

# Modeling the Dynamics of Corruption in Police and Judiciary Systems Using Fractional Order Differential Equations and the Laplace-Adomian Transform Method

Seno Hannington & Abdullahi Mohammed Baba

Department of Mathematics/Statistics, School of Mathematics and Computing, Kampala International University, Kampala, Uganda

Corresponding author's email: [abdullahi.baba@kiu.ac.ug](mailto:abdullahi.baba@kiu.ac.ug)

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## ABSTRACT

This study develops a model to better understand how corruption spreads within Police and the judiciary, using fractional-order differential equations and the Laplace-Adomian transform method to capture the complex, non-linear nature of corruption. Corruption in these sectors weakens the rule of law, erodes public trust, and stifles socio-economic growth, yet many existing models oversimplify its causes. By considering the memory-dependent nature and delayed effects of interventions, this research offers a more accurate representation of how corruption evolves over time. The findings highlight the importance of early intervention, sustained accountability, and systemic reforms, showing that while short-term policy changes can help, lasting success requires deeper institutional change. External factors like international oversight or whistleblowing can also play a crucial role, though their effectiveness depends on trust in local institutions. This approach provides a more detailed framework for designing targeted anti-corruption strategies within police and judicial systems, offering valuable insights for policymakers and reformers.

**Keywords:** Modeling, dynamics, corruption, police, judiciary, fractional order differential equations and the laplace-adomian transform method

## 1.0 INTRODUCTION

Corruption within police and judicial systems remains a profound global issue, severely undermining public trust, the rule of law, and effective governance. When corruption becomes entrenched in these institutions, it fosters unsafe, unjust, and inequitable societies. Practices such as bribery and favoritism create a vicious cycle that diminishes institutional effectiveness and fuels public disillusionment (Hough & Jackson, 2020; Peiffer & Clements, 2021). Due to the complex, non-linear, and historical nature of corruption—where past behaviors influence future outcomes—traditional modeling approaches often fall short in capturing its true dynamics. To address this gap, the current study adopts fractional-order differential equations (FODEs), which are particularly adept at incorporating memory effects and delayed responses commonly observed in corruption processes (Zhao et al., 2019). These equations offer a nuanced representation of how corruption spreads and persists over time.

To solve the proposed FODEs, the study utilizes the Laplace-Adomian transform method, a powerful technique for analyzing non-linear systems. This method allows for efficient handling of the complex interdependencies inherent in corruption dynamics (Siddiqi et al., 2020). By modeling interactions between law enforcement and the judiciary, the study reveals how corrupt actors in both sectors reinforce each other—police officers influencing judicial outcomes and the judiciary shielding corrupt officers—thereby perpetuating the problem. Such feedback loops make it difficult to eradicate corruption without considering the interconnectedness of these systems. Consequently, the model not only provides a theoretical

understanding of corruption dynamics but also offers a practical framework for devising targeted, long-term interventions.

This research contributes significantly to policymaking by proposing a data-driven mathematical model that can inform more effective strategies to combat corruption, improve institutional transparency, and strengthen governance structures (Transparency International, 2023). Supporting this work, other researchers have also applied fractional-order models to study corruption. Nwajeri and Oname (2023) explored how corruption spreads across different demographic and occupational groups, including children, youth, civil servants, and government officials, stressing the role of public awareness and behavioral change. Mokaya et al. (2021) focused on preventing moral corruption in adolescents through education and counseling, highlighting the preventive power of early guidance. Similarly, Ngondiep (2024) developed a model linking corruption with poverty, demonstrating their mutual reinforcement and the resulting societal degradation. Doko et al. (2024) applied the Laplace-Adomian Decomposition Method to examine corruption dynamics in Nigeria, offering an effective approach for understanding and mitigating corruption. Together, these studies underscore the multifaceted and persistent nature of corruption and advocate for comprehensive solutions that combine structural reforms, education, and public engagement.

## 2.0 MATERIALS AND METHOD

### 2.1 Fractional Order Differential Equations

The FODEs have been widely used to model various dynamical systems. However, these models often struggle to capture the complex and memory-dependent nature of real-world



phenomena, including corruption dynamics. Fractional calculus, with its ability to describe processes with memory effects, offers a powerful alternative for modeling corruption in police and judiciary systems.

