



Enhanced Robust Estimators for Population Mean Accounting for Non-Response and Measurement Error

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ABSTRACT

Estimators play a crucial role in statistics, quality assurance, and survey methodology, which estimate key population characteristics like: the mean, variance, and standard deviation based on sample values. However, data collection in medical and social sciences often faces significant challenges due to non-response and measurement errors, which complicate the data compilation, computation, and estimation processes. This study proposes a robust class of estimators that address these challenges by incorporating call-back and imputation schemes and parameters accounting for measurement errors, thereby enhancing estimator efficiency despite the adverse effects of non-response and measurement errors. The properties of these proposed estimators, including bias and mean square errors (MSEs), were derived using a second-degree Taylor series approximation, and a consistency test was performed. The research also established the conditions under which the proposed estimators are more efficient than existing ones. An empirical study utilizing simulated data from distributions—normal, exponential, chi-square, uniform, gamma, and Poisson—demonstrated that the modified estimators under the imputation scheme are significantly more efficient and reliable than the existing alternatives. Consequently, these modified classes of estimators are recommended for practical application, particularly in scenarios involving non-response and measurement errors during data analysis and estimation.

Keywords: Bias, Mean Square Error, Auxiliary variables, Non-response, Measurement error

1.0 INTRODUCTION

Non-response, a frequent issue in medical and social science data collection, complicates data compilation and estimation. It occurs when respondents are absent, refuse, or are inaccessible. Hansen and Hurwitz (1946) first addressed this problem, with a common solution being callbacks, as suggested by Audu *et al.* (2021, 2022), who developed estimators for parameters like the mean and variance. However, this method increases costs, time, and logistical challenges.

Imputation schemes provide an alternative to callbacks by filling in missing data with substitutes, enabling standard analysis. Common methods include mean imputation, hot deck, nearest neighbour, worm deck, and mean-cum-nearest neighbour imputation. Researchers such as Singh (2009), and others have recommended mean imputation for addressing non-response and missing values.

Measurement errors are another common issue in data collection, as survey sampling often assumes error-free data, which is rarely the case. These errors can lead to biased results and incorrect mean square errors. Researchers like Singh *et al.*, (2021), and Audu *et al.*

(2020) have studied their effects on ratio, product, and regression estimators across various sampling schemes.

This study focuses on modifying the estimators proposed by Zaman *et al.* (2021) to account for both non-response and measurement errors affecting the study and auxiliary variables.

1.1 Some Existing Related Estimators in the Presence of Non-Response and Measurement Error.

Consider a finite population $(V = V_1, V_2, \dots, V_N)$ of size N such that Y be a study variable and X be any auxiliary variable and draw a sample of size n from a population by using simple random sampling without a replacement scheme. Suppose that N_1 units respond for the survey questions and N_2 units do not respond. Then by Hansen Hurwitz's (1946) sampling plan, a sub-sample of size $g = \frac{z_2}{k} (k > 1)$ from N_2 non-respondents is selected at random and re-contacted for their direct interview. Here, it is assumed that r units respond to the survey.

Let (x_h^*, y_h^*) be the observed values and (X_h^*, Y_h^*) be the true values of the study Y and auxiliary variable X, where $(h=1,2,\dots,n)$ units in the sample. Then measurement error is given by:

$$eu_h^* = y_h^* - Y_h^* \text{ and } ev_h^* = x_h^* - X_h^*$$

where (eu_h^*, ev_h^*) are random and both are uncorrelated

with mean zero and variance S_{eu}^2 and S_{ev}^2 are associated with measurement error in Y and X respectively for the responding part of the population. $S_{eu(2)}^2$ and $S_{ev(2)}^2$ are the variances associated with measurement error in Y and X respectively for the non-responding part of the population.

S_y^2 and S_x^2 are the population variances of Y and X respectively for the responding part of the population.

$S_{Y(2)}^2$ and $S_{X(2)}^2$ are the variances of X and Y

respectively for the non-responding part of the population.

ρ and $\rho_{(2)}$ are the population correlation coefficients between X and Y for the responding and non-responding parts of the population respectively. W_y and $W_{y(2)}$ are the coefficients of variation of Y for the responding and non-responding part of the population respectively. Similarly, W_x and $W_{x(2)}$ are the coefficients of variation of X for responding and non-responding parts of the population respectively.

Let $f_Y^* = \sum_{h=1}^n (Y_h^* - \bar{Y})$ and

$$f_X^* = \sum_{h=1}^n (X_h^* - \bar{X}), \text{ then}$$

$$f_{eu}^* = \sum_{h=1}^n eu_h^* \text{ and } f_{ev}^* = \sum_{h=1}^n ev_h^*$$

Therefore,

$$\frac{1}{n}(\beta_{ev}^* + \beta_{eu}^*) = \frac{1}{n} \sum_{h=1}^n (Y_h^* - \bar{Y}) + \frac{1}{n} \sum_{h=1}^n (y_h^* - \bar{y}_h^*)$$

