

Optimal Control Analysis of Isolation and Treatment Strategies on the Transmission Dynamics and Control of Lassa Fever

G.H. Anka¹, S. Abdulrahman², A.A. Haliru¹, S.S. Manga¹ and S.F. Abubakar¹

¹Kebbi State University of Science and Technology, Aleiro, Nigeria

²Federal University Birnin Kebbi, Kebbi, Nigeria

Corresponding Author's Email: gazalianka@gmail.com

Received on: August, 2024

Revised and Accepted on: September, 2024

Published on: October, 2024

ABSTRACT

This study constructed and examined a mathematical model on the dynamics of Lassa fever transmission and control. We took into account two interacting rodent and human populations. There are six groups within the human population, one of which is made up of people who underwent complicated recovery. Additionally, there are three divisions within the rodent population. The reduction of susceptible, latent, and infectious individuals by treatment and isolation is the main goal of this work. The goal of optimality is to reduce the number of latent and infectious individuals through isolation and treatment. The amount of influence of the controls in lowering the population of causal pathogens and susceptible, latent, and infectious people is determined by applying the optimal control theory. The model's optimal solution was established using Pontryagin's maximum principle, and the optimal system was then generated and solved numerically. Graphs were used to create simulations showing how the controls affected susceptible, latent, and infected persons. The results demonstrated that applying the two controls simultaneously can be one of the quick and efficient strategies to regulate Lassa Fever.

Keywords: Lassa fever, rodents, isolation, treatment, optimal control.

1.0 INTRODUCTION

Acute viral hemorrhagic fever, or Lassa fever, was originally identified in 1969 in Lassa town, in the Yedseran river valley in what is now Borno State in Northern Nigeria (Tara, 2004). According to Marien *et al.* (2019) and Musa *et al.* (2020), Lassa fever is a zoonotic illness that can spread from an infected animal to a human. *Mastomys natalensis*, a multimammate rat species, is the natural reservoir of the Lassa-Fever virus (Mahmoud, A.I. and Attila, D. (2021)). The virus comes into touch with rat urine, feces, and saliva directly; by contact with the excretion or secretion of infected materials; or through ingestion or excretion of contaminated food, as some forms of *Mastomys* frequently reside in human households. Additionally, skin fractures and mucous membranes from dust-borne particle aerosol transfer might infect victims. In certain places, rats are kept as food, which exposes people to rat blood that may be tainted on top of that and permits eating of possibly contaminated meat. Lassa fever is one of many infectious diseases that are emerging or reappearing in some west African countries. It has caused widespread and serious health problems in west African countries i.e Nigeria, Liberia, Ghana, Guinea and Sierra Leone (Onah and Collins, 2020). In contrast to other arena viruses, the Lassa virus is fairly and readily spread from person to person (WHO, 2022).

When interacting with an infected individual, health professionals are recommended to use goggles, face masks, laboratory coats, and hand gloves to prevent coming into touch with bodily fluids like blood.

Rats are the primary vector for the transmission of the Lassa virus, an antisense RNA virus belonging to the *Arenaviridae* family that causes Lassa Fever. There have also been reports of sexual transmissions (WHO, 2022).

The Lassa-Fever virus infects many people, although many of them show no symptoms. Symptoms that often manifest when they do include fever, weakness, headaches, nausea, and sore muscles. Less often, there might be oral or gastrointestinal bleeding. Once infected, there is a one percent chance of mortality, which usually happens within two weeks of the commencement of symptoms. Roughly 25% of survivors experience hearing loss complications; in about half of cases, these become better with time (WHO, 2022). The CDC and WHO report that between 100,000 and 300,000 cases of Lassa Fever occur each year, with an estimated 5,000 fatalities occurring in west Africa (Marien *et al.*, 2019; Musa *et al.*, 2020). The Lassa-fever cases spread from 19 states in 2017 to 32 states and Federal Capital Territory (FCT) in 2020. Ondo state (25.39%) had the highest of death rate from Lassa-fever over the four years (CDC, 2020). From December 2016 to September 2020, 516 fatalities and 2787 confirmed cases were recorded in Nigeria.

Numerous experts have created numerous mathematical models in recent years to illustrate the fatal Lassa Fever disease's mechanism of transmission and to offer long-term projections on the frequency and containment of the deadly virus in different regions. The aim of this research is to formulate mathematical

model on the transmission dynamics and control of Lassa Fever by incorporating isolated and recovered with complications and hence carryout an optimal control analysis on the model. These elements are crucial to the dynamics of Lassa Fever transmission and its management, particularly in underdeveloped nations where the illness is widespread.

2.0 MATERIALS AND METHODS

2.1 Material

In order to better understand the role that treatment and isolation of sick persons play in the dynamical spread of Lassa Fever resulting from effective interaction between infected rats and humans, Adewale *et al.* (2016) developed a six-compartment model. Human population size $N_h(t)$ is further subdivided into subclasses of isolated $J_h(t)$, susceptible $S_h(t)$, exposed $E_h(t)$, and infected people $I_h(t)$. Additionally, the number of rodents $N_r(t)$ is separated into infectious $I_r(t)$ and susceptible $S_r(t)$ groups. The model is properly posed both mathematically and epidemiologically, as demonstrated by the positivity of the solution. Using the next-generation matrix approach, the basic reproduction number ' R_0 ' which decides whether the Lassa Fever disease spreads or dies out was determined. The outcome revealed that the illness spreads when its reproduction number surpasses unity, i.e. $R_0 > 1$ and dies out otherwise.