The total population  $N(t)$ , is divided in to nine classes of individuals which is the susceptible individuals,  $S(t)$ , who are not corrupt but can become corrupt due to other factors, exposed individuals  $E(t)$ , these are individuals who can interact with the corrupt individuals but eventually can be attracted to be corrupt, corrupt individuals  $C(t)$ , these are persons who can be found guilty of being corrupted, honest police  $P_h(t)$ , these interact with criminals who are corrupt and send them to the courts of law without as required of the law,

corrupt police  $P_c(t)$  in normal situations police is not supposed to be corrupt but due to interactions with corrupt individuals and unfavorable working conditions like law salary payments, inadequate training, disrespect of the law, and others leads to the introduction of this class. Honest judiciary  $J_h(t)$  who uphold the rule of law and accountability, corrupt judiciary  $J_c(t)$  who involve in bribery and abuse of public trust in the courts of law and Convicted individuals  $I(t)$  .who are found guilty of engaging in corrupt act after been proved in the courts of law.

$$N = S(t) + E(t) + C(t) + R(t) + P_h(t) + P_c(t) + J_h(t) + J_c(t) + I(t) \tag{1}$$

The fractional equations are generated as below;

**Susceptible population S(t)**

The susceptible population ( $S$ ) decreases due to interaction with honest police and honest judiciary and transition to the corrupt class ( $\beta_1 SC$ ) with natural death  $\mu$

$${}_0^C D_t^\alpha S(t) = \Lambda - \beta_1 SC(t) - (\alpha_1 + \alpha_2 + \mu)S(t) \tag{2}$$

**The exposed individuals E(t))**

Exposed individuals ( $E$ ) increase due to interaction with susceptible ( $\beta_1 SC$ ) and decrease due to transition to corrupt class with natural death

$${}_0^C D_t^\alpha E(t) = \beta_1 SC(t) - (\beta_2 + \beta_3 + \mu)E(t) \tag{3}$$

**The corrupt individuals C(t)**

Corrupt individuals ( $C$ ) increase due to transition from exposed and decrease due to arrest by honest police ( $\beta_2 E$ ) and judiciary corruption

$${}_0^C D_t^\alpha C(t) = \beta_2 E(t) - (\beta_3 + \mu)C(t) \tag{4}$$

**The Convicted individuals V(t)**

Convicted individuals  $V(t)$  increase due to arrests of corrupt individuals  $\beta_3 C$  sent to court by honest police and decrease due to recovery from corruption after being convicted and natural death

$${}_0^C D_t^\alpha V(t) = \beta_3 C - (\beta_4 + \mu)V \tag{5}$$

**The recovery class R**

The recovery class increases due to individuals who have been convicted learn a lesson and leave corrupt behavior but decrease due to natural death

$${}_0^C D_t^\alpha R(t) = \beta_4 V - \mu R \tag{6}$$

**The honest judiciary**

Honest judiciary decreases due to increase in convicted individuals  $\beta_7 VJ_h$ , corrupt individuals  $\beta_9 CJ_h$  and natural death  $\mu J_h$

$${}_0^C D_t^\alpha J_h(t) = \alpha_1 S - \beta_7 VJ_h - \beta_9 CJ_h - \mu J_h \tag{7}$$

**The Corrupt judiciary (Jc)**

Corrupt judiciary ( $Jc$ ) increases due to corruption by corrupt individuals  $\beta_9 CJ_h$  and decreases due to death  $\mu J$

$${}_0^C D_t^\alpha J_c(t) = \beta_7 V J_h + \beta_9 C J_h - \mu J_c \tag{8}$$

**The Honest police ( $P_h$ )**

Honest police ( $P_h$ ) decrease due to Interaction with corrupt police  $\beta_{10} P_h C(t)$ , convicted individuals  $\beta_{11} V P_h$  and natural death  $\mu P_h(t)$  and increase due to release by corrupt police ( $\eta P_c$ )

$${}_0^C D_t^\alpha P_h(t) = \alpha_2 S(t) - \beta_{10} P_h C(t) - \beta_{11} V P_h - \mu P_h(t) \tag{9}$$

**The Corrupt police ( $P_c$ )**

Corrupt police ( $P_c$ ) increase due to interaction with corrupt individuals  $\beta_{10} P_h C(t)$  and decrease due to release by themselves and natural death  $\mu P_c(t)$

$${}_0^C D_t^\alpha P_c(t) = \beta_{10} P_h C(t) + \beta_{11} V P_h - \mu P_c(t) \tag{10}$$

The new system of the differential equation is represented by the fractional system of differential equations is given as follows.