Singh *et al.* (2018) considered the effect of Measurement error and Non-response on the estimation of population mean and suggested estimator defined in (2.59)

$$y_1 = \left[\frac{1}{2} \left\{ \bar{y}^* e^{\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*} \right)} + \bar{y}^* e^{\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}} \right)} \right\} + \partial_1 (\bar{X} - \bar{x}^*) + \partial_2 \bar{y}^* \right] e^{\left[\frac{\bar{X}^* - \bar{x}^{**}}{\bar{X}^* + \bar{x}^{**}} \right]} \quad (1)$$

$$\bar{X}^* = \bar{X} \rho, \quad \bar{x}^{**} = \bar{x}^* + \bar{X} (d - 1), \quad d = \frac{\rho_{xy} + 1}{4},$$

where:

$$\partial_1 = \frac{PR - QL}{JR - Q^2}$$

$$\partial_2 = \frac{JL - QR}{JP - Q^2}$$

$$\text{Bias}(y_1) = \partial_2 \bar{Y} + \left(\frac{\bar{Y}}{8\bar{X}^2} + \frac{\partial_1}{2\bar{X}\rho} + \frac{3}{8} \frac{\bar{Y}}{\bar{X}^2 \rho^2} + \frac{3}{8} \frac{\partial_2 \bar{Y}}{\bar{X}^2 \rho^2} \right) \lambda_1^2 - \left(\frac{1}{2\bar{X}\rho} + \frac{\partial_2}{2\bar{X}\rho} \right) \lambda_0 \lambda_1 \quad (2)$$

where:

$$\lambda_0^2 = E(\theta_0^2) = g_1 (S_Y^2 + S_{eu}^2) + g_2 (S_{Y(2)}^2 + S_{eu(2)}^2)$$

$$\lambda_1^2 = E(\theta_1^2) = g_1 (S_X^2 + S_{ev}^2) + g_2 (S_{X(2)}^2 + S_{ev(2)}^2)$$

$$\lambda_0 \lambda_1 = E(\theta_0 \theta_1) = g_1 \rho_{YX} S_Y S_X + g_2 \rho_{YX(2)} S_{Y(2)} S_{X(2)}$$

$$MSE(y_1) = \partial_1^2 J + \partial_2^2 P - 2\partial_1\partial_2 Q - 2\partial_1 R + 2\partial_2 L + M \quad (3)$$

$$\text{where } J = \lambda_1^2, \quad P = \left(\bar{Y}^2 + \lambda_0^2 + \frac{\bar{Y}^2}{\bar{X}^2 \rho^2} \lambda_1^2 - 2 \frac{\bar{Y}}{\bar{X} \rho} \lambda_0 \lambda_1 \right), \quad Q = \left(\lambda_0 \lambda_1 - \frac{\bar{Y}}{\bar{X} \rho} \lambda_1^2 \right),$$

$$R = \left(\lambda_0 \lambda_1 - \frac{\bar{Y}}{2\bar{X} \rho} \lambda_1^2 \right), \quad L = \left(\frac{\bar{Y}}{8\bar{X}} \lambda_1^2 - \frac{3}{2} \frac{\bar{Y}}{\bar{X} \rho} \lambda_0 \lambda_1 + \lambda_1^2 + \frac{5}{8} \frac{\bar{Y}^2}{\bar{X}^2 \rho^2} \lambda_1^2 \right)$$

$$M = \lambda_0^2 - \frac{\bar{Y}}{\bar{X} \rho} \lambda_0 \lambda_1 + \frac{\bar{Y}^2}{4\bar{X}^2} \frac{\lambda_1^2}{\rho^2}, \quad \partial_1 = \frac{PR - QL}{JP - Q^2}, \quad \partial_2 = \frac{JL - QR}{JP - Q^2}$$

$$MSE(y_1)_{\min} = \bar{Y}^2 + \left[\frac{2QRL - PR^2 - JL^2}{JP - Q} \right] \quad (4)$$

Munneer *et al.* (2018) proposed exponential-type estimators of population mean in the presence of non-response as in (5) – (7)

$$y_2 = \bar{y}_{(\text{exp})}^* \left(\frac{(\bar{X} - \bar{x}^*)}{(\bar{X} + \bar{x}^*)} \right) e^{\left(\frac{(\bar{Z} - \bar{z}^*)}{\bar{Z} + \bar{z}^*} \right)} \quad (5)$$

$$y_3 = \bar{y}_{(\text{exp})}^* \left(\frac{(\bar{X} - \bar{x}^*)}{(\bar{X} + \bar{x}^*)} \right) e^{\left(\frac{(\bar{z}^* - \bar{Z})}{\bar{z}^* + \bar{Z}} \right)} \quad (6)$$

$$y_4 = \bar{y}_{(\text{exp})}^* \left(\frac{(\bar{X} - \bar{x}^*)}{(\bar{X} + \bar{x}^*)} \right) e^{\left(\frac{n(\bar{Z} - \bar{z}^*)}{2N\bar{Z} - n(\bar{z}^* + \bar{Z})} \right)} \quad (7)$$

$$y_5 = \bar{y}_{(\text{exp})}^* \left(\frac{(\bar{x}^* - \bar{X})}{(\bar{x}^* + \bar{X})} \right) e^{\left(\frac{(\bar{Z} - \bar{z}^*)}{(\bar{Z} + \bar{z}^*)} \right)} \quad (8)$$

$$y_6 = \bar{y}_{(\text{exp})}^* \left(\frac{(\bar{x}^* - \bar{X})}{\bar{x}^* + \bar{X}} \right) e^{\left(\frac{(\bar{z}^* - \bar{Z})}{\bar{z}^* + \bar{Z}} \right)} \quad (9)$$

$$y_7 = \bar{y}_{(\text{exp})}^* \left(\frac{(\bar{x}^* - \bar{X})}{\bar{x}^* + \bar{X}} \right) e^{\left(\frac{n(\bar{Z} - \bar{z}^*)}{2N\bar{Z} - n(\bar{z}^* + \bar{Z})} \right)} \quad (10)$$

$$y_8 = \bar{y}_{(\text{exp})}^* \left(\frac{n(\bar{X} - \bar{x}^*)}{2N\bar{X} - n(\bar{x}^* + \bar{X})} \right) e^{\left(\frac{(\bar{Z} - \bar{z}^*)}{\bar{Z} + \bar{z}^*} \right)} \quad (11)$$

$$y_9 = \bar{y}_{(\text{exp})}^* \left(\frac{n(\bar{X} - \bar{x}^*)}{2NZ\bar{X} - n(\bar{x}^* + \bar{X})} \right) e^{\left(\frac{\bar{z}^* - \bar{Z}}{\bar{z}^* + \bar{Z}} \right)} \quad (12)$$

$$y_{10} = \bar{y}_{(\text{exp})}^* \left(\frac{n(\bar{X} - \bar{x}^*)}{2N\bar{X} - n(\bar{x}^* + \bar{X})} \right) e^{\left(\frac{\bar{Z} - \bar{z}^*}{\bar{z}^* + \bar{Z}} \right)} \quad (13)$$