Using the comparison approach, the global stability of the disease-free equilibrium was examined. According to numerical simulation, isolating infected individuals would help prevent the dynamical spread of Lassa Fever throughout society. It would also help the public avoid coming into contact with infected people and allow infected individuals who are isolated to receive appropriate medical care. The model equations are listed below:

$$\begin{aligned} \frac{dS_h(t)}{dt} &= \Lambda_h + \phi(1-v)I_h + \psi R_h - \beta_1 \sigma I_r S_h - \beta_2 \varepsilon I_h S_h - \eta(1-e^{-rt})S_h - \mu_h S_h \\ \frac{dE_h(t)}{dt} &= \beta_1 \sigma I_r S_h + \beta_2 \varepsilon I_h S_h + \eta(1-e^{-rt})S_h - (\kappa + \mu_h)E_h \\ \frac{dI_h(t)}{dt} &= \kappa E_h - \phi(1-v)I_h - \phi v I_h - (\delta + \mu_h)I_h \\ \frac{dR_h(t)}{dt} &= \phi v I_h - (\psi + \mu_h)R_h \\ \frac{dS_r(t)}{dt} &= \Lambda_r - \beta_3 \vartheta I_h S_r - \mu_r S_r \\ \frac{dE_r(t)}{dt} &= \beta_3 \vartheta I_h S_r - (\alpha + \mu_r)E_r \\ \frac{dI_r(t)}{dt} &= \alpha E_r - \mu_r I_r \end{aligned}$$

A model for the dynamics of lassa fever spread and control was developed by Anorue and Okeke (2020)

$$\begin{aligned} S_h' &= (1-\rho)\pi_h - (\beta_1 I_R + \beta_2 I_h)S_h - \mu_h S_h \\ E_h' &= (\beta_1 I_R + \beta_2 I_h)S_h - (\kappa_h + \mu_h)E_h + U_1 I_h + U_2 J_h \\ I_h' &= \rho\pi_h + \kappa_h E_h - (\mu_h + \sigma + \delta_h + U_1)I_h \\ J_h' &= \sigma I_h - (\mu_h + \delta_h + U_2)J_h \\ S_R' &= \pi_R - \beta_1 I_R S_R - \mu_R S_R \\ I_R' &= \beta_1 I_R S_R - \mu_R I_R \end{aligned}$$

A mathematical model of the dynamics of the Lassa Fever sickness was developed by Obasi and Mbah in 2019. The whole human population $N_h(t)$ at time t is subdivided by the model to indicate the number of susceptible individuals $s_h(t)$, the number of non-infectious persons $e_h(t)$ exposed, the number of infectious individuals (infected and symptomatic or diagnosed) $i_h(t)$, and the number of recovered individuals $r_h(t)$. There are three types of rodents in the community at any given time: susceptible $s_r(t)$, exposed $e_r(t)$, and infectious $i_r(t)$.

The presence and stability of the endemic and disease-free equilibrium points in the model were qualitatively examined. It was demonstrated that the disease-free equilibrium is globally and locally asymptotically stable. The next generation approach was used to compute the fundamental reproduction number. The result's implications for public health demonstrated that Lassa Fever might be eradicated from the population if the management method forced the reproduction number to be less than one. The ordinary differential equation system is expressed as follows:

and took into account the population of humans and vectors at a certain period.

The total humans population is given by $N_H(t)$ and is divided into five compartments; susceptible humans (S_H), exposed humans (E_H), infectious asymptomatic humans (I_{HA}), infectious symptomatic human (I_{HS}) and recovered humans (R_H). The total vector population is given by $N_V(t)$ and is divided into three compartments; susceptible vectors (S_V), exposed vectors (E_V) and the infectious vectors (I_V).

Analysis was done on the disease-free equilibrium. Reproduction number was calculated using the total generation matrix approach, and the results indicated that when locally asymptotically stable $R_0 < 1$, this will reduce the spread of the illness. The disease-free equilibrium's global asymptotic stability was established using the Castillo Chavez theorem. The model's numerical simulations demonstrated that the virus exhibits both symptomatic and asymptomatic behavior. These first-order differential equations can be used to represent the transmission across model classes:

$$\left. \begin{aligned} \frac{dS_H(t)}{dt} &= \Lambda_H - \beta_1 I_{HA} S_H - \beta_2 I_{HS} S_H \\ &\quad - \beta_3 I_{HQ} S_H - \beta_4 I_V S_H - \mu_1 S_H \\ \frac{dE_H(t)}{dt} &= \beta_1 I_{HA} S_H + \beta_2 I_{HS} S_H \\ &\quad + \beta_3 I_{HQ} S_H + \beta_4 I_V S_H - (\alpha + \mu_1) E_H \\ \frac{dI_{HA}(t)}{dt} &= \alpha E_H - (\mu_H + \omega_1 + \delta) I_{HA} \\ \frac{dI_{HS}(t)}{dt} &= (1-a) \alpha E_H - (\mu_H + \omega_2 + \delta) I_{HS} \\ \frac{dR_H(t)}{dt} &= \eta I_{HQ} - \mu_H R_H \\ \frac{dS_V(t)}{dt} &= \Lambda_V - (\beta_4 I_V + \mu_H + \phi_V) S_V \\ \frac{dE_V(t)}{dt} &= \beta_4 I_V S_V - (\gamma_V + \mu_V + \phi_V) E_V \\ \frac{dI_V(t)}{dt} &= \gamma_V E_V - (\mu_V + \phi_V) I_V \end{aligned} \right\}$$

By taking into account a model on the transmission dynamics of Lassa-Fever and adding isolated and recovered with complications compartment while taking into account all the model analysis methods presented by the aforementioned authors, this research attempted to enhance the work of Obasi and Mbah (2019).

2.2 Method

Two interacting populations of hosts and rodents. The host population is divided into six compartments of susceptible humans $S_h(t)$, latently infected humans $L_h(t)$, Infectious humans $I_h(t)$, isolated humans $J_h(t)$, recovered without complication human $R(t)$ and Recovered with complication human $R_k(t)$ while the rodents community is partitioned into three compartments of Non-carrier rodents $N_r(t)$, latently infected rodents $L_r(t)$ and Carrier rodents $C_r(t)$ where $t \geq 0$ is the time. The host is the human population while the rodents is the multimamate rat population. Figure 1 is a schematized diagram of the system.

2.2.1 The Assumptions of the model

The model is built based on the following assumptions:

- (i) The human population is mixed uniformly, meaning that everyone has an equal chance of contracting an infectious disease from another person should they come into touch with them.
- (ii) The natural recovery of the infectious individuals largely depends on the strongness of the immune system.
- (iii) Individuals who make rodents as delicacy are assumed to being infected by the virus.
- (iv) Government provides centres where individuals with lassa fever symptoms are isolated and treatment is administered to them.