$$\begin{aligned} {}_0^C D_t^\alpha S(t) &= \Lambda - \beta_1 S C(t) - (\alpha_1 + \alpha_2 + \mu) S(t) \\ {}_0^C D_t^\alpha E(t) &= \beta_1 S C(t) - (\beta_2 + \beta_3 + \mu) E(t) \\ {}_0^C D_t^\alpha C(t) &= \beta_2 E(t) - (\beta_3 + \mu) C(t) \\ {}_0^C D_t^\alpha I(t) &= \beta_3 C - (\beta_4 + \mu) V \\ {}_0^C D_t^\alpha R(t) &= \beta_4 V - \mu R \\ {}_0^C D_t^\alpha J_h(t) &= \alpha_1 S - \beta_7 V J_h - \beta_9 C J_h - \mu J_h \\ {}_0^C D_t^\alpha J_c(t) &= \beta_7 V J_h + \beta_9 C J_h - \mu J_c \\ {}_0^C D_t^\alpha P_h(t) &= \alpha_2 S(t) - \beta_{10} P_h C(t) - \beta_{11} V P_h - \mu P_h(t) \\ {}_0^C D_t^\alpha P_c(t) &= \beta_{10} P_h C(t) + \beta_{11} V P_h - \mu P_c(t) \end{aligned} \tag{11}$$

**Table 1 State variables of the model**

Var	Description	Values	Ref
$S(t)$	Susceptible individuals	0.9527	Mataru & Malonza (2023)
$E(t)$	Exposed individuals	3.823	Mataru & Malonza (2023)
$C(t)$	Corrupt individuals	0.1344	Mataru & Malonza (2023)
$v(t)$	Convicted corrupt individuals	0.5097	Mataru & Malonza (2023)
$R(t)$	Recovered individuals from corruption	0.025	Mataru & Malonza (2023)
$P_h(t)$	Honest judiciary	0.0162	Mataru & Malonza (2023)
$J_h(t)$	Corrupt judiciary	0.2485	Mataru & Malonza (2023)
$P_c(t)$	Corrupt police	0.001	Mataru & Malonza (2023)
$J_c(t)$	Corrupt police	0.001	Mataru & Malonza (2023)



**Table 2** Parameters of the model

Par.	Description	Values	Reference
$\beta_{12}$	Interaction between honest judiciary	0.096.	Assumed
$\beta_{13}$	Interaction between honest police	0.039	Assumed
$\beta_1$	Transition from susceptible to exposed	0.5800	Partohaghighi & Akgül, A. (2022)
$\beta_2$	Transition from exposed to corrupt individual	0.001	Partohaghighi & Akgül, A. (2022)
$\beta_3$	Transition from corrupt to convicted individual	0.1265	Partohaghighi & Akgül, A. (2022)
$\beta_4$	Transition from convicted to recovery	0.004	Assumed
$\beta_5$	Transition from recovery back to exposed	0.44	Assumed
$\beta_6$	Interaction between corrupt and honest judiciary	0.015	Partohaghighi & Akgül, A. (2022)
$\beta_7$	Interaction between corrupt and honest police	0.5	Partohaghighi & Akgül, A. (2022)
$\beta_8$	Interaction between corrupt and corrupt judiciary	0.015	Assumed
$\beta_9$	Interaction between corrupt and corrupt police	0.35	Partohaghighi & Akgül, A. (2022)
$\beta_{10}$	Interaction between convicted and corrupt police	0.1265	Partohaghighi & Akgül, A. (2022)
$\beta_{11}$	Interaction between convicted and corrupt police	0.54	Partohaghighi & Akgül, A. (2022)
$\mu$	Death rate	0.115	Partohaghighi & Akgül, A. (2022)
$\alpha_1$	First control measure	1	Zhao et al. (2019)
$\alpha_2$	Second control measure	0.9	Zhao et al. (2019)
$\alpha_3$	Third control measure	0.8	Zhao et al. (2019)
$\alpha_4$	Fourth control measure	0.7	Zhao et al. (2019)

We based the assumed parameter values on a mix of research, expert insights, and real-world corruption data, ensuring the model reflects both theory and actual trends.

**Analysis of the model**

The model is analyzed to check if the control measures have any impact on the corruption control, that is, whether the corruption can be controlled (eliminated) or not. The model parameters which determine persistence or elimination of corruption will be determined and studied. Therefore, we start by determining the invariant region to check whether the SEIR-SEI corruption model is in biologically feasible region for both police and judiciary and showing that all solutions of equations (1) are positive at all  $t \geq 0$  and are attracted in that region that is to show that the model equations are mathematically well posed.