The estimators y_h , $h = 1, 2, \dots, 9$ can be generally written as in

$$y = \bar{y}^* \left[e^{\left(\frac{U_1 - R_1}{U_1 + R_1} \right)} \right] e^{\left(\frac{U_2 - R_2}{U_2 + R_2} \right)} \quad (14)$$

$$U_1 = (J_1 + Q_1)\bar{X} + IP_1\bar{x}^* \quad U_2 = (J_2 + Q_2)\bar{Z} + IP_2\bar{z}^* \quad R_1 = (J_1 + IP_1)\bar{X} + Q_1\bar{x}^* \\ R_2 = (J_2 + IP_2)\bar{Z} + Q_2\bar{z}^* \quad J_h = (g_h - 1)(g_h - 2) \quad P_h = (g_h - 1)(g_h - 4) \\ Q_h = (g_h - 2)(g_h - 3)(g_h - 4) \\ h = 1, 2, 3, 4, \dots, 9$$

$$MSE(y)_{\min} = \bar{Y}^2 \left[W_y^* - \frac{W_{yx}^{*2} W_z^* + W_{yz}^{*2} W_x^* - 2W_{yx}^{*2} W_{yz}^* W_{xz}^*}{W_x^* W_z^* - W_{xz}^{*2}} \right] \quad h = 1, 2, \dots, 9 \quad (15)$$

$$W_y^* = \alpha Q_y^2 + \tau Q_{y(2)}^2, \quad W_x^* = \alpha Q_x^2 + \tau Q_{x(2)}^2, \quad W_z^* = \alpha Q_z^2 + \tau Q_{z(2)}^2$$

$$W_{yx}^* = \alpha Q_{yx} + \tau Q_{yx(2)}, \quad W_{yz}^* = \alpha Q_{yz} + \tau Q_{yz(2)}, \quad W_{xz}^* = \alpha Q_{xz} + \tau Q_{xz(2)}$$

$$W_{yz}^* = \alpha Q_{yz} + \tau Q_{yz(2)}, \quad Q_{yx} = \rho_{yx} Q_y Q_x, \quad Q_{yz} = \rho_{yz} Q_y Q_z,$$

$$Q_{xz} = \rho_{xz} Q_x Q_z, \quad Q_{y(2)}^2 = \frac{S_{y(2)}^2}{\bar{Y}^2}, \quad Q_{x(2)}^2 = \frac{S_{x(2)}^2}{\bar{X}^2}, \quad Q_{z(2)}^2 = \frac{S_{z(2)}^2}{\bar{Z}^2}$$

$$Q_{yx(2)} = \rho_{yx(2)} Q_{y(2)} Q_{x(2)}, \quad Q_{yz(2)} = \rho_{yz(2)} Q_{y(2)} Q_{z(2)}, \quad Q_{xz(2)} = \rho_{xz(2)} Q_{x(2)} Q_{z(2)},$$

$$\alpha = \left(\frac{1-l}{n} \right), \quad l = \frac{n}{N}, \quad \tau = C_2 \left(\frac{j-1}{n} \right), \quad C_2 = \frac{N_2}{N}$$

Salim and Onyango (2022) proposed a modified estimator for single-phase sampling in the presence of observational errors as in

$$x_1 = \left[\left(\bar{y}_e^* + \varphi(\bar{X} - \bar{x}_e^*) \right) \left(\vartheta e^{\left(\frac{\bar{Z} - \bar{z}_e^*}{\bar{Z} + \bar{z}_e^*} \right)} + (1 - \vartheta) e^{\left(\frac{\bar{Z} - \bar{z}_e^*}{\bar{Z} + \bar{z}_e^*} \right)} \right) \right] \quad (16)$$

$$MSE(x_1) = \tau \bar{Y}^2 (Q_Y^2 + Q_U^2) + \bar{Y} \left(\frac{1}{2} - \vartheta \right) \frac{1}{2} \tau \bar{Y} \bar{Z} \rho_{YZ} Q_Y Q_Z - \beta \tau \bar{Y} \bar{X} \rho_{XY} Q_Y Q_X \quad (17)$$

$$\mathcal{G} = \frac{1}{2}, \quad \varphi = \frac{\rho S_Y}{S_X}$$

2.0 PROPOSED ESTIMATORS

In this paper, four classes of robust estimators of population mean were proposed in the presence of non-response and measurement error using an imputation scheme as defined in (18) – (21)

$$K_1 = \bar{y}_{z(e)} \frac{z}{n} + \left(1 - \frac{z}{n}\right) \left[\bar{y}_{z(e)} \left(\frac{\bar{X}_1}{\bar{x}_{1z(e)}} \right)^{\alpha_{11}} \left(\frac{\bar{X}_2}{\bar{x}_{2z(e)}} \right)^{\alpha_{21}} + q_{1(LTS)} (\bar{X}_1 - \bar{x}_{1z(e)}) \right] + q_{2(LTS)} (\bar{X}_2 - \bar{x}_{2z(e)}) \quad (18)$$