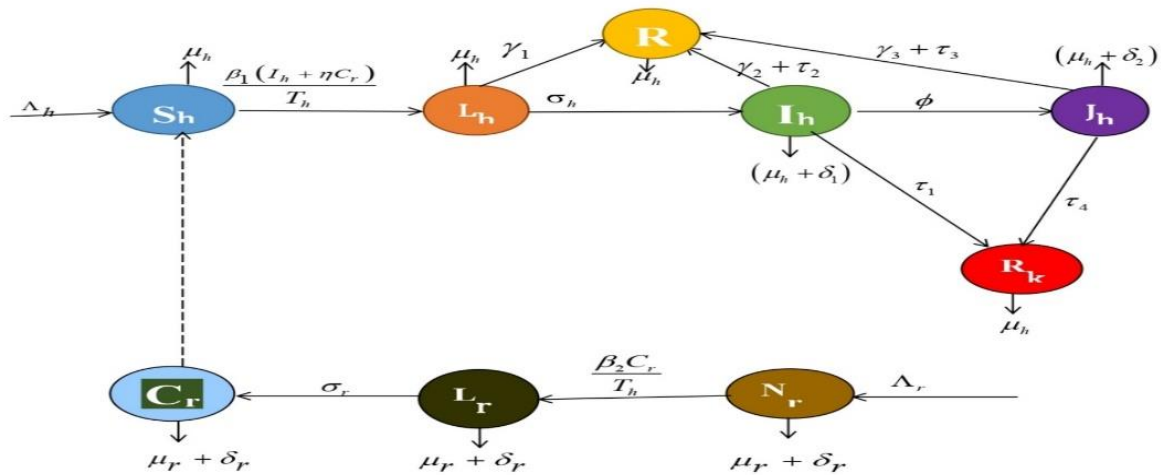


Fig. 1: A Schematized Diagram of Lassa Fever

A system of ordinary differential equations provided below can be used to describe the relevant mathematical equations of the schematic diagram.

2.2.2 MODEL EQUATIONS

$$\frac{dS_h}{dt} = \Lambda_h - \frac{\beta_1(I_h + \eta C_r)}{T_h} S_h - \mu_h S_h \quad (1)$$

$$\frac{dL_h}{dt} = \frac{\beta_1(I_h + \eta C_r)}{T_h} S_h - (\sigma_h + \gamma_1 + \mu_h) L_h \quad (2)$$

$$\frac{dI_h}{dt} = \sigma_h L_h - (\phi + \gamma_2 + \tau_1 + \tau_2 + \mu_h + \delta_1) I_h \quad (3)$$

$$\frac{dJ_h}{dt} = \phi I_h - (\tau_3 + \tau_4 + \gamma_3 + \mu_h + \delta_2) J_h \quad (4)$$

$$\frac{dR_k}{dt} = \tau_1 I_h + \tau_4 J_h - \mu_h R_k \quad (5)$$

$$\frac{dR}{dt} = \gamma_1 L_h + (\gamma_2 + \tau_2) I_h + (\gamma_3 + \tau_3) J_h - \mu_h R \quad (6)$$

$$\frac{dN_r}{dt} = \Lambda_r - \frac{\beta_2 C_r}{T_h} N_r - (\mu_r + \delta_r) N_r \quad (7)$$

$$\frac{dL_r}{dt} = \frac{\beta_2 C_r}{T_h} N_r - (\sigma_r + \mu_r + \delta_r) L_r \quad (8)$$

$$\frac{dC_r}{dt} = \sigma_r L_r - (\mu_r + \delta_r) C_r \quad (9)$$

where

$$T_h = S_h + L_h + I_h + J_h + R_k + R \quad (10)$$

$$T_r = N_r + L_r + C_r \quad (11)$$

$$\lambda_h = \frac{\beta_1(I_h + \eta C_r)}{T_h} \quad (12)$$

$$\lambda_r = \frac{\beta_2 C_r}{T_r} \quad (13)$$

$$k_1 = (\sigma_h + \gamma_1 + \mu_h), \quad k_2 = (\phi + \gamma_2 + \tau_1 + \tau_2 + \mu_h + \delta_1), \quad k_3 = (\tau_3 + \tau_4 + \gamma_3 + \mu_h + \delta_2)$$

$$k_4 = (\gamma_2 + \tau_2), \quad k_5 = (\gamma_3 + \tau_3), \quad k_6 = (\mu_r + \delta_r), \quad k_7 = (\sigma_r + \mu_r + \delta_r)$$

Hence, equations (1) - (9) becomes

$$\frac{dS_h}{dt} = \Lambda_h - \lambda_h S_h - \mu_h S_h \quad (14)$$

$$\frac{dL_h}{dt} = \lambda_h S_h - k_1 L_h \quad (15)$$

$$\frac{dI_h}{dt} = \sigma_h L_h - k_2 I_h \quad (16)$$

$$\frac{dJ_h}{dt} = \phi I_h - k_3 J_h \quad (17)$$

$$\frac{dR_k}{dt} = \tau_1 I_h + \tau_4 J_h - \mu_h R_k \quad (18)$$

$$\frac{dR}{dt} = \gamma_1 L_h + k_4 I_h + k_5 J_h - \mu_h R \quad (19)$$

$$\frac{dN_r}{dt} = \Lambda_r - \lambda_r N_r - k_6 N_r \quad (20)$$

$$\frac{dL_r}{dt} = \lambda_r N_r - k_7 L_r \quad (21)$$

$$\frac{dC_r}{dt} = \sigma_r L_r - k_6 C_r \quad (22)$$

The differential equation may now be used to calculate the size of the human population overall $T_h(t)$.

$$\frac{dT_h}{dt} = \Lambda_h - \mu_h T_h - \delta_1 I_h - \delta_2 J_h \quad (23)$$