**Stability analysis and equilibrium**

**Disease free equilibrium (DFE)**

In this section, we assume that the control parameters are constant and determine the basic reproductive number, the steady state solutions or equilibrium points and their stabilities are well as the bifurcation behavior of the system. The corruption model (1) has a DFE, obtained by setting the right hand sides of the equations in the model (1) equal to zero, given by the following;

$$\begin{cases} {}^C D_t^\alpha S(t) = 0 \\ \vdots \\ {}^C D_t^\alpha P_c(t) = 0 \end{cases} \quad (12)$$

**Equilibrium point without corruption**

Disease free equilibrium points (DFE) are steady states solutions where there is no corruption in the susceptible population, honest judiciary, and recovered and honest police population. Let’s define the “diseased” classes as the exposed, corrupt, convicted, corrupt judiciary and corrupt police that is  $(E, C, I, J_c, P_c)$  also

$$S = 0, E = 0, C = 0, I = 0, J_c = 0, P_c = 0$$

Since there is no corruption to recover from, which implies that?

$$J_h = \frac{\alpha_1 \Lambda}{(\alpha_1 + \alpha_2 + \mu)(\beta_1 + \mu)}$$

$$P_h = \frac{\Lambda \alpha_2}{\mu^2 + \mu \alpha_1 + \mu \alpha_2 + \beta_2 (\mu + \alpha_1 + \alpha_2)} \tag{13}$$

Therefore, the disease free equilibrium point of the corruption model (1) is given by,

$$E_0 = (S, E, C, I, J_h, J_c, P_h, P_c) = \left( \frac{\Lambda}{(\alpha_1 + \alpha_2 + \mu)}, 0, 0, 0, \frac{\alpha_1 \Lambda}{(\alpha_1 + \alpha_2 + \mu)(\beta_1 + \mu)}, 0, \frac{\Lambda \alpha_2}{\mu^2 + \mu \alpha_1 + \mu \alpha_2 + \beta_2 (\mu + \alpha_1 + \alpha_2)}, 0 \right) \tag{14}$$

This represents the state in which there is no corruption in the society

$$E_0 = (S, E, C, I, R, J_h, J_c, P_h, P_c) = \left( \frac{\Lambda}{\alpha_1 + \alpha_2 + \mu}, 0, 0, 0, 0, \frac{\alpha_1 \Lambda}{(\alpha_1 + \alpha_2 + \mu)(\beta_1 + \mu)}, 0, \frac{\Lambda \alpha_2}{\mu^2 + \mu \alpha_1 + \mu \alpha_2 + \mu \beta_2 + \alpha_1 \beta_2 + \alpha_2 \beta_2}, 0 \right) \tag{15}$$

From the five Eigen values, the dominant Eigen value of the matrix is  $\lambda = \frac{\beta_1 \Lambda \beta_2}{(\alpha_1 + \alpha_2 + \mu)(\beta + \mu)(\beta_3 + \mu)}$  (15)

Therefore the basic reproductive number of the model equations denoted by  $R_0$ , is given by

$$\mathfrak{R}_0 = \frac{\beta_1 \Lambda \beta_2}{(\alpha_1 + \alpha_2 + \mu)(\beta + \mu)(\beta_3 + \mu)} \tag{16}$$

The basic reproductive number can be used to determine the local stability of the disease free equilibrium point.

**Local stability of disease free equilibrium**

The local stability of the disease equilibrium can be analyzed using the Jacobian matrix of the corruption model at the disease free equilibrium point. The DFE of model (1) is locally asymptotic stable (LAS) if  $\mathfrak{R}_0 < 1$  and unstable if  $\mathfrak{R}_0 > 1$  (Sule & Abdullahi, 2022)

**Existence of an endemic equilibrium point (EEP)**

Next conditions for the existence of endemic equilibrium for the model (1) are explored see (Abdullahi and Sule (2024).

Let  $\varepsilon = (S, E, C, I, R, J_h, J_c, P_h, P_c)$  be the arbitrary endemic equilibrium of model (1) in which at least one of the corrupted components of the model is non-zero. Setting the right hand side of the equations in (1) to zero gives the expressions for locally asymptotically stable if  $\mathfrak{R}_0 < 1$  and unstable if  $\mathfrak{R}_0 > 1$

