



Mathematical Assessment on the Impact of Insecticide Treated Net on the Dynamics of Malaria Transmission Using Non Linear Mathematical Model

Alhassan Charity Jumai¹ and Ogbiti Temitope John²

Department of Mathematics, Edo State University, Iyamho
Department of Computer Science, Edo State University, Iyamho

Corresponding Author's Email: charity.alhassan@edouniversity.edu.ng ¹ogbiti.john@edouniversity.edu.ng²

Received on: January, 2025

Revised and Accepted on: February, 2025

Published on: March, 2025

ABSTRACT

Malaria remains a major threat in many part of the tropical and subtropical region of the world, even though significant effort have been implemented with the used of Insecticide-treated net, the disease remains chronic. In this work, we used nonlinear mathematical model to study the transmission dynamics of malaria disease. An insecticide- treated net control strategy was incorporated, using system of ordinary differential equations, A basic reproduction number was determine from the disease free equilibrium (DFE) and it was found that, the model is locally asymptotically stable when the associated reproduction number is less than unity. Numerical simulation was carried out and findings were displayed graphically. It was found that insecticide -treated net has a significant impact on malaria transmission.

Keywords: Insecticide- Treated Net, Associated Basic Reproduction Number, locally asymptotically Stable, ODE, Parasites

1.0 INTRODUCTION

1.1 Introduction to the problem

Malaria is a life-threatening disease endemic in many African countries. It is transmitted when an infected female anopheles mosquito bite a susceptible human [Temidayo et al 2022]. The parasite inform of a sporozoite is injected into the blood stream and that is where infection started, and can also be transmitted through blood transfusion and contaminated needles. Infections by malaria parasites may result in a wide variety of symptoms, mild symptoms are fever, chills and headache. Severe symptoms include fatigue, confusion, seizures, and difficulty breathing. In general, malaria is a curable disease if diagnosed and treated promptly and correctly (CDC, 2021). The latest world malaria report, released in 2022 shows that there were about 247 million malaria cases in 2021 compared to 245 million cases in 2020. In 2021, 617 000 people died of malaria compared to 62500 in 2020, most were young children, in sub-saharan Africa, 608,000 malaria deaths occurred in 2022, with 95% of them occurring in the African Region. (WHO, 2022).

Children are mostly affected, where the large majority of malaria deaths occur, the victims are chiefly young children, and the main vectors such as Anopheles gambiae preferentially bite humans at night, indoors and rest inside the home. In many of the highest burden malaria-endemic countries, Plasmodium falciparum transmission is strongly seasonal.(Thomas 2024).

According to 2022 malaria report, Nigeria has the highest number of global malaria cases (26.6% of global malaria cases) and the highest number of

deaths (31.3% of global malaria deaths) in 2021 (United to beat malaria, 2022). Nigeria malaria cases was at level of 313.8 cases per 100 000 people in 2020, up from 300.4 cases per 100 000 people in previous year, this is a change of 4.43% (WHO, 2022).

However the economical burden for households includes personal expenditure which includes spending on insecticide-treated net ITN, doctor's fees, antemalaria drugs, transport to health facilities etc.

The malaria burden as (Kalula. *et al*, 2021) depend on the age-structure of the human population as children under five years old bears more burden than adults (WHO, 2019) Since they have not developed immunity to infection (Abid et al 2013 and Augusto et al 2012).

Sessional Malaria Chemoprevention (SMC) is "the intermittent administration of full treatment administration of full treatment of anti-malarial medicine during the malaria season to prevent malaria illness with the objective of maintaining therapeutic drug concentrations in the blood throughout the period of greatest malarial risk".(World Health Organization(WHO, 2019; World Malaria Report, 2019). Mathematical model have been used to study the impact of Insecticide Treated net on the transmission dynamics of malaria. An insecticide control strategy was incorporated, using system of ordinary differential equations and established that the model is locally asymptotically stable when the associated reproduction number is less than unity.