And also, the total rodent's population size $T_r(t)$ is written as

$$\frac{dT_r}{dt} = \Lambda_r - \mu_r T_r \quad (24)$$

2.3 Optimal Control

The goal of optimal control is to look for optimality for the control(s) that minimizes or maximizes an objective functional which depends on the goal for inputting the

control into the model in the first place. Now, minimizing susceptible, latent and infectious humans through, impact of isolation and treatment. The objective of optimizing are: to bring down the

$$\frac{dS_h}{dt} = \Lambda_h - \frac{\beta_1 (I_h + \eta C_r)}{T_h} S_h - \mu_h S_h \quad (25)$$

$$\frac{dL_h}{dt} = \frac{\beta_1 (I_h + \eta C_r)}{T_h} S_h - (\sigma_h + \gamma_1 + \mu_h) L_h \quad (26)$$

$$\frac{dI_h}{dt} = \sigma_h L_h - (u_1 + \gamma_2 + \tau_1 + \tau_2 + \mu_h + \delta_1) I_h \quad (27)$$

$$\frac{dJ_h}{dt} = u_1 I_h - (u_2 + \tau_4 + \gamma_3 + \mu_h + \delta_2) J_h \quad (28)$$

$$\frac{dR_k}{dt} = \tau_1 I_h + \tau_4 J_h - \mu_h R_k \quad (29)$$

number of susceptible, latent and infectious humans through two controls, impact of isolation and treatment. The model (1)-(9) is reformulated to identify time dependent controls, $u_1(t)$ is the impact of isolation and $u_2(t)$ is the treatment, which are now function of time in the reformulated model below.

$$\frac{dR}{dt} = \gamma_1 L_h + (\gamma_2 + \tau_2) I_h + (\gamma_3 + u_2) J_h - \mu_h R \quad (30)$$

$$\frac{dN_r}{dt} = \Lambda_r - \frac{\beta_2 C_r}{T_r} N_r - (\mu_r + \delta_r) N_r \quad (31)$$

$$\frac{dL_r}{dt} = \frac{\beta_2 C_r}{T_r} N_r - (\sigma_r + \mu_r + \delta_r) N_r \quad (32)$$

$$\frac{dC_r}{dt} = \sigma_r L_r - (\mu_r + \delta_r) C_r \quad (33)$$

where

$$\lambda_h = \frac{\beta_1 (I_h + \eta C_r)}{S_h + L_h + I_h + J_h + R_k + R} \quad \text{and} \quad \lambda_r = \frac{\beta_2 C_r}{N_r + L_r + C_r}$$

Now, the biological reasons, initial values of the state variables are assumed as follows:

Let $S_h(0) = S_h 0$, same applies to the remaining state variables

$S_h 0 > 0$, $L_h 0 \geq 0$, $I_h 0 \geq 0$, $J_h 0 \geq 0$, $R_k 0 \geq 0$, $R 0 \geq 0$, $N_r 0 \geq 0$, $L_r 0 \geq 0$, and $C_r 0 \geq 0$

Assume that the terminal time t_f is such that the objective functional is optimized as follows.

$$J(u_1, u_2) = \int_0^{t_f} [C_1 I_h + C_2 J_h + \frac{\theta_1}{2} u_1^2 + \frac{\theta_2}{2} u_2^2] dt \quad (34)$$

where t_f is the final time. We need to find an optimal control u_1^* and u_2^* such that

$$J(u_1, u_2) = \min [J(u_1^*, u_2^*) | u_1, u_2 \in U] \quad (35)$$

where $U = [u_1^*, u_2^*]$ is the control set such that u_1^* , u_2^* are measurable with $0 \leq u_1^*$ and $0 \leq u_2^* \leq 0.9$ for $t \in [0, t_f]$.

Now, the Hamiltonian is given by

$$\begin{aligned} \Upsilon = & C_1 I_h + C_2 J_h + \frac{\theta_1}{2} u_1^2 + \frac{\theta_2}{2} u_2^2 \\ & + \lambda_1 \left\{ \Lambda_h - \frac{\beta_1 (I_h + \eta C_r)}{T_h} S_h - u_h S_h \right\} + \lambda_2 \left\{ \frac{\beta_1 (I_h + \eta C_r)}{T_h} S_h - (\sigma_h + \gamma_1 + \mu_h) L_h \right\} \\ & + \lambda_3 \left\{ \sigma_h L_h - (u_1 + \gamma_2 + \tau_1 + \tau_2 + \mu_h + \delta_1) I_h \right\} + \lambda_4 \left\{ u_1 I_h - (u_2 + \tau_4 + \gamma_3 + \mu_h + \delta_2) J_h \right\} \\ & + \lambda_5 \left\{ \tau_1 I_h + \tau_4 J_h - \mu_h R_k \right\} + \lambda_6 \left\{ \gamma_1 L_h + (\gamma_2 + \tau_2) I_h + (\gamma_3 + u_2) J_h - \mu_h R \right\} \\ & + \lambda_7 \left\{ \Lambda_r - \frac{\beta_2 C_r}{T_r} N_r - (\mu_r + \delta_r) N_r \right\} + \lambda_8 \left\{ \frac{\beta_2 C_r}{T_r} N_r - (\sigma_r + \mu_r + \delta_r) L_r \right\} \\ & + \lambda_9 \left\{ \sigma_r L_r - (\mu_r + \delta_r) C_r \right\} \end{aligned} \quad (36)$$

In this case, the adjoint or co-state variables are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$ and λ_9 . By considering the proper partial derivatives of the Hamiltonian (37) with respect to the related state variable, the system of equations may be constructed.

2.3.1 EXISTENCE OF OPTIMAL CONTROL

We now prove that there is a solution for the best control that minimizes the function of the objectives J.

Theorem 1: Suppose J is defined on the control set U subject to system (25)-(33) with non-negative initial conditions at $t=0$ then, there exist an optimal control $u^* = (u_1, u_2)$ such that

$J(u^*) = \min [J(u_1, u_2) | u_1, u_2 \in U]$ if the following conditions given in Abubakar and Ibrahim (2022) hold:

- (i) U is a closed, convex acceptable control set.
- (ii) A linear function in the state and control variables bounds the state system.
- (iii) With regard to the controls, the integrand of the objective function in (36) is convex.
- (iv) Below, the Lagrangian is surrounded by

$\varpi_1 |u|^{\varpi_3} - \varpi_2$, where $\varpi_1 > 0, \varpi_2 > 0, \varpi_3 > 1$.