$$K_2 = \bar{y}_{z(e)} \frac{z}{n} + \left(1 - \frac{z}{n}\right) \left[\bar{y}_{z(e)} \left(\frac{\bar{X}_1}{\bar{x}_{1z(e)}} \right)^{\alpha_{21}} \left(\frac{\bar{X}_2}{\bar{x}_{2z(e)}} \right)^{\alpha_{22}} + q_{1(S)} (\bar{X}_1 - \bar{x}_{1z(e)}) \right] + q_{2(S)} (\bar{X}_2 - \bar{x}_{2z(e)}) \quad (19)$$

$$K_3 = \bar{y}_{z(e)} \frac{z}{n} + \left(1 - \frac{z}{n}\right) \left[\bar{y}_{z(e)} \left(\frac{\bar{X}_1}{\bar{x}_{1z(e)}} \right)^{\alpha_{31}} \left(\frac{\bar{X}_2}{\bar{x}_{2z(e)}} \right)^{\alpha_{32}} + q_{1(LMS)} (\bar{X}_1 - \bar{x}_{1z(e)}) \right] + q_{2(LMS)} (\bar{X}_2 - \bar{x}_{2z(e)}) \quad (20)$$

$$K_4 = \bar{y}_{z(e)} \frac{z}{n} + \left(1 - \frac{z}{n}\right) \left[\bar{y}_{z(e)} \left(\frac{\bar{X}_1}{\bar{x}_{1z(e)}} \right)^{\alpha_{41}} \left(\frac{\bar{X}_2}{\bar{x}_{2z(e)}} \right)^{\alpha_{42}} + q_{1(HUBM)} (\bar{X}_1 - \bar{x}_{1z(e)}) \right] + q_{2(HUBM)} (\bar{X}_2 - \bar{x}_{2z(e)}) \quad (21)$$

$$\text{where } \bar{y}_e^* = \frac{n_1 \bar{y}_{1(e)} + n_2 \bar{y}_{2(e)}}{n_1 + n_2}, \quad \bar{x}_{1(e)}^* = \frac{n_1 \bar{x}_{1j_1(e)} + n_2 \bar{x}_{1j_2(e)}}{n_1 + n_2}, \quad \bar{x}_{2(e)}^* = \frac{n_1 \bar{x}_{2(e)} + n_2 \bar{x}_{2j_2(e)}}{n_1 + n_2}$$

2.1 Bias and MSEs of the proposed estimators $K_h, h = 1, 2, 3, 4$

$$\text{Bias}(K_h) = \left(1 - \frac{z}{n}\right) \left(\frac{1}{z} + \frac{1}{N} \right) \left[\frac{\alpha_{h2} (\lambda_{h2} - 1)}{2} Q_{x_2}^2 + \frac{\lambda_{h1} (\lambda_{h1} - 1)}{2} Q_{x_1}^2 + \alpha_{h1} \alpha_{h2} \rho_{x_1 x_2} - \alpha_{h2} \rho_{y x_2} Q_y Q_{x_2} \right] - \alpha_{h1} \rho_{y x_1} Q_y Q_{x_1} \quad (22)$$

$$\text{MSE}(K_h)_{(\min)} = T_{00} + 2J\alpha_{h1(opt)} + 2P\alpha_{h2(opt)} + 2Q\alpha_{h1(opt)}^2 + 2R\alpha_{h2(opt)}^2 + 2L\alpha_{h1(opt)}\alpha_{h2(opt)} + M \quad (23)$$

$$\text{where } \alpha_{h1} = \frac{JR - PL}{QR - L^2}, \quad \alpha_{h2} = \frac{PQ - JL}{QR - L^2}$$

$$J = \left(1 - \frac{z}{n}\right) \bar{Y} [T_{01} - q_1 T_{11} - q_2 T_{12}], P = \left(1 - \frac{z}{n}\right) \bar{Y} [T_{02} - q_2 T_{22} - q_1 T_{12}], Q = \left(1 - \frac{1}{n}\right) \bar{Y}^2 T_{11}$$

$$R = \left(1 - \frac{1}{n}\right) \bar{Y}^2 T_{22}, L = \bar{Y}^2 \left(1 - \frac{z}{n}\right) T_{12}, M = q_1^2 T_{11} + q_2^2 T_{22} - 2q_1 T_{01} - 2q_2 T_{02} + 2q_1 q_2 T_{12}$$

2.2. Efficiency comparisons of the proposed Estimators K_h

In this subsection, efficiency conditions of the estimators of the proposed ratio type over some existing estimators were established.

$$MSE(K_h) - MSE(\tau) < 0 \quad (24)$$

where τ are the existing estimators $MSE(K_h) - MSE(x_1) < 0$

$$\left(\begin{array}{l} T_{00} - 2J\alpha_{h1} - 2P\alpha_{h2} + 2Q\alpha_{h1}^2 + 2R\alpha_{h2}^2 + \\ 2L\alpha_{h1}\alpha_{h2} + F \end{array} \right) - \left[\begin{array}{l} \tau \bar{Y}^2 (Q_Y^2 + Q_Z^2) \bar{Y} \left(\frac{1}{2} - \rho \right) \tau \bar{Y} \bar{Z} \rho_{YZ} Q_Y Q_Z \\ - \phi \tau \bar{Y} \bar{X} \rho_{XY} Q_Y Q_X \end{array} \right] < 0 \quad (25)$$