$$\begin{aligned}
 S &= \frac{\beta\mu + \beta\beta_3 + \mu^2 + \mu\beta_3}{\beta_1\beta_2} \\
 E &= \frac{-\mu^3 + (-\beta - \beta_3 - \alpha_1 - \alpha_2)\mu^2 + ((-\beta_3 - \alpha_1 - \alpha_2)\beta - \beta_3(\alpha_1 + \alpha_2))\mu - \beta_3(\alpha_1 + \alpha_2)\beta + \Lambda\beta_1\beta_2}{\beta_3\beta_1(\beta + \mu)} \\
 C &= \frac{-\mu^3 + (-\beta - \beta_3 - \alpha_1 - \alpha_2)\mu^2 + ((-\beta_3 - \alpha_1 - \alpha_2)\beta - \beta_3(\alpha_1 + \alpha_2))\mu - \beta_3(\alpha_1 + \alpha_2)\beta + \Lambda\beta_1\beta_2}{(\beta_3 + \mu)(\beta + \mu)\beta_1} \\
 I &= \frac{\beta_3(\mu^3 + (\beta + \beta_3 + \alpha_1 + \alpha_2)\mu^2 + ((\beta_3 + \alpha_1 + \alpha_2)\beta_3 + \beta(\alpha_1 + \alpha_2))\mu + \beta_3(\alpha_1 + \alpha_2)\beta - \Lambda\beta_1\beta_2}{\beta_1(\beta_4 + \mu)(\beta_3 + \mu)(\beta + \mu)} \\
 R &= \frac{\beta_3(\mu^3 + (\beta + \beta_3 + \alpha_1 + \alpha_2)\mu^2 + ((\beta_3 + \alpha_1 + \alpha_2)\beta_3 + \beta(\alpha_1 + \alpha_2))\mu + \beta_3(\alpha_1 + \alpha_2)\beta - \Lambda\beta_1\beta_2}{\beta_1(\beta_4 + \mu)(\beta_3 + \mu)(\beta + \mu)\mu} \\
 J_h &= \frac{\alpha_1(\beta_3 + \mu)(\beta + \mu)}{\beta_1\beta_2(\beta_1 + \mu)} \\
 &\quad (\mu^3 + (\beta + \beta_3 + \alpha_1 + \alpha_2)\mu^2 + ((\beta + \alpha_1 + \alpha_2)\beta_3 + \beta(\alpha_1 + \alpha_2))\mu + \beta_3(\alpha_1 + \alpha_2)\beta) \\
 J_c &= \frac{-\Lambda\beta_1\beta_2(\mu\beta_9 + \beta_3\beta_7 + \beta_4\beta_9)\alpha_1}{\mu(\beta_1 + \mu)\beta_2(\beta_4 + \mu)\beta_1^2} \\
 P_h &= \frac{\alpha_2(\beta_3 + \mu)(\beta + \mu)}{\beta_1\beta_2(\beta_2 + \mu)} \\
 P_c &= \frac{\alpha_2\beta_3(\mu^3 + (\beta + \beta_3 + \alpha_1 + \alpha_2)\mu^2 + ((\beta + \alpha_1 + \alpha_2)\beta_3 + \beta(\alpha_1 + \alpha_2))\mu + \beta_3(\alpha_1 + \alpha_2)\beta - \Lambda\beta_1\beta_2}{\mu(\beta_2 + \mu)(\beta_4 + \mu)\beta_1^2}
 \end{aligned} \tag{17}$$

Thus,  $\Omega$  is positively invariant moreover, if and  $N(t) \leq \frac{\Lambda}{\mu}$  either the solution enter  $\Omega$  in finite time, or  $N(t)$  approaches

$\frac{\Lambda}{\mu}$ , and the exposed and the corrupt variables  $E, C, P_h, P_c, J_h, J_c$  and  $C_c$  approached zero. Hence  $\Omega$  is attracting (that is

all solution in  $\mathfrak{R}_+^8$  eventually enter  $\Omega$ ).

Thurs in  $\Omega$ , the model has been thoroughly validated both epidemiologically and mathematically, making it sufficient to focus on studying its dynamics. See (Abdullahi & Sule 2023)

The closed set

$$\Omega = \left\{ (S, E, C, P_h, P_c, J_p, J_c, I) \in \mathfrak{R}^8 : S + E + C + P_h + P_c + J_h + J_c + I \leq \frac{\Lambda}{\mu} \right\}$$

### Analysis of the Fractional Order Model of Corruption

#### The Laplace–Adomian Decomposition Method

This section focuses on the model 11 overall operation under specified initial conditions. When both sides of the model are transformed using the Caputo fractional derivative system (11), the following results are obtained:

$$\left. \begin{aligned} L\left\{C D_t^\alpha S(t)\right\} &= \Lambda - \beta_1 SC(t) - (\alpha_1 + \alpha_2 + \mu)S(t) \\ \vdots \\ L\left\{C D_t^\alpha P_c(t)\right\} &= \beta_{10} P_h C(t) + \beta_{11} VP_h - \mu P_c(t) \end{aligned} \right\} \quad (18)$$

Thus indicates

$$\left. \begin{aligned} S^\alpha L\left\{C D_t^\alpha S(t)\right\} - S^{\alpha-1} S(0) &= L\left\{\Lambda - \beta_1 SC(t) - (\alpha_1 + \alpha_2 + \mu)S(t)\right\} \\ \vdots \\ S^\alpha L\left\{C D_t^\alpha P_c(t)\right\} - S^{\alpha-1} P_c &= \beta_{10} P_h C(t) + \beta_{11} VP_h - \mu P_c(t) \end{aligned} \right\} \quad (19)$$