Proof: Let $U = [0, 1]$ be the control set consisting of $u = (u_1, u_2) \in U$, $x = (S_h, L_h, I_h, J_h, R_k, R, N_r, L_r, C_r)$ and $f(t, x, u)$ the right hand of (25)-(33), that is

$$f(t, x, u) = \begin{pmatrix} \Lambda_h - \frac{\beta_1(I_h + \eta C_r)}{T_h} S_h - u_h S_h \\ = \frac{\beta_1(I_h + \eta C_r)}{T_h} S_h - (\sigma_h + \gamma_1 + \mu_h) L_h \\ \sigma_h L_h - (u_1 + \gamma_2 + \tau_1 + \tau_2 + \mu_h + \delta_1) I_h \\ u_1 I_h - (u_2 + \tau_4 + \gamma_3 + \mu_h + \delta_2) J_h \\ \tau_1 I_h + \tau_4 J_h - \mu_h R_k \\ \gamma_1 L_h + (\gamma_2 + \tau_2) I_h + (\gamma_3 + u_2) J_h - \mu_h R \\ \Lambda_r - \frac{\beta_2 C_r}{T_r} N_r - (\mu_r + \delta_r) N_r \\ \frac{\beta_2 C_r}{T_r} N_r - (\sigma_r + \mu_r + \delta_r) N_r \\ \sigma_r L_r - (\mu_r + \delta_r) C_r \end{pmatrix} \quad (38)$$

In other to prove theorem 1, we need to note the following:

- (i) It is clear that $U = [0, u_{\max}]$ bounded. Furthermore, considering any given two elements $v, w \in U$ where $v = (v_1, v_2)$, $w = (w_1, w_2)$. Then, $\Phi v + (1 - \Phi) w \in [0, u_{\max}] : \forall \Phi \in [0, u_{\max}]$. That is $\Phi v + (1 - \Phi) w \in U$ and hence U is a convex set.

- (ii) A linear function of control variables u_1, u_2 with coefficients representing functions of time and state variables may be used to define the control system (25)–(33).

$$f(t, x, u) = \mathcal{G} = (t, x) + \phi(t, x)u \quad (39)$$

$$\mathcal{G}(t, x) = \begin{pmatrix} \Lambda_h - \frac{\beta_1(I_h + \eta C_r)}{T_h} S_h - u_h S_h \\ = \frac{\beta_1(I_h + \eta C_r)}{T_h} S_h - (\sigma_h + \gamma_1 + \mu_h) L_h \\ \sigma_h L_h - (\gamma_2 + \tau_1 + \tau_2 + \mu_h + \delta_1) I_h \\ - (\tau_4 + \gamma_3 + \mu_h + \delta_2) J_h \\ \tau_1 I_h + \tau_4 J_h - \mu_h R_k \\ \gamma_1 L_h + (\gamma_2 + \tau_2) I_h + \gamma_3 J_h - \mu_h R \\ \Lambda_r - \frac{\beta_2 C_r}{T_r} N_r - (\mu_r + \delta_r) N_r \\ \frac{\beta_2 C_r}{T_r} N_r - (\sigma_r + \mu_r + \delta_r) N_r \\ \sigma_r L_r - (\mu_r + \delta_r) C_r \end{pmatrix} \quad (40)$$

$$\phi(t, x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -I_h & 0 \\ I_h & -J_h \\ 0 & 0 \\ 0 & J_h \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (41)$$

As the parameters and variables of the model are positive, we have

$$\begin{aligned} \|f(t, x, u)\| &\leq \|\mathcal{G}(t, x)\| + \|\phi(t, x)\| \|u\| \\ &\leq a + b \|u\| \text{ where } a > 0, b > 0. \end{aligned}$$

(ii) The optimal control problem's Lagrangian is given by

$$L = C_1 I_h + C_2 J_h + \frac{1}{2} \sum_{i=1}^2 \theta_i u_i^2 \quad (42)$$

Let us consider two arbitrary elements $v, w \in U$ where $v = (v_1, v_2)$, $w = (w_1, w_2)$ and $\Phi \in [0, u_{\max}]$. We now show that, $L[t, x, (1-\Phi)v + \Phi w] \leq (1-\Phi)L(t, x, v) + \Phi L(t, x, w)$

It follows from (42) that,

$$L[t, x, (1-\Phi)v + \Phi w] = C_1 I_h + C_2 J_h + \frac{1}{2} \sum_{i=1}^2 \theta_i [(1-\Phi)v_i + \Phi w_i]^2 \quad (43)$$

$$(1-\Phi)L(t, x, v) + \Phi L(t, x, w) = C_1 I_h + C_2 J_h + \frac{1}{2} (1-\Phi) \sum_{i=1}^2 \theta_i v_i^2 + \frac{1}{2} \Phi \sum_{i=1}^2 \theta_i w_i^2 \quad (44)$$

Taking the difference of (43) and (44) equations given above, we have

$$\begin{aligned} L[t, x, (1-\Phi)v + \Phi w] - [(1-\Phi)L(t, x, v) + \Phi L(t, x, w)] \\ = \frac{1}{2} (\Phi^2 - \Phi) \sum_{i=1}^2 \theta_i (v_i - w_i)^2 < 0, \text{ since } \Phi \in [0, u_{\max}]. \end{aligned} \quad (45)$$

Thus,

$$L[t, x, (1-\Phi)v + \Phi w] \leq [(1-\Phi)L(t, x, v) + \Phi L(t, x, w)],$$

Gives

$$L[t, x, (1-\Phi)v + \Phi w] - [(1-\Phi)L(t, x, v) + \Phi L(t, x, w)] \leq 0$$

And hence convexity of L .

(iii) There exist constants ϖ_1, ϖ_2 and ϖ_3 such that $L \leq \varpi_1 |u|^{\varpi_3} - \varpi_2$,
 $\varpi_1 > 0, \varpi_2 > 0, \varpi_3 > 1$.