$$M < 2 \left(\begin{array}{l} -T_{00} + J\alpha_{h1} + P\alpha_{h2} - Q\alpha_{h1}^2 \\ -R\alpha_{h2}^2 - L\alpha_{h1}\alpha_{h2} \end{array} \right) + \left[\begin{array}{l} \tau \bar{Y}^2 (Q_Y^2 + Q_Z^2) + \bar{Y} \left(\frac{1}{2} - \rho \right) \tau \bar{Y} \bar{Z} \rho_{YZ} Q_Y Q_Z \\ - \phi \tau \bar{Y} \bar{X} \rho_{XY} Q_Y Q_X \end{array} \right] \quad (26)$$

$$MSE(K_h) - MSE(y) < 0$$

$$\left(\begin{array}{l} T_{00} - 2J\alpha_{h1} - 2P\alpha_{h2} + 2Q\alpha_{h1}^2 + 2R\alpha_{h2}^2 + \\ 2L\alpha_{h1}\alpha_{h2} + M \end{array} \right) - \left[\bar{Y}^2 \left(W_y^* - \frac{W_{yx}^{*2} W_x^* + W_{yz}^* W_x^* - W_{yx}^* W_{yz}^* W_{xz}^*}{W_x^* W_z^* - W_{xz}^{*2}} \right) \right] < 0 \quad (27)$$

$$M < 2 \left(\begin{array}{l} -T_{00} + J\alpha_{h1} + P\alpha_{h2} - Q\alpha_{h1}^2 \\ -R\alpha_{h2}^2 - L\alpha_{h1}\alpha_{h2} \end{array} \right) + \left[\bar{Y}^2 \left(W_y^* - \frac{W_{yx}^{*2} W_x^* + W_{yz}^* W_x^* - W_{yx}^* W_{yz}^* W_{xz}^*}{W_x^* W_z^* - W_{xz}^{*2}} \right) \right] \quad (28)$$

3.0 RESULTS AND DISCUSSION

This section presents empirical studies that evaluate the performance of the proposed estimators in comparison to the existing ones under review. The biases, MSEs (Mean Squared Errors), and PREs (Percentage Relative Efficiencies) of the estimators were calculated using equations (29), (30), and (31), respectively.

$$Bias(\hat{\tau}_h) = \frac{1}{1000} \sum_{h=1}^{1000} (\hat{\tau}_h - \bar{Y}), \quad \hat{\tau}_h = \bar{y}_h \quad (29)$$

$$MSE(\hat{\tau}_h) = \frac{1}{1000} \sum_{h=1}^{1000} (\hat{\tau}_h - \bar{Y})^2, \quad \hat{\tau}_h = \bar{y}_h \quad (30)$$

$$PRE(\hat{\tau}_h) = \frac{MSE(K_h)}{MSE(\hat{\tau}_h)} \times 100\%, \quad \hat{\tau}_h = \bar{y}_h, K_h, h = 1, 2, 3, 4 \quad (31)$$

**Table 1: Population used for Simulation Study**

Population	Auxiliary Variable (X)	Study Variable (Y)
1	$X_1 = r \exp(N, 2)$ $X_2 = r \exp(N, 1)$	$Y = 0.3 * X_1 + 5 * X_2 + e_i$ Where, $e \sim \text{Normal}(0, 1)$
2	$X_1 = rchisq(N, 2)$ $X_2 = rchisq(N, 3)$	$U \sim \text{Normal}(0, 25)$ $V_1 \sim \text{Normal}(0, 16)$ $V_2 \sim \text{Normal}(0, 9)$
3	$X_1 = r\text{gamma}(N, 0.5, 2.5)$ $X_2 = r\text{gamma}(N, 1.5, 3.5)$	
4	$X_1 = r\text{pois}(N, 3)$ $X_2 = r\text{pois}(N, 1.5)$	

Table 2: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Exponential Distribution

Estimator	Bias	MSE	PRE
Sample mean	-0.009989655	3.670668	100
Singh <i>et al.</i> (2018) y_1	3.023097e+111	1.606164e+224	2.285364e-222
Salim and Onyango (2022) x_1	0.1601833	1.593409	378.4095
Muneer <i>et al.</i> (2018) y_2	0.4332807	10.93393	230.3658
Muneer <i>et al.</i> (2018) y_3	-0.009989655	3.670668	33.57135
Muneer <i>et al.</i> (2018) y_4	0.1738815	3.833255	95.75853
Muneer <i>et al.</i> (2018) y_5	-0.1675946	1.865367	196.7799
Muneer <i>et al.</i> (2018) y_6	72.44146	6090.717	0.0602666
Muneer <i>et al.</i> (2018) y_7	-0.1404735	4.09145	89.71559
Muneer <i>et al.</i> (2018) y_8	-0.06705114	0.9670699	379.566
Muneer <i>et al.</i> (2018) y_9	0.2097258	9.696611	37.85517
Muneer <i>et al.</i> (2018) y_{10}	-0.06705114	0.9670699	379.566
Proposed Estimators under imputation schemes			



K_1	-0.0123064	0.4026394	911.6515
K_2	-0.008408729	0.4209914	871.9106
K_3	-0.03844668	0.2309159	1589.612
K_4	-0.1847394	4.459426	864.6427

