exponential Distribution (MED). In Figure 3a ( $\Phi_{12}$ ), which represents the transition from state 1 to state 2, both ED and MED show increasing transition probabilities over time. However, the ED curve rises sharply, approaching a probability of 1 within 15 years, indicating a rapid transition. In contrast, the MED curve increases more gradually, reaching about 0.8 over 60 years, suggesting a slower, long-term transition process.

Figure 3b ( $\Phi_{11}$ ) depicts the probability of remaining in state 1 over time (self-transition). Here, both methods show a decreasing trend. The ED curve drops steeply and quickly stabilizes near zero, indicating a rapid exit from state 1. The MED curve, however, declines more slowly and stabilizes around a probability of 0.2 over a longer period, indicating a more prolonged stay.

Figures 3c ( $\Phi_{21}$ ) and 3d ( $\Phi_{22}$ ) mirror the interpretations of Figures 3a and 3b, but for transitions involving state 2. In Figure 3c, representing the transition from state 2 to state 1, ED again shows a fast rise to near certainty, while MED reflects a more gradual increase. Figure 3d shows the probability of remaining in state 2, where ED rapidly declines to near zero, and MED decreases more slowly, stabilizing at a higher value. Overall, Figures 3a to 3d demonstrate that the Exponential Distribution (ED) is only suited for modelling short-term transitions, where state changes occur quickly. In contrast, the Modified Exponential Distribution (MED) is more appropriate for long-term analyses, capturing more gradual transitions and extended durations within states. This distinction is particularly useful in studying labour market dynamics, such as transitions between employment and unemployment, where timing and persistence play crucial roles.

#### 4.2 Nigeria's Unemployment and Employment Probabilities across Geopolitical Zones

Nigeria's unemployment and employment transition probabilities are computed across the six geopolitical

zones, with the duration of stay or transition in each labour state governed by the Modified Exponential Distribution (MED). Based on the data presented in

Table 3, the results presented in Figure 4 and Figure 5 are obtained.

**Table 3: Parameter Values for Figure 4 and Figure 5**

Parameter	Working population ( $n$ )	age	Unemployed ( $J_U$ )	Unemployment Probability ( $p_U$ )	Employed ( $J_E$ )	Employment Probability ( $p_E$ )
NorthEast	14815258		755578	0.040	1405968	0.040
NorthWest	28267065		847449	0.042	2707984	0.042
NorthCentral	16882155		725933	0.043	1615622	0.043
SouthSouth	17643804		909516	0.072	1637345	0.072
SouthEast	13546221		878325	0.090	1232706	0.090
SouthWest	25450357		967352	0.051	2707984	0.042
Source	NBS, 2023		NBS, 2023	Assumed	NBS, 2023	Assumed

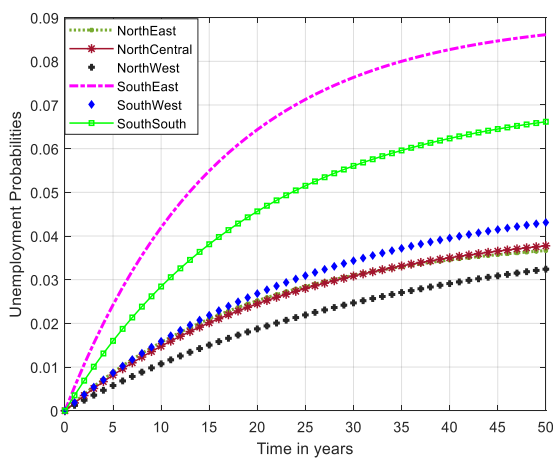


Figure 4: Nigeria's Unemployment Probabilities by Zone

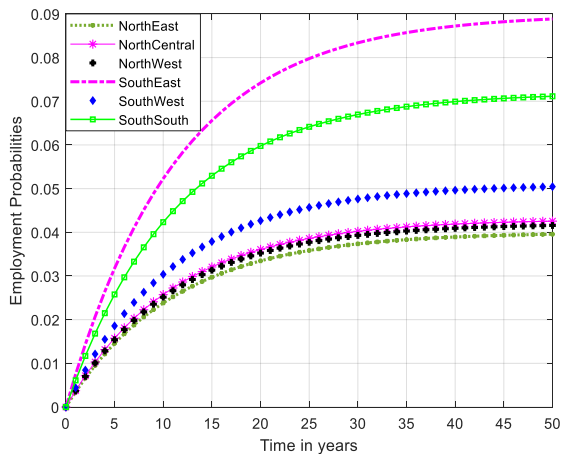


Figure 5: Nigeria's Employment Probabilities by Zone

Figure 4 (titled: Nigeria's Unemployment Probabilities by Zone) presents a graph showing how unemployment probabilities evolve over time across Nigeria's six geopolitical zones. The graph indicates the South-east