Using the initial conditions and taking inverse Laplace transform to system (28), we have

$$\left. \begin{aligned} S(t) &= S_0 + L^{-1}\left\{\Lambda - \beta_1 SC(t) - (\alpha_1 + \alpha_2 + \mu)S(t)\right\} \\ \vdots \\ P_c(t) &= P_{c0} + L^{-1}\left\{\beta_{10} P_h C(t) + \beta_{11} VP_h - \mu P_c(t)\right\} \end{aligned} \right\} \quad (20)$$

Using the values of initial condition in (29), we get

$$\left. \begin{aligned} S(t) &= n_1 + L^{-1}\left\{\Lambda - \beta_1 SC(t) - (\alpha_1 + \alpha_2 + \mu)S(t)\right\} \\ \vdots \\ P_c(t) &= n_9 + L^{-1}\left\{\beta_{10} P_h C(t) + \beta_{11} VP_h - \mu P_c(t)\right\} \end{aligned} \right\} \quad (21)$$

Assume that the solutions,  $S(t), E(t), C(t), V(t), R(t), J_h(t), J_c(t), P_h(t), P_c(t)$  in the form of infinite series are given by

$$\begin{aligned} S(t) &= \sum_{n=0}^{\infty} S_n(t), \quad E(t) = \sum_{n=0}^{\infty} E_n(t), \quad C(t) = \sum_{n=0}^{\infty} C_n(t), \quad V(t) = \sum_{n=0}^{\infty} V_n(t) \\ R(t) &= \sum_{n=0}^{\infty} R_n(t), \quad J_h(t) = \sum_{n=0}^{\infty} J_h(t), \quad J_c(t) = \sum_{n=0}^{\infty} J_{cn}(t), \quad P_h(t) = \sum_{n=0}^{\infty} P_{hn}(t) \\ P_c(t) &= \sum_{n=0}^{\infty} P_{cn}(t) \end{aligned} \quad (22)$$

While the nonlinear term involved in the model are  $S(t)C(t), V(t)J_h(t), P_h(t)C(t), V(t)P_h(t)$  and are decomposed as follows where  $X_n, Y_n, W_n$  and  $Z_n$  are the Adomian polynomials defined as are

$$\left\{ \begin{aligned} X_n &= \frac{1}{\Gamma(n+1)} \frac{d^n}{dt^n} \left[ \sum_{k=0}^{\infty} \lambda^k S_k \sum_{k=0}^{\infty} \lambda^k C_k \right] \Big|_{\lambda=0} \\ Y_n &= \frac{1}{\Gamma(n+1)} \frac{d^n}{dt^n} \left[ \sum_{k=0}^{\infty} \lambda^k V_k \sum_{k=0}^{\infty} \lambda^k J_{hk} \right] \Big|_{\lambda=0} \\ Z_n &= \frac{1}{\Gamma(n+1)} \frac{d^n}{dt^n} \left[ \sum_{k=0}^{\infty} \lambda^k P_{hk} \sum_{k=0}^{\infty} \lambda^k C_k \right] \Big|_{\lambda=0} \\ W_n &= \frac{1}{\Gamma(n+1)} \frac{d^n}{dt^n} \left[ \sum_{k=0}^{\infty} \lambda^k V_k \sum_{k=0}^{\infty} \lambda^k P_{hk} \right] \Big|_{\lambda=0} \end{aligned} \right. \quad (23)$$

The first three polynomials are given by

$$\left\{ \begin{aligned} X_0 &= S_0(t)C_0(t), \\ X_1 &= S_0(t)C_1(t) + S_1(t)C_0(t) \\ X_2 &= 2S_0(t)C_2(t) + 2S_1(t)C_1(t) + 2S_2(t)C_0(t) \\ Y_0 &= V_0(t)J_{h0}(t), \\ Y_1 &= V_0(t)J_{h1}(t) + V_1(t)J_{h0}(t) \\ Y_2 &= 2V_0(t)J_{h2}(t) + 2V_1(t)J_{h1}(t) + 2V_2(t)J_{h0}(t) \\ Z_0 &= P_{h0}(t)C_0(t), \\ Z_1 &= P_{h0}(t)C_1(t) + P_{h1}(t)C_0(t) \\ Z_2 &= 2P_{h0}(t)C_2(t) + 2P_{h1}(t)C_1(t) + 2P_{h2}(t)C_0(t) \\ W_0 &= V_0(t)P_{h0}(t), \\ W_1 &= V_0(t)P_{h1}(t) + V_1(t)P_{h0}(t) \\ W_2 &= 2V_0(t)P_{h2}(t) + 2V_1(t)P_{h1}(t) + 2V_2(t)P_{h0}(t) \end{aligned} \right. \quad (24)$$