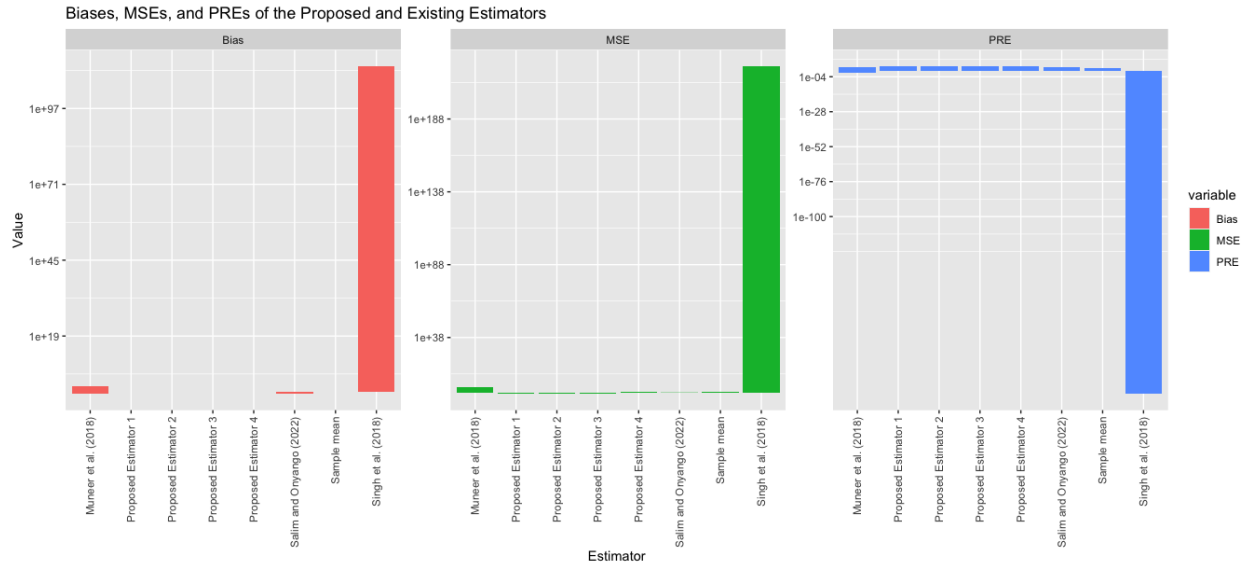


Fig. 1: Plot of Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Exponential Distribution

Table 3: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Chi-square Distribution

Estimator	Bias	MSE	PRE
Sample mean	-0.08410128	21.12014	100
Singh <i>et al.</i> (2018) y_1	-6.712408e+113	1.103315e+229	1.914243e-226
Salim and Onyango (2022) x_1	0.4067392	10.83238	449.5515
Muneer <i>et al.</i> (2018) y_2	0.9687119	68.01479	194.9722
Muneer <i>et al.</i> (2018) y_3	-0.08410128	21.12014	31.05228
Muneer <i>et al.</i> (2018) y_4	0.4246086	25.7296	82.085
Muneer <i>et al.</i> (2018) y_5	-0.2983239	13.9069	151.868
Muneer <i>et al.</i> (2018) y_6	217.3016	52068.63	0.04056212
Muneer <i>et al.</i> (2018) y_7	-0.293289	27.66242	76.34957
Muneer <i>et al.</i> (2018) y_8	-0.1910325	4.67781	451.4963
Muneer <i>et al.</i> (2018) y_9	0.3349565	54.84342	38.50989
Muneer <i>et al.</i> (2018) y_{10}	-0.1910325	4.67781	451.4963
Proposed Estimators under imputation schemes			



K_1	-0.05648113	2.141613	986.179
K_2	-0.05570801	2.175715	970.7216
K_3	-0.2050515	3.528707	598.5234
K_4	-1.994144	29.3358	380.305

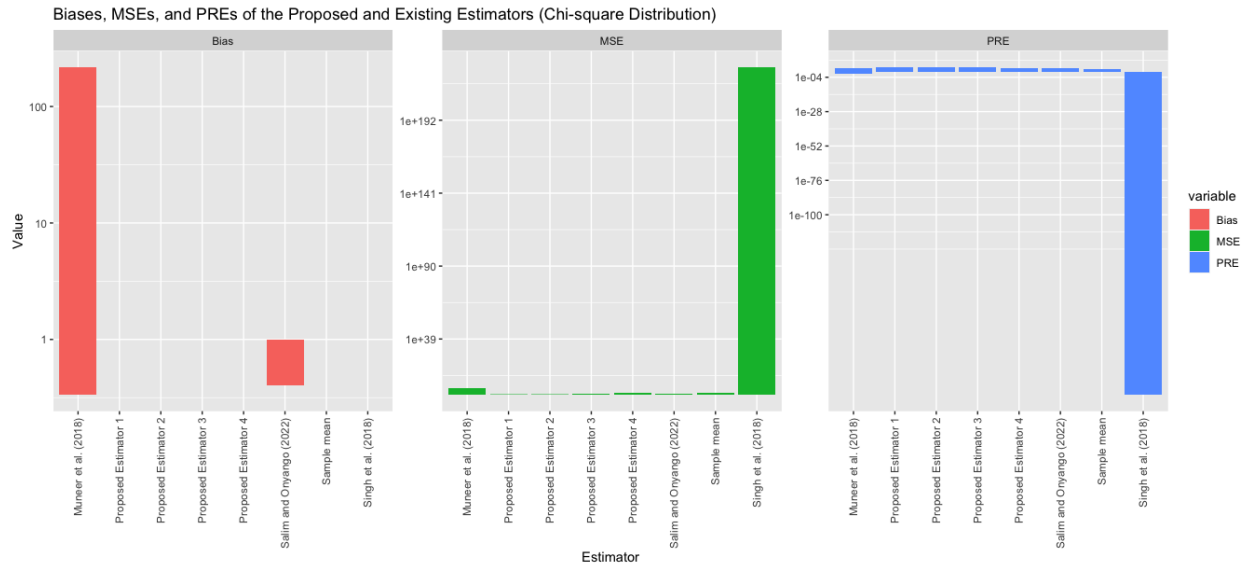


Fig. 2: Plot of Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Chi-square Distribution

Table 4: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from gamma Distribution