2.0 METHODOLOGY

In this section, we formulated our model by first introducing the model of Segun *et al* (2020).

2.1 The existing model

The model at time *t*, denoted by $N_h(t)$, is subdivided into susceptible humans (S_h), infected humans (I_h) and recovered humans (R_h). Thus, the total human population is given by

$$N_h(t) = S_h + I_h + R_h(t) \tag{1}$$

Furthermore, the model at time *t*, denoted by $N_v(t)$, is subdivided into immature mosquitoes (A_v) susceptible mosquitoes (S_v), and infected mosquitoes (I_v). Thus, the total vector population is given by

$$N_v(t) = A_v + S_v + I_v \tag{2}$$

2.2 The variables and parameters of the existing model

Table 2.1 below shows the state variables and parameters of the existing model with their descriptions

Table 2.1: variables of the existing model with their definitions

Variable	Description
S_h	Susceptible humans
I_h	Infected humans
R_h	Recovered humans
A_v	Immature mosquitoes
S_v	Susceptible mosquitoes
I_v	Infected mosquitoes

Table 2.2.1: Parameters of the existing model with their definitions

Parameters	Description
Π_h	Recruitment rate of humans
θ	Per capitial rate of loss of immunity in humans
μ_h	Natural mortality rate of humans
σ_h	Rate of vaccinated humans
ρ_h	Rate of waning vaccine or relapse rate
w_h	Disease – induced death rate of humans
δ_h	Probability of effective transmission from human to mosquito
β_{hv}	Probability of effective transmission from mosquito to human
β_{vh}	Contact rate of mosquito - human
$\varepsilon(b)$	Maximum mosquito biting rate
$\varepsilon(max)$	Minimum mosquito biting rate
$\varepsilon(min)$	Proportion of treated net usage
b	Proportion of insecticide spray on the environment
q	Modification parameter
η	Egg deposition rate of mosquitoes
π_v	Maturation rate of immature mosquitoes
μ_v	Natural mortality rate of mosquitoes
γ_v	Mortality rate of mosquitoes due to treated net
$(\mu v(b))$	Mortality rate of mosquitoes due to insecticide spray

2.3 The equation of the existing model

$$\left. \begin{aligned} \frac{dS_h}{dt} &= \pi_h + \theta R_h - \lambda h(b)S_h - \mu_h S_h \\ \frac{dI_h}{dt} &= \lambda h(b)S_h - (\omega_h + \mu_h + \delta_h)I_h \\ \frac{dR_h}{dt} &= \omega_h I_h - (\theta + \mu_h)R_h \\ \frac{dA_v}{dt} &= \pi_v(q) - (\gamma_v + \mu_v(q))A_v \\ \frac{dS_v}{dt} &= \gamma_v A_v - \lambda v(b)S_v - (\mu_v(b) + \mu_v(q))S_v \\ \frac{dI_v}{dt} &= \lambda v(b)S_v - (\mu_v(b) + \mu_v(q))I_v \end{aligned} \right\} \tag{3}$$

where,

$$\varepsilon(b) = \varepsilon_{max} - b(\varepsilon_{max} - \varepsilon_{min}), \quad \mu_v(b) = \mu_v + \mu_{max}b, \quad \text{for } 0 \leq b \leq 1$$

$$\pi_v(q) = \pi_{vmax} - q(\pi_{vmax} - \pi_{vmin}), \quad \mu_v(q) = \mu_v + \mu_{max}q, \quad \text{for } 0 \leq q \leq 1$$