We now establish the bound on L . Note that $\zeta_1 u_1^2 \leq \zeta_1$. As $u_1 \in [0, 1]$, $\frac{1}{2} \zeta_1 u_1^2 \leq \frac{1}{2} \zeta_1$.

$$L > \frac{\theta_1}{2} u_1^2 + \frac{\theta_2}{2} u_2^2$$

$$\text{Now, } \geq \frac{\theta_1}{2} u_1^2 + \frac{\theta_2}{2} u_2^2 - \frac{\theta_1}{2} \quad (46)$$

$$\geq \min\left(\frac{\theta_1}{2}, \frac{\theta_2}{2}\right) (u_1^2 + u_2^2) - \frac{w_1}{2}$$

$$\geq \min\left(\frac{\theta_1}{2}, \frac{\theta_2}{2}\right) |u_1 + u_2|^2 - \frac{w_1}{2}$$

Hence,

$$L \leq \varpi_1 |u|^{\varpi_3} - \varpi_2, \text{ where } \varpi_1 = \min\left(\frac{\theta_1}{2}, \frac{\theta_2}{2}\right) > 0, \varpi_2 = \frac{w_1}{2} > 0 \text{ and } \varpi_3 = 2 > 1$$

Theorem 2: Suppose the set $u = \{u_1, u_2\}$ minimizes J over U , then adjoint variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$ and λ_9 satisfies the adjoint equations

$$\frac{d\lambda_i}{dt} = -\frac{\delta\chi}{\delta j} \quad (47)$$

where $j = S_h, L_h, I_h, J_h, R_k, R, N_r, L_r, C_r, \quad i = 1, 2, 3, \dots, 9$ with transversality condition $\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = \lambda_5(t_f) = \lambda_6(t_f) = \lambda_7(t_f) = \lambda_8(t_f) = \lambda_9(t_f) = 0$ (3.574)

Furthermore,

$$u_1^* = \min \left\{ 1, \max \left(0, \frac{J_h(\lambda_6 - \lambda_4)}{2C} \right) \right\} \quad (48)$$

$$u_2^* = \min \left\{ 1, \max \left(0, \frac{N_r C_r \beta_2 (\lambda_8 - \lambda_7)}{2DT_h} \right) \right\} \quad (49)$$

Proof: Consider $U^* = (u_1^*, u_2^*)$ and the associated solutions $S_h^*, L_h^*, I_h^*, J_h^*, R_k^*, R^*, N_r^*, L_r^*, C_r^*$.

Pontryagin's Maximum Principle is applied to obtain the following

$$\frac{d\lambda_1}{dt} = -\frac{\delta\chi}{\delta S_h}, \lambda_1(t_f) = 0, \quad \frac{d\lambda_2}{dt} = -\frac{\delta\chi}{\delta L_h}, \lambda_2(t_f) = 0, \quad \frac{d\lambda_3}{dt} = -\frac{\delta\chi}{\delta I_h}, \lambda_3(t_f) = 0 \quad (50)$$

$$\frac{d\lambda_4}{dt} = -\frac{\delta\chi}{\delta J_h}, \lambda_4(t_f) = 0, \quad \frac{d\lambda_5}{dt} = -\frac{\delta\chi}{\delta R_k}, \lambda_5(t_f) = 0, \quad -\frac{d\lambda_6}{dt} = \frac{\delta\chi}{\delta R}, \lambda_6(t_f) = 0,$$

$$-\frac{d\lambda_7}{dt} = \frac{\delta\chi}{\delta N_r}, \lambda_7(t_f) = 0, \quad -\frac{d\lambda_8}{dt} = \frac{\delta\chi}{\delta L_r}, \lambda_8(t_f) = 0, \quad -\frac{d\lambda_9}{dt} = \frac{\delta\chi}{\delta C_r}, \lambda_9(t_f) = 0$$

On the interior of the set, where $0 < u_j < 1, \quad \forall (j = 1, 2)$ we have

$$0 = \frac{\delta\chi}{\delta u_1} = \theta_1 u_1^* - \lambda_h S_h (\lambda_2 - \lambda_1) \quad (51)$$

$$0 = \frac{\delta\chi}{\delta u_2} = \theta_2 u_2^* - S_h (\lambda_4 - \lambda_3) \quad (52)$$

Therefore,

$$u_1^* = \frac{\lambda_h S_h (\lambda_2 - \lambda_1)}{\theta_1} \quad (53)$$

$$u_2^* = \frac{S_h (\lambda_4 - \lambda_3)}{\theta_2} \quad (54)$$

Thus, we have that

$$u_1^* = \min \left\{ 1, \max \left(0, \frac{S_h \beta_1 (I_h + \eta C_r) (\lambda_2 - \lambda_1)}{2AT_h} \right) \right\}$$

$$u_2^* = \min \left\{ 1, \max \left(0, \frac{I_h (\lambda_4 - \lambda_3)}{2B} \right) \right\}$$

This completes the proof.

3.0 RESULTS AND DISCUSSION

3.1 Variables and Population Dependent Parameters Values

It is often necessary to determine the values of population-dependent parameters and model variables based on the epidemiology of Lassa fever and the

demographics of the affected community. Table 1 lists hypothetical values as well as Nigerian demographic values for the model's population-dependent parameters and variables. The reasons for each of these values are provided in detail, with the exception of the hypothetical variable values, which were assumed.

Table 1: Hypothetical, Nigeria (2021) Model Variables and population dependent parameters Values

S/N	Variable/ parameters	Nigeria Values
1	$S_h(t)$	202,713,887

2	$L_h(t)$	32,000
3	$I_h(t)$	16,000
4	$J_h(t)$	12,000
5	$R_k(t)$	20,000
6	$R(t)$	5,000
7	$S_r(t)$	10,946,000
8	$L_r(t)$	6,998,000
9	$I_r(t)$	4,000
10	$T_h(t)$	202,802,887
11	$T_r(t)$	11,000,000
12	μ_h	0.00005
13	μ_r	0.0009
14	Λ_h	10,140
15	Λ_r	3,666,300

3.2 Population Independent Parameters Value

Values for population independent factors must typically be determined using published data and the Lassa Fever epidemiology. The parameter values are listed in Table 2.