Estimator	Bias	MSE	PRE
Sample mean	-0.003964409	0.6811538	100
Singh <i>et al.</i> (2018) y_1	1.41007e+111	7.553066e+222	9.018242e-222
Salim and Onyango (2022) x_1	0.1328968	0.55617	221.8051
Muneer <i>et al.</i> (2018) y_2	0.2428026	2.080598	122.4722
Muneer <i>et al.</i> (2018) y_3	-0.003964409	0.6811538	32.73836
Muneer <i>et al.</i> (2018) y_4	0.1419581	0.920482	73.99968
Muneer <i>et al.</i> (2018) y_5	-0.09282761	0.5294192	128.6606
Muneer <i>et al.</i> (2018) y_6	30.5277	1088.365	0.06258505
Muneer <i>et al.</i> (2018) y_7	-0.08356495	0.7941444	85.77203
Muneer <i>et al.</i> (2018) y_8	-0.02729135	0.3115916	218.6047
Muneer <i>et al.</i> (2018) y_9	0.07586364	1.525341	44.65584
Muneer <i>et al.</i> (2018) y_{10}	-0.02729135	0.3115916	218.6047
Proposed Estimators under imputation schemes			

K_1	-0.05599152	0.08630086	789.2781
K_2	-0.05498659	0.08482839	802.9786
K_3	-0.05425017	0.0593816	1147.079
K_4	-0.09695563	1.022151	768.975

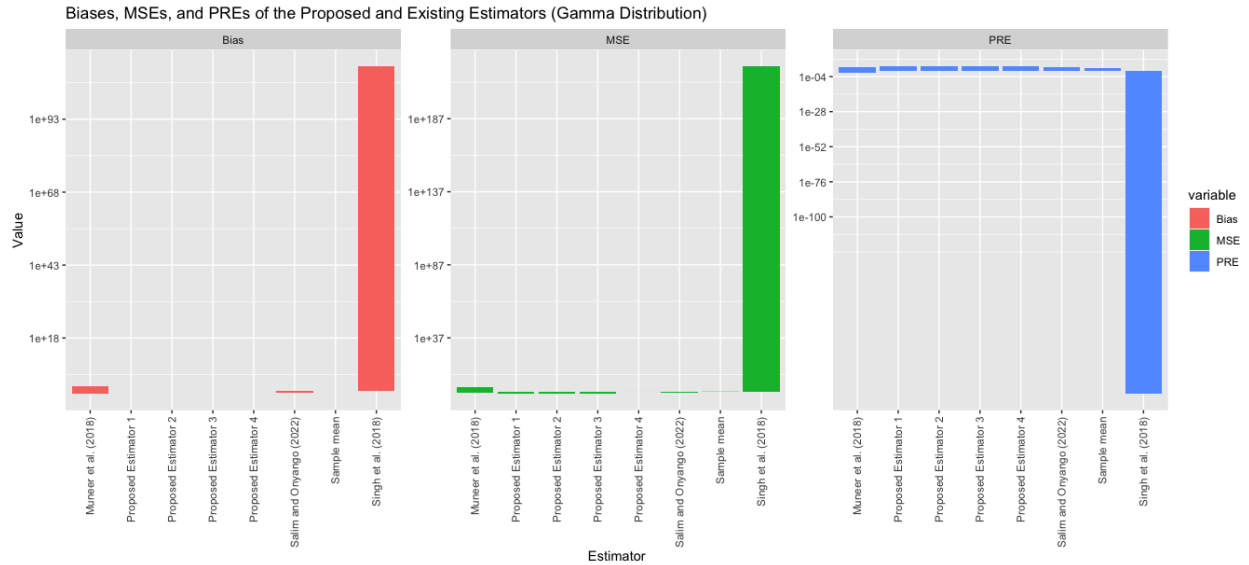


Fig. 3: Plot of Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Gamma Distribution

Table 5: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from poison Distribution

Estimator	Bias	MSE	PRE
Sample mean	0.02252968	6.508619	100
Singh <i>et al.</i> (2018) y_1	-1.840903e+113	5.938039e+227	1.096089e-225
Salim and Onyango (2022) x_1	0.09002835	2.20565	460.7558
Muneer <i>et al.</i> (2018) y_2	0.4742451	19.57555	295.0884
Muneer <i>et al.</i> (2018) y_3	0.02252968	6.508619	33.24872
Muneer <i>et al.</i> (2018) y_4	0.1198424	6.75827	96.306
Muneer <i>et al.</i> (2018) y_5	-0.06961472	2.750671	236.6193
Muneer <i>et al.</i> (2018) y_6	121.6848	16301.8	0.03992577
Muneer <i>et al.</i> (2018) y_7	-0.05585562	6.508619	96.91564
Muneer <i>et al.</i> (2018) y_8	-0.05232447	6.715757	461.1598
Muneer <i>et al.</i> (2018) y_9	0.3083664	1.411359	38.16082
Muneer <i>et al.</i> (2018) y_{10}	-0.05232447	17.05576	461.1598
Proposed Estimators under imputation schemes			



K_1	0.271166	7.465962	894.3244
K_2	0.276815	0.7277694	861.1318
K_3	0.1049767	0.7558215	1867.092
K_4	-0.1308369	7.835589	1048.67