(Magenta Dashed Line) is having the Highest unemployment probability over time: it starts low but grows rapidly and consistently by year 50, and the steep initial growth rate suggests early vulnerability to unemployment. Also, the South-south (Green Line with Circles) which is the second highest unemployment trend rises sharply in the first 10 years, then increases more slowly. This is followed by the South-west (Blue Diamonds) which has moderate unemployment growth that shows a smoother increase over time, reaching just under 0.05 (5%) by year 50 indicates relatively better unemployment management compared to South-east and South-south. The North-central (Brown Asterisks) is showing lower unemployment probabilities compared to southern zones. This is followed by the North-west (Black Dots) which is growing steadily, and North-east (Green Dotted Line) which is showing the lowest unemployment probability throughout the time span. All regions show an increasing trend in unemployment probabilities over time, though at different rates. Southern zones (South-east, South-south) exhibit higher probabilities and steeper growth curves. While, Northern zones (especially North-east and North-west), have lower probabilities and flatter curves, suggesting slower growth in unemployment. The Implications could be that South-east and South-south require urgent attention and targeted employment policies to mitigate long-term unemployment risks, North-east's low trajectory suggests less exposure to unemployment pressures—this could be due to structural economic differences or security instability in the zone, and the significant regional disparities in unemployment trends, underscoring the need for zone-specific policy interventions.

Figure 5 (Nigeria's Employment Probabilities by Zone) illustrates the projected employment probabilities across Nigeria's six geopolitical zones over a 50-year period. The results here follow a similar trend to that of Figure 4 thus: The South-east (Magenta Dash-Dot Line) exhibits the highest employment probability consistently across all years starting fast and continues to grow steeply which indicates strong employment trends or optimistic projections. Followed by South-



south (Green Circles), South-west (Blue Diamonds), North-central (Purple Stars), North-west (Black Dots), and North-east (Olive Dotted Line). All zones show an upward trend in employment probability, but at different growth rates. The South-east and South-south are the leaders in employment growth, suggesting economic vibrancy or more favourable labour dynamics. North-east and North-west show slowest growth, pointing to structural employment challenges. In comparison of Figures 4 and 5, South-east, which has the highest unemployment probabilities, also shows the highest employment probabilities suggesting a bimodal distribution, where the zone experiences both high job turnover and opportunity. North-east consistently ranks lowest in both employment and unemployment suggesting stagnation rather than volatility. The implications are that though South-east and South-south are leading in employment probabilities, their corresponding high unemployment (from Figure 4) suggests a dynamic but unstable labour market possibly due to migration, informal sector transitions, or economic inequality. North-east requires urgent intervention to stimulate job creation, training, and investment. North-central and North-west show moderate trends, but would benefit from strategic economic policies to push them into higher-performing tiers. South-west reflects a balanced and sustainable employment trajectory, ideal for benchmarking. Conclusively, Figures 4 and 5 together offer valuable dual insights into Nigeria's labour economy: High employment does not guarantee low unemployment, especially in unstable or informal economies. Low unemployment may conceal deeper structural problems, like labour market exclusion. Thus, employment and unemployment probabilities must be jointly analysed for effective labour policy formulation.

### 4.3 Public and Private Sector Employment Probabilities

Here, a comparison of employment probabilities in Nigeria between the public and private sectors is carried out using the data of Table 3 and the result is presented in Figure 6.

Table 3: Parameter Values for Figure 6

Parameter	Value	Source
Public Sector ( $J_{pu}$ )	332011	NBS, 2023
Private Sector ( $J_{pr}$ )	9637736	NBS, 2023
Wage Employment Population ( $n$ )	12957747	NBS, 2023