Table 4.2 The Values for Population independent parameters of the model

S/N	Parameter	Value	Unit	Source(s)
1.	β_1	0.5	-	Adewale et al. (2016)
2.	β_2	0.6	-	Adewale et al. (2016)
3.	δ_1, δ_2	0.01, 0.15	Individuals/day	WHO (2022)
4.	δ_r	0.5	Rodents/day	Assumed
5.	γ_1	0.04	Individuals/day	Obasi and Mbah (2019)
6.	γ_2	0.006	Individuals/day	Obasi and Mbah (2019)
7.	γ_3	0.37	Individuals/day	Obasi and Mbah (2019)
8.	σ_h	0.048	Individuals/day	Adewale et al. (2016)
9.	σ_r	0.7	Rodents/day	Assumed
10.	τ_1	0.05	Individuals/day	Obasi and Mbah (2019)
11.	τ_2	0.01	Individuals/day	Obasi and Mbah (2019)
12.	τ_3	(0,1)	-	Control parameter
13.	η	0.16	-	Adewale et al. (2016)
14.	ϕ	(0,1)	-	Control parameter

3.3 Validation and Extension of Analytical Results by Numerical Method

Because real issues frequently result in differential equations that cannot be precisely solved by simple analytical approaches, numerical methods of solving

differential equations are extremely important to engineers, epidemiologists, and physicists. Furthermore, certain differential equations have formulaic solutions that are so complex that it is preferable to use a numerical approach to solve them since numerical methods are unconstrained by these kinds of restrictions. To show that the numerical findings agree with the theoretical results, a numerical experiment is also carried out. In other words, it provides evidence for, expands upon, and clarifies the different theoretical (analytical) conclusions obtained. To get the numerical simulation of the model (1) to (9), we utilized the Maple program. Tables 1 and 2 provide the values for the initial variables and parameters. In order to validate our analytical results on the global stability of the disease-free equilibrium and demonstrate the global stability of the endemic equilibrium, we then presented some numerical simulations to track the dynamics of the full model for various values and associate it with the effective basic reproduction number.

3.4 Optimal Control Plotting

In order to reduce the cost of eliminating LFV in a society, model (25)–(33) must be simulated in order to ascertain the model's optimality.

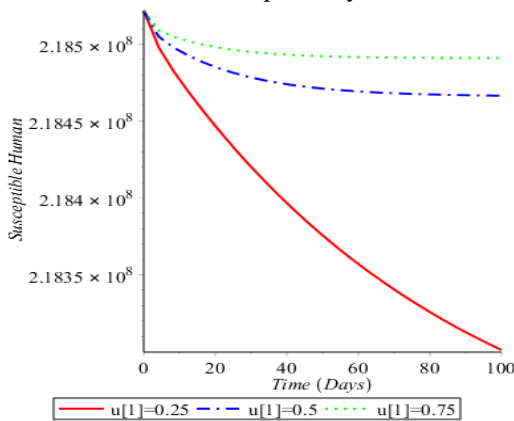


Fig. 2: is a comparison of effectiveness of the Susceptible humans' population. Variables and parameters used are as in Table 1 and 2

Fig. 2 clearly shows the level at which the susceptible humans vary the isolation rate. The increase of control measures put in place the increase in the number of individuals who are not infected but prone to being infected.

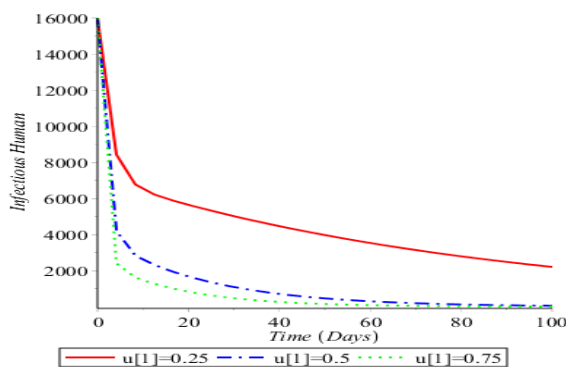


Fig. 3: is a comparison of effectiveness of the Infectious humans' population with different strategies. Variables and parameters used are as in Table 1 and 2

Fig. 3 shows the Infectious human varying the rate of isolation in preventing LFV, this clearly shows the reduction of number of cases of LFD in the society if proper isolation centers are put in place and proper and adequate care is giving to such patients. Increase in the number of infectious individuals who are isolated to a conducive atmospheric center leads to decrease in the spread of the virus in the society. It reaches to an extent where the disease will die out completely in the society i.e as the dotted lines are approaching zero. So, isolation is an important tool towards curbing the LFD in the society.

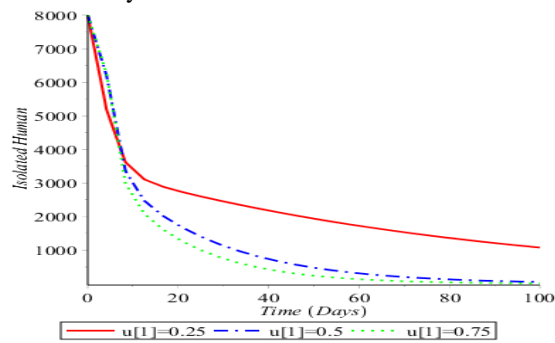


Fig. 4: is a comparison of effectiveness of the Isolated humans' population with different strategies. Variables and parameters used are as in Table 1 and 2

Fig. 4 shows the level of spread of the virus within the isolation center. Individuals who are isolated may likely experience a hike in the spread of the virus within the environment, but will with time scroll down and approaching zero. This may be due to proper measures put in place to curtail the deadly virus.