Using (27), (29) in model (11), yields

$$\left. \begin{aligned} &\left\{ \sum_{n=0}^{\infty} S_k(t) \right\} = \frac{S_0}{s} + \left[ \frac{1}{S^\alpha} L \{ \Lambda - \beta_1 SC(t) - (\alpha_1 + \alpha_2 + \mu)S(t) \} \right] \\ L: & \\ &L \left\{ \sum_{n=0}^{\infty} P_{ck}(t) \right\} = \frac{P_{c0}}{s} + \left[ \frac{1}{S^\alpha} L \{ \beta_{10} P_h C(t) + \beta_{11} VP_h - \mu P_c(t) \} \right] \end{aligned} \right\} \quad (25)$$

Now, comparing like terms on both sides, yields

$$\left. \begin{aligned}
 &L[S_0(t)] = \frac{n_1}{s}, L[E_0(t)] = \frac{n_2}{s}, L[C_0(t)] = \frac{n_3}{s}, L[V(t)] = \frac{n_4}{s}, \\
 &L[R(t)] = \frac{n_5}{s}, L[J_{h0}(t)] = \frac{n_6}{s}, L[J_{c0}(t)] = \frac{n_7}{s}, L[P_{h0}(t)] = \frac{n_8}{s}, \\
 &L[P_{c0}(t)] = \frac{n_9}{s}, \\
 &L(S_1) = (\Lambda - \beta_1 SC(t) - (\alpha_1 + \alpha_2 + \mu)S(t)) \frac{1}{s^{\alpha+1}} \\
 &\vdots \\
 &L\{P_{c1}\} = (\beta_{10}P_h C(t) + \beta_{11}VP_h - \mu P_c(t)) \\
 &\vdots \\
 &L(S_{n+1}) = (\Lambda - \beta_1 S_{n+1} C_{n+1}(t) - (\alpha_1 + \alpha_2 + \mu)S_{n+1}(t)) \frac{1}{s^{\alpha+1}} \\
 &\vdots \\
 &L\{P_{cn+1}\} = (\beta_{10}P_{hn+1} C_{n+1}(t) + \beta_{11}V_{n+1}P_{hn+1} - \mu P_{cn+1}(t))
 \end{aligned} \right\} \tag{26}$$

Taking Laplace inverse of (34) and considering first two terms at different values of  $\alpha = 0.9, 0.8, 0.7$  and  $0.6$  : and using the following values in table 2

**Numerical Simulation of the LADM**

The simulation of the basic model was carried out to explore the dynamics of corruption within the police and judiciary systems of the population, assuming no interventions to reduce corruption. The plots below illustrate the population dynamics of each compartment for various values of the model parameter  $\alpha_i (i = 1, 2, 3, 4)$ .

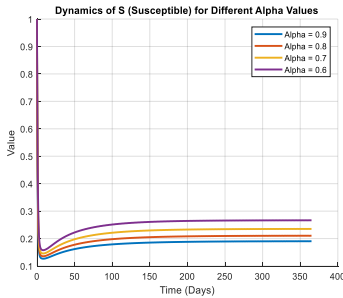


Figure 2: The plot shows the population of susceptible at  $\alpha_i (i = 1, 2, 3, 4)$ .

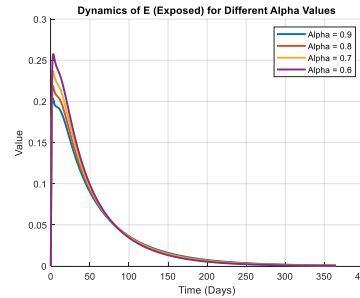


Figure 3: The plot shows the population of the exposed at  $\alpha_i (i = 1, 2, 3, 4)$

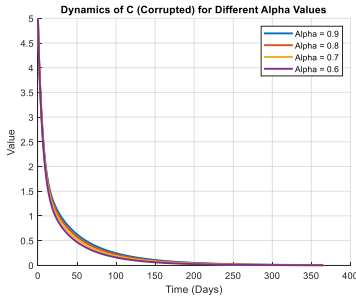


Figure 4: The plot shows the population of corrupt at  $\alpha_i (i = 1,2,3,4)$ .