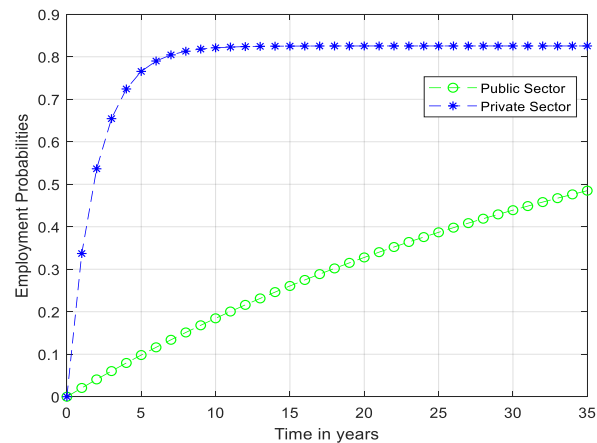


Figure 6: Public versus Private Sector Employment Probabilities

The graph of Figure 6 shows Employment Probabilities over time (in years) for both the Public Sector and Private Sector. The general trends are that the Private Sector Employment Probability increases rapidly in the first few years and then tapers off, approaching a plateau, while Public Sector Employment Probability increases slowly and linearly over time with much lower probabilities throughout the 10-year period. It can be seen from this figure that Private sector consistently has a significantly higher probability of employment at all points in time. By year ten, the private sector will have about 82% chance of employment. While the public sector will have about 18% chance of employment. This is implying that the Nigerian labour market is fostering faster absorption into the workforce through the private sector than the public sector. This is because, the public sector employment may be more competitive or limited in availability. The private sector shows a much faster growth rate in employment probability compared to the public sector. The private sector dominance in terms of quicker and higher employment probabilities suggests that individuals seeking employment may find more opportunities and faster placement in the private sector compared to the public sector. So, persons entering the Nigerian's labour market are strongly advised to consider the private sector for quicker employment opportunities, especially in the early to mid-career stages.

### 4.4 Formal and Informal Sector Employment Probabilities

Here, a comparison of employment probabilities in Nigeria between the formal and informal sector sectors is carried out using the data of Table 4 and the result is presented in Figure 6.

Table 4: Parameter Values for Figure 7

Parameter	Value	Source
Formal Sector ( $J_F$ )	6587174	NBS, 2023
Informal Sector ( $J_I$ )	77561393	NBS, 2023
Employment Population ( $n$ )	84148567	NBS, 2023

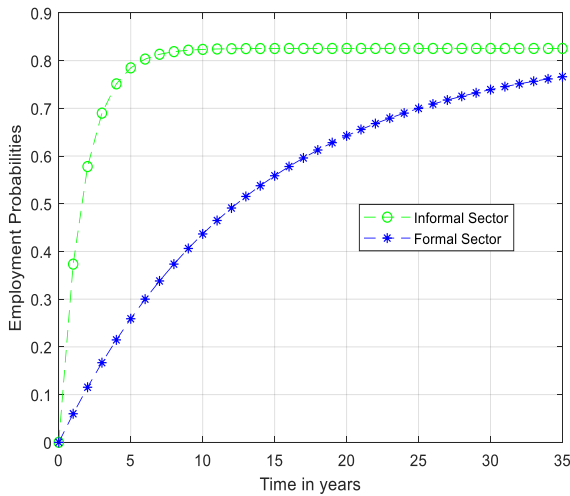


Figure 7: Formal versus Informal Sector Employment Probabilities

Figure 7 presents a comparison of Formal and Informal Sector Employment Probabilities over 10 years. There is a rapid rise in employment probability for the informal sector, reaching 0.6 within 2 years. Informal Sector’s employment probability continues to increase but at a slower pace, reaching about 0.8 by year 6. It almost plateaued, reaching a maximum employment probability around 0.83 by year 10, indicating limited capacity for further absorption. The formal sector is having a gradual increase, reaching only 0.15 by year 2. The informal sector absorbs labour much faster in the initial phase compared to the formal sector. The formal sector probability is having a linear and consistent growth, reaching around 0.3 by year 6. Formal Sector continues with linear growth, reaching 0.45 by year 10, no signs of plateauing, implying potential for continued expansion. This could be interpreted thus that over the long term, the formal sector may offer more stable and growing opportunities, even if it starts slowly. The key takeaways here are that the informal sector acts as a quick absorber of labour in the early years of job seeking, provides short-term employment options or survival jobs, and plateaus early, indicating limited long-term scalability. On the other hand, the formal sector starts slower in the beginning, possibly due to structural requirements (education, training, and regulation), offers long-term growth potential and possibly better job security, appears to be more sustainable for long-term career development. Thus, the informal sector can be leveraged for immediate employment, but economic resilience depends on strengthening the formal sector.