2.2.1 The flow diagram of the existing model

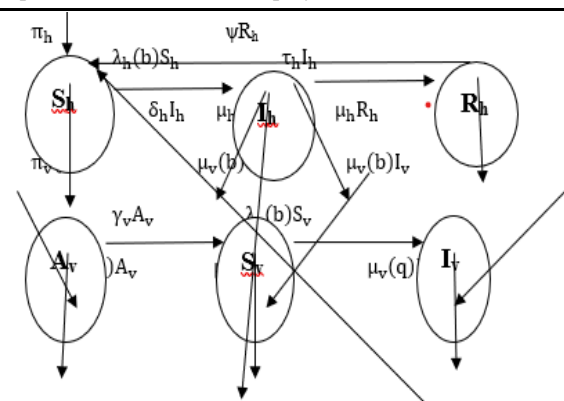


Figure 2.1: Flow Diagram of Existing Model

2.5 The Modified Model

2.5.1 The Variables and Parameters of the Modified Model

Table 2.2.2 Variables of the modified model with their description

Variables	Descriptions
-----------	--------------

S_h	Susceptible humans
E_h	Exposed humans
I_h	Infected humans
R_h	Recovered humans

S_v	Susceptible mosquitoes
E_v	Exposed mosquitoes
I_v	Infected mosquitoes

Table 2.2.3: Parameters of the modified model with their description

Parameters	Description
Λ_h	Recruitment rate for humans
Λ_m	Recruitment rate for mosquitoes
β_{hv}	Probability of effective transmission from human to mosquito
β_{vh}	Probability of effective transmission from human to mosquito
α_h	Rate of recovered humans
μ_h	Natural death rate for humans
q_h	Developing rate of exposed (humans)becoming infectious
δ_h	Induced death rate for humans
ξ_m	Insecticide rate on mosquitoes
μ_m	Natural death rate for mosquito
b	Proportion of insecticide spray on the environment
q_m	Developing rate of exposed (mosquitoes)becoming infectious

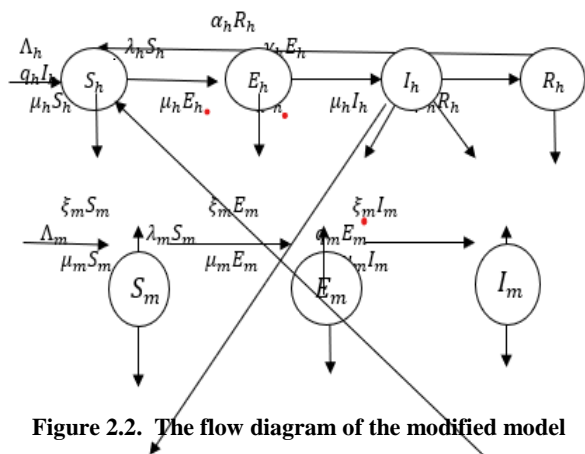


Figure 2.2. The flow diagram of the modified model

2.4 Equations of the modified model

$$\left. \begin{aligned}
 \frac{dS_h}{dt} &= \Lambda_h - \lambda_h S_h - \mu_h S_h + \alpha_h R_h \\
 \frac{dE_h}{dt} &= \lambda_h S_h - (\gamma_h + \mu_h) E_h \\
 \frac{dI_h}{dt} &= \gamma_h E_h - (q_h + \delta_h + \mu_h) I_h \\
 \frac{dR_h}{dt} &= q_h I_h - (\alpha_h + \mu_h) R_h \\
 \frac{dS_m}{dt} &= \Lambda_m - \lambda_m S_m - \mu_m S_m - \xi_m S_m \\
 \frac{dE_m}{dt} &= \lambda_m S_m - (q_m + \mu_m) E_m - \xi_m E_m \\
 \frac{dI_m}{dt} &= q_m E_m - (\xi_m + \mu_m) I_m
 \end{aligned} \right\} \quad (4)$$

where

$$\left. \begin{aligned}
 \lambda_h &= \beta_{vh} \frac{b I_m}{N_h} \\
 \lambda_m &= \beta_{hv} \frac{b I_h}{N_h}
 \end{aligned} \right\} \quad (5)$$

With the total human population defined as $N_h = S_h + E_h + I_h + R_h$ (6)

Taking the time derivative of (6) we get; $\frac{dN_h}{dt} = \frac{dS_h}{dt} + \frac{dE_h}{dt} + \frac{dI_h}{dt} + \frac{dR_h}{dt}$ (7)

Substituting the respective values of (4) into (7) and simplifying we have;

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h(S_h + E_h + I_h + R_h) - \delta_h I_h \quad (8)$$

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h N_h - \delta_h I_h \quad (9)$$

And $\frac{dN_m}{dt} = \Lambda_m - \mu_m N_m$ (10)

Invariant region:

This region will be obtained by considering the following proposition.