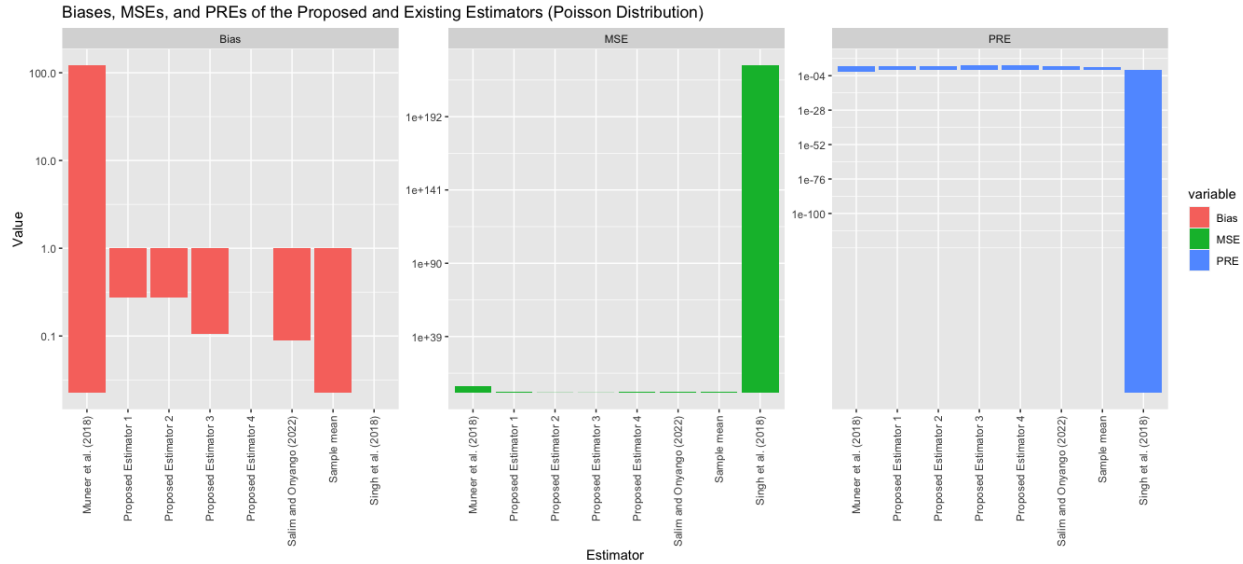


Fig. 4: Plot of Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Poisson Distribution

Table 1 shows that the proposed estimators under imputation schemes have lower MSEs and higher PREs compared to existing estimators, making them more efficient for estimating population parameters with non-response and measurement errors using the exponential distribution.

Similarly, Table 2 demonstrates that the proposed estimators outperform most existing ones under the chi-square distribution, except in a few cases. Thus, the proposed estimators provide more accurate estimates in the presence of non-response and measurement errors.

In Table 3, the proposed imputation-based estimators have lower MSEs and higher PREs, making them more efficient than existing estimators for gamma distribution. Call-back estimators, however, show higher MSEs.

Table 4 shows similar results for the uniform distribution, where the proposed imputation estimators outperform most others, except for a few cases.

4.0 CONCLUSION

The results from both the theoretical and empirical studies indicate that the new classes of proposed estimators using the imputation scheme exhibit the lowest MSEs and higher PREs compared to all competing estimators reviewed in the literature. As a result, these suggested estimators demonstrate greater efficiency than those analyzed in the study. Consequently, they are more likely to yield estimates closer to the true population mean when faced with non-response and measurement errors. Additionally, the consistency test results confirm that the proposed classes of estimators are consistent.

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REFERENCES

Audu, A., Singh, R. & Khare, S. (2021). New Regression type compromised Imputation class of estimators with known parameters of Auxiliary variable. *Communication in statistics- Simulation and*



- Computation.24(3), 573-589.
<https://doi.org/10.1080/03610918.2021.1970182>.
- Audu, A., Abdullazeez S.A., Danbaba A., Ahijjo Y.M., Gidado A. and Yunusa M.A. (2022). Modified Classes of Regression-Type Estimators of Population Mean in the presence of Auxiliary Attributes. *Asian Research Journals of Mathematics*. 18(1): 65-89. DOI: 10.9734/ARJOM/2022/v18i130355.
- Hansen, M.H & Hurwitz, W.N (1946). The Problem of non-response in sample survey. *Journal of the American Statistical Association*, 41(236), 517-529.
- Muneer, S., Shabbir, J., & Khalil, A. (2018). A generalized exponential type estimator of population mean in the presence of non-response. *Statistics in Transition New Series*, 19(2).
- Salim, N. M., & Onyango, C. O. (2022). A Modified Regression Estimator for Single Phase Sampling in the Presence of Observational Errors. *Open journal of Statistics*, 12(2), 175-187.
<https://doi.org/10.4236/ojs.2022.122012>.
- Singh, P., Singh, R., & Bouza, C. N. (2018). Effect of measurement error and non-response on estimation of population mean. *Investigación Operacional*, 39(1).
<https://revistas.uh.cu/invoperacional/article/view/4082>
- Singh, S. (2009): A new method of imputation in survey sampling, *Statistics: A Journal of Theoretical and Applied Statistics*, 43:5, 499-51..
<https://doi.org/10.1080/02331880802605114>.
- Audu, A., Ishaq, O.O Mulli, J.O., Abukabar, A., Rashida, A., Akintola, A & Isah, U. (2020). Modified Estimation of Population Mean using Robust Multiple Regression.Methods. *NIPES Journal of Science and Technology Research* 2(4), 2682-5821.
- Singh, N., Vishwakarma, G. K., & Gangele, R. K. (2021). Variance estimation in the presence of measurement errors under stratified random sampling. *REVSTAT-Statistical Journal*, 19(2), 275-290.
<https://doi.org/10.57805/revstat.v19i2.343>.
- Zaman, T., Dünder, E., Audu, A., Alilah, D. A., Shahzad, U., & Hanif, M. (2021). Robust regression-ratio-type estimators of the mean utilizing two auxiliary variables: a simulation study. *Mathematical Problems in Engineering*, 2021(1), 6383927.
<https://doi.org/10.1155/2021/6383927>