#### 4.5 Informal Employment Probabilities by Zones

Table 5: Parameter Values for Figure 8

Zone	Parameter	Value	Source
North-East	$J$	10476492	NBS, 2023
	$n$	10982088	NBS, 2023
North-West	$J$	20882115	NBS, 2023
	$n$	21740969	NBS, 2023
North-Central	$J$	11937125	NBS, 2023
	$n$	12811609	NBS, 2023
South-South	$J$	10549251	NBS, 2023
	$n$	11653217	NBS, 2023
South-East	$J$	8270782	NBS, 2023
	$n$	8849398	NBS, 2023
South-West	$J$	15445628	NBS, 2023
	$n$	18111286	NBS, 2023
	$p_l$	0.8255	Assumed

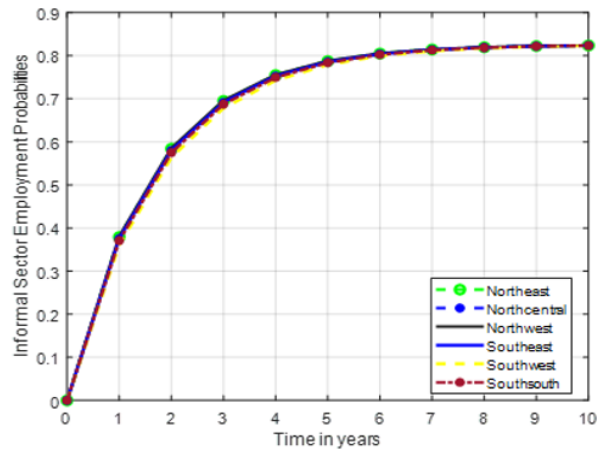


Figure 8: Informal Sector Employment Probabilities by Zones

Figure 8 is showing Informal Sector Employment Probabilities, and it provides a visual representation of how the probability of employment in the informal sector evolves over a 10-year period across the six geopolitical zones in Nigeria. The trend over time shows that all zones demonstrate a steep increase in informal sector engagement in the first 3 to 4 years. The steep curve in the early years suggests that most individuals who will enter the informal sector do so within the first few years of entering the labour market in Nigeria. After that, the probability curve flattens, indicating a saturation point or diminishing increase in engagement rates. North-east starts lower but catches up quickly. North-central and North-west show almost identical trajectories. South-east, South-west, and South-south also closely follow the general trend with minimal divergence. All zones converge around a probability of 0.82 by year 10, indicating similar long-term engagement levels across zones. The convergence suggests that despite possible socioeconomic or policy differences, the long-term probability of informal sector employment is fairly uniform across zones in Nigeria.

#### 5 SUMMARY AND CONCLUSION



The results presented in this work are the transition probabilities between employment and unemployment states in Nigeria using the proposed two-state stochastic model considering that the stay in a state is governed by two holding time distributions: Exponential Distribution (ED) and Modified Exponential Distribution (MED). Key findings include the following: transition Probabilities when a stay in a state is governed by ED predicts rapid transitions between states (e.g., unemployment to employment), saturating quickly, while the use of MED shows gradual changes over a longer period. This shows that using MED in studying transition probabilities between the two labour market state is more suitable for long-term analysis, whereas ED is better for short-term; analysis of unemployment and employment probabilities by zones (when the stay in a state is governed by MED) shows that South-South and South-East zones exhibit the highest unemployment and employment probabilities, indicating dynamic but potentially unstable labour markets. North-East has the lowest employment and unemployment probabilities, suggesting economic inactivity or labour force exclusion. South-West shows stability, likely due to a more formalized economy and better governance; analysis of public and private sector employment probabilities in Nigeria (when the stay in a state is governed by MED) indicates that the private sector dominates, offering higher and faster employment probabilities (82% by year 10) compared to the public sector (18% by year 10). Job seekers are advised to prioritize the private sector for quicker opportunities; analysis of formal and informal sector employment (when the stay in a state is governed by MED) shows that the informal sector absorbs labour quickly initially but plateaus early (0.83 by year 10), while the formal sector grows steadily (45% by year 10) with long-term potential suggesting that the informal sector is ideal for immediate employment, but the formal sector offers sustainable career growth; also, the analysis of informal sector employment by zones (when the stay in a state is governed by MED) show that all zones have a steep rise in informal sector engagement in the first few years, converging to similar probabilities (0.82 by year 10). This uniformity suggests consistent informal sector reliance across Nigeria, regardless of regional differences.

Conclusively, the analysis highlights significant disparities in employment dynamics across Nigeria's geopolitical zones and sectors. The private and informal sectors play crucial roles in absorbing labour quickly, but long-term economic resilience depends on strengthening the formal sector. Regional variations underscore the need for targeted policies, such as addressing economic inactivity in the North-East and leveraging the stability of the South-West as a model for other zones. For job seekers, prioritizing the private sector and balancing short-term informal opportunities with long-term formal sector growth is recommended.

## 6 DATA AVAILABILITY

The data used in this work are available as cited.

## 7 ACKNOWLEDGMENT

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