Proposition 1: the solution of the system (4) are feasible for all $t > 0$ if they enter the invariant region.

$$\Gamma = \{(S_h, E_h, I_h, R_h)\} \in \mathcal{R}^4; S_h \geq 0, E_h \geq 0, I_h \geq 0, R_h \geq 0, N_h \leq \frac{\Lambda_h}{\mu_h}$$

Proof: let $\epsilon_0 = (S_h, E_h, I_h, R_h)$ be any solution of the system (4). From equation (8) we can observe that, in the absence of malaria induced death rate $\delta_h = 0$ we have;

$$\frac{dN_h}{dt} \leq \Lambda_h - \mu_h N_h \quad (11)$$

Rearranging (9) we have; $\frac{dN_h}{dt} + \mu_h N_h \leq \Lambda_h$

Solving (10) we have ;

$$\begin{aligned}
 IF &= e^{\int \mu_h dt} = e^{\mu_h t} \\
 \Rightarrow e^{\mu_h t} \frac{dN_h}{dt} + e^{\mu_h t} \mu_h N_h &\leq \Lambda_h e^{\mu_h t} \\
 \Rightarrow \frac{d}{dt} (e^{\mu_h t} N_h) &\leq \Lambda_h e^{\mu_h t} \quad (12)
 \end{aligned}$$

Integrating eqn (12) with respect to dt we have

$$\begin{aligned}
 \int \frac{d}{dt} (e^{\mu_h t} N_h) &\leq \int \Lambda_h e^{\mu_h t} dt \\
 e^{\mu_h t} N_h(t) &\leq \frac{\Lambda_h}{\mu_h} e^{\mu_h t} + C \\
 \Rightarrow N_h(t) &\leq \frac{\Lambda_h}{\mu_h} + C e^{-\mu_h t} \quad (13)
 \end{aligned}$$

Applying the initial condition $N(0) = N_0$ in (13) gives;

$$N_0 - \frac{\Lambda_h}{\mu_h} \leq C \rightarrow C = N_0 - \frac{\Lambda_h}{\mu_h} \quad (14)$$

Substitute (13) into (12) so that;

$$N_h(t) \leq \frac{\Lambda_h}{\mu_h} + (N_0 - \frac{\Lambda_h}{\mu_h}) e^{-\mu_h t} \quad (15)$$

Solving (10) in the similar manner we have

$$N_m(t) = \frac{\Lambda_m}{\mu_m} + (N_0 - \frac{\Lambda_m}{\mu_m}) e^{-\mu_m t} \quad (16)$$

Hence, Γ is a positively invariant set and solution enters Γ in finite time or $N_h(t) \rightarrow \frac{\Lambda_h}{\mu_h}$ and $N_m(t) \rightarrow$

$\frac{\Lambda_m}{\mu_m}$ as $t \rightarrow \infty$.

Thus, it is sufficient to consider the dynamics of system (4) in Γ . In this region, the system (4) is

considered as being mathematically and epidemiologically well posed.

3.0 RESULTS AND DISCUSSION

The numerical simulation and discussion are presented in this section.

3.1 Numerical Simulations

In this section, we present the numerical simulation of our models. All the parameters used in this section are displayed in Table 3.