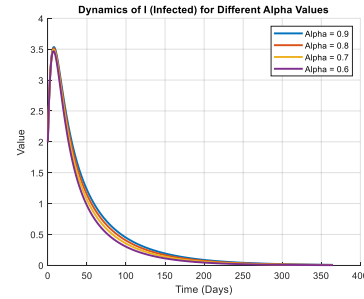


Figure 5: The plot shows the population of convicted at  $\alpha_i (i = 1,2,3,4)$ .

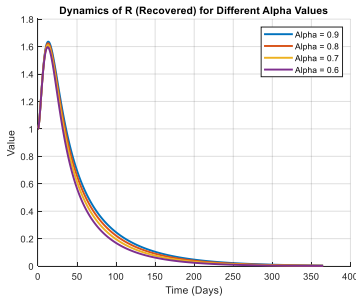


Figure 6: The plot shows the population of recovered at  $\alpha_i (i = 1,2,3,4)$ .

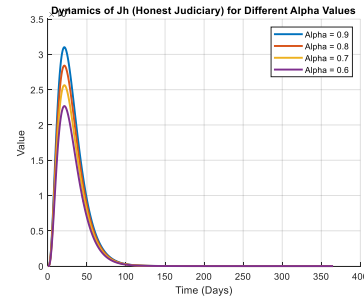


Figure 7: The plot shows the population of Honest Judiciary at  $\alpha_i (i = 1,2,3,4)$ .

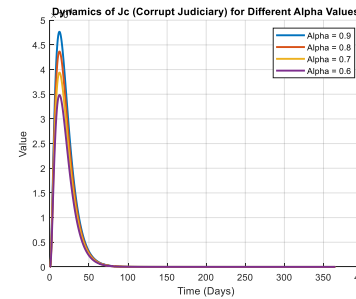


Figure 8: The plot shows the population of Corrupt Judiciary at  $\alpha_i (i = 1,2,3,4)$ .

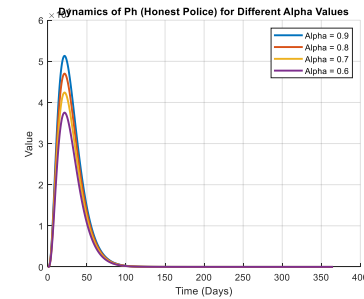


Figure 9: The plot shows the population of Honest Police at  $\alpha_i (i = 1,2,3,4)$ .

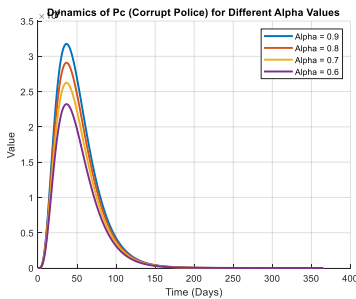


Figure 10: The plot shows the population of Corrupt Police at  $\alpha_i (i = 1,2,3,4)$ .

Based on the numerical simulations from the Law Enforcement and Judiciary Corruption Model (LADM), several strategic interventions can be employed to tackle corruption. Key actions include promoting transparency through audits, independent

oversight, and clear codes of conduct for officials. Using digital tools, anonymous reporting, and AI-driven detection can improve corruption identification, while stronger legal penalties can deter wrongdoing. Ethics training, mentorship

programs, and better whistleblower protections encourage reporting of corrupt activities. Merit-based recruitment, background checks, and digitizing case management help prevent corruption and reduce bribery opportunities. Community engagement through awareness campaigns and citizen-led policing can foster trust between law enforcement and the public. Lastly, safeguarding judicial independence and rotating officials can prevent entrenched corruption. These strategies offer a clear path to a fairer, more accountable system.

#### Conclusion

This study presents a fresh approach to understanding corruption within law enforcement and the judiciary systems by using fractional-order differential equations and the Laplace-Adomian decomposition method. Unlike traditional models, which often oversimplify the complexities of corruption, this model accounts for how corruption builds up over time, influenced by past actions and the delayed effects of interventions. The simulations conducted in the study reveal how corruption spreads and persists across various sectors, illustrating different stages such as susceptible individuals, corrupt officers, and those who have either been convicted or recovered from corruption. By capturing the dynamics of both police and judicial corruption, the model provides a clearer picture of how these systems become entrenched in corrupt behavior.

What makes this model particularly valuable is its ability to show how past behaviors influence future outcomes, offering key insights for policymakers. It suggests that effective interventions must be strategic and focus on systemic change, targeting areas where corruption is most likely to spread or persist. The study emphasizes the importance of fostering transparency, accountability, and integrity within both law enforcement and the judiciary. Ultimately, this research provides a more realistic framework for modeling corruption, which can help design more informed, targeted strategies for reducing corruption in these critical areas. In doing so, it contributes to the development of a fairer, more just system that can strengthen public trust and uphold the rule of law.

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