Table 2.2.3: Parameter values of the Modified Model Equations

Parameters	Values	References
Λ_h	20	Mojeeb et al.(2017)
Λ_m	200	Mojeeb et al.(2017)
β_{hv}	0.25	Segun <i>et al</i> (2020)
β_{vh}	0.322	Segun <i>et al</i> (2020)
α_h	0.0035	Mojeeb et al.(2017)
μ_h	0.01147	Mojeeb et al.(2017)
q_h	0.2	Linsday etal.(2021)
δ_h	0.068	Linsday etal.(2021)
μ_m	0.05	Mojeeb et al.(2017)
ξ_m	1.5	Linsday etal.(2021)
q_m	0.083	Otieno et al.(2016)
b	0.53	Okuneye et al. (2017)

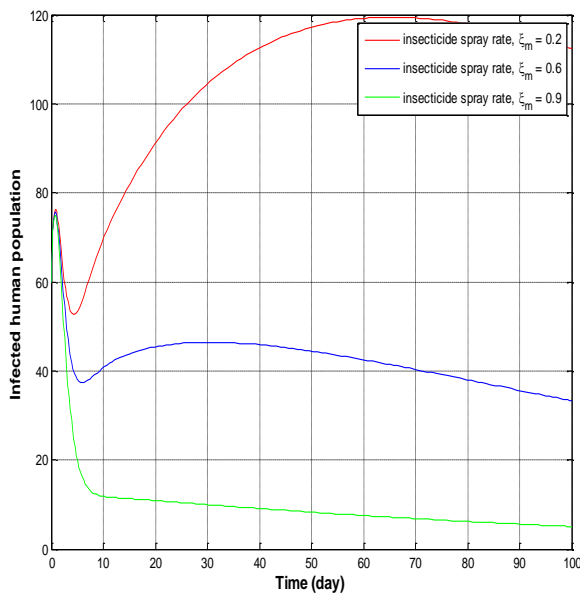


Figure 3.1: Varying rate of insecticide spray parameter

In Figure 3.1, we investigate the impact of insecticides spray on malaria transmission. It was observed that the higher the insecticides spray rate the lower the transmission of malaria and vice versa

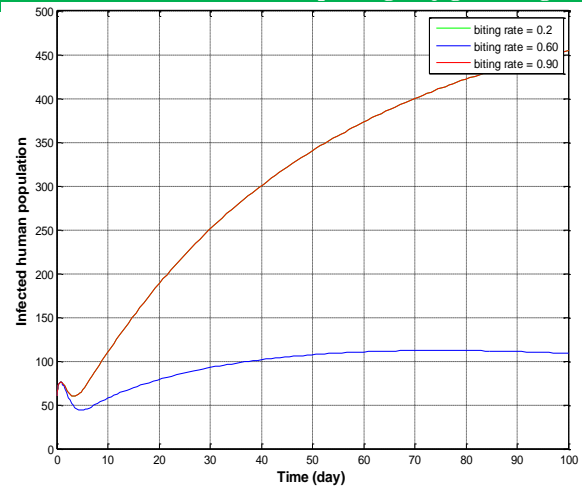


Figure 3.2: Varying rate of biting parameter

In Figure 3.2, we also investigate the impact of mosquito biting rate on malaria transmission. It was observed that the higher the mosquito bite the higher the transmission of malaria and vice versa.

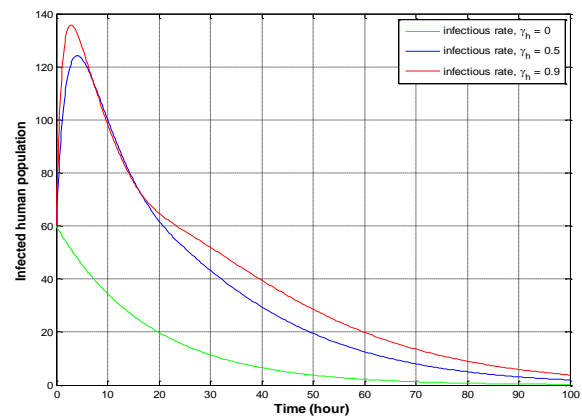


Figure 3.3: Varying rate of infectious parameter

In Figure 3.3, we simulate the infectious rate parameter on malaria transmission. It was observed that the higher the infectious rate parameter the higher the transmission of malaria and vice versa.

3.4 DISCUSSION

The equilibrium, ϵ_0 is the equilibrium state where there is no disease in the environment. The basic reproduction number was computed and it was found that when R_0 is less than unity and at this point there will be no disease in the population. The numerical analysis of the nonlinear model shows that the decrease in disease induced death rate, recovered rate, increase in insecticide net, will drastically reduce malaria disease.

4.0 CONCLUSION

In this work, we analyzed the transmission dynamics of malaria disease in a population using a nonlinear mathematical model. An insecticide control strategy



was incorporated, using system of ordinary differential equations and established that the model is locally asymptotically stable when the associated reproduction number is less than unity. Numerical simulation was carried out and pertinent findings are displayed graphically. It was found that insecticide net has a significant impact on malaria transmission reduction.

REFERENCES

- Kalula, A.S., Eunice M., Theresia M., and Isambi M. (2021). An Age-Structured Model for Transmission Dynamics of Malaria with Infected Immigrants and Asymptomatic Carriers. DOI: <https://dx.doi.org/10.4314/tjs.v47i3.7>
- Mojeeb A.R., Osman E.N., and Isaac K.A. (2017). Simple Mathematical Model for Malaria Transmission. *Journal of Advances in Mathematics and Computer Science* 25(6): 1-24.
- Okuneye, K., and Gumel, A.B. (2017) Analysis of a temperature- and rainfall-dependent model for malaria transmission dynamics, *Mathematical Bioscience*. 287: 72–92.
- Otieno, J. K., Koske, J. M., and Mutiso (2016) Transmission dynamics and optimal control of malaria in Kenya. *Discrete Dynamics and Natural Society* 20: 874.
- Segun, I. O., Micheal, M.O., Maba, B.M. (2020). Mathematical Modelling of Malaria disease with control strategy. Available on line at <http://scik.org> *Commun Math. Bio. Neurosci.* <https://doi.org/10.28919/cmbn/4513> ISSN: 2052-254.
- World Health Organization (WHO), World Malaria Report, 2022.
- Center for disease control (2021). Fact sheets.
- Lou, Y., and Zhao, X.Q. (2010). A Climate-based malaria transmission model with structured vector population *SIAMJ. Appl. Math.* 70(6)(2010), 2023-2044.
- Temidayo, O., and Emmanuel, A. (2022). Impact of Hygiene on Malaria Transmission Dynamics: A Mathematical Model. *Journal of Multidisciplinary Applied Natural Science*.
- Abid, A. L., Hyo, J., Kazeem, O. O., and Young, I. S. (2013) Mathematical Analysis of a Malaria Model with Partial Immunity to Reinfection. *Research Article | Open Access Volume 2013 | Article ID 405258 | <https://doi.org/10.1155/2013/405258>*
- Emmanuel A., Orverem J. I., Kura, Y. M., and Dahiru, U.J. (2020). Mathematical Approach for Modelling Malaria Disease in the Presence of Drug Therapy and Treatment. *9(3)*
- Agusto, F.B., Marcus, N., and Okosun, K.O. (2012). Application of optimal control to the epidemiology of malaria, *Electron. Journal of Differential Equation* 81.
- Lindsay, W, Thomas, M.B, Kleinschmidt I. (2021). Threats to the effectiveness of insecticide-treated bednets for malaria control: thinking beyond

- insecticide resistance. *Lancet Glob Health*;9(9): 1325–31
- WHO. World Malaria Report. Geneva, Switzerland. 2023.
- Thomas J. P. (2024). Insecticide-treated bednets and chemoprophylaxis reduce malaria mortality and parasite prevalence. *Trans R Soc Trop Med Hyg* 119: 187–188