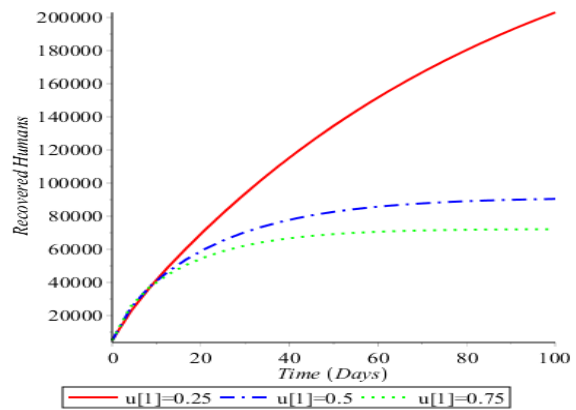


Fig. 5: is a comparison of effectiveness of the Recovered humans' population with different strategies. Variables and parameters used are as in Table 1 and 2

Table 5 presented a different recovery stage as a result of isolating the infectious individuals and proper control that include preventing measures and other control strategies are put in place.

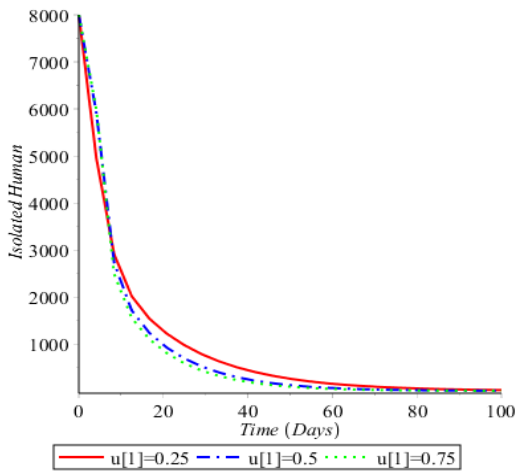


Fig. 6: shows impact of Isolation in humans' population with different strategies. Variables and parameters used are as in Table 1 and 2

This shows clearly the role played by ribavirin treatment towards curtailing the virus in the society. Treatment is one of the best control measures of curtailing a disease. Increase in the level of treatment administered to LFV patients leads to measure massive reduction of the spread and transmission of the virus. The graph shows how all controls are approaching zero and at the end of this level of control, the disease will die out completely in the society.

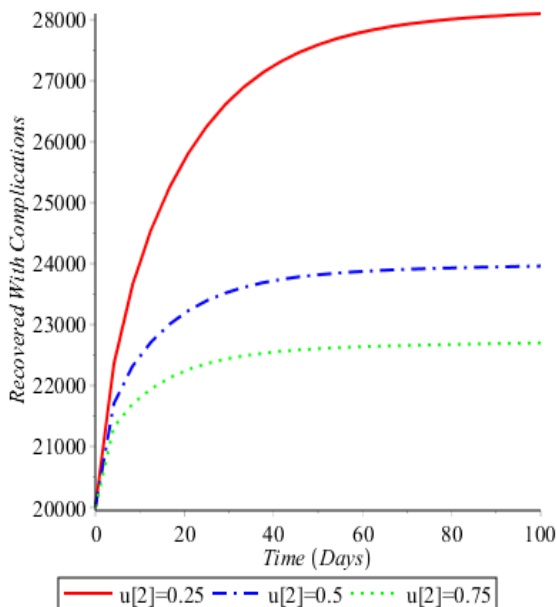


Fig. 7: is a comparison of effectiveness of the Infectious humans' population. Variables and parameters used are as in Table 1 and 2

When proper treatment measures are put in place then recovery is paramount. Increase in treatment leads to a rapid increase in the chances of a complete recovery. Despite some recoveries may come with complications, but normal and complete recovery may be ascertained.

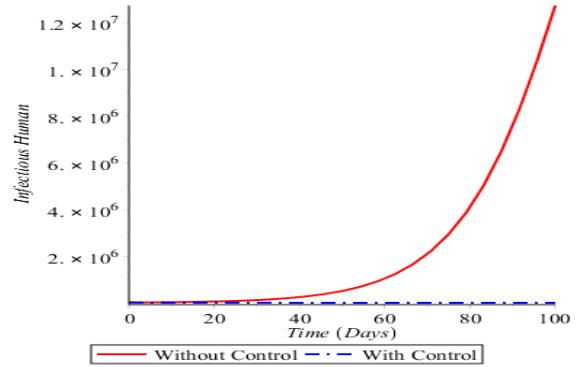


Fig. 8: is a comparison of without controls and the effectiveness of the two controls in infectious humans' population. Variables and parameters used are as in Table 1 and 2

Fig 8 is showing the comparison of without controls and the effectiveness of the two controls in Infectious humans' population. It is clear from the above figure that combining the two controls helps a lot in controlling the susceptible humans in the society. The red solid line is increasing since there is no control while the blue dash dot line is approaching zero which signifies the different control strategies if properly enhance will curtail the menace.

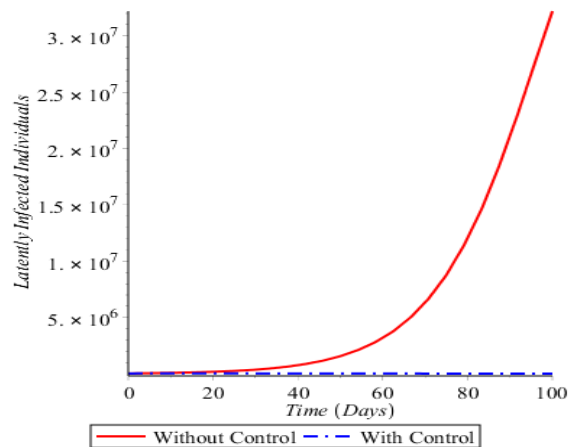


Fig. 9: is a comparison of without controls and the effectiveness of the two controls in latently infected humans' population. Variables and parameters used are as in Table 1 and 2

Fig 9 is showing the comparison of without controls and the effectiveness of the two controls in latently infected humans' population. It is clear from the above figure that combination of all the two controls helps a lot in controlling the latently infected humans in the society. The red solid line is increasing since there is no control while the blue dash dot line is approaching zero which signifies control is effective and the disease will completely die in the society.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

This work has been carried out, using deterministic model for the Lassa Fever Virus transmission

dynamics in a population. The model has various control strategies that could be implemented to reduce the transmission of Lassa fever in a society. The following findings and conclusions were made:

- i. Applying all the control parameters in this model simultaneously leads to a smooth and unique way of curbing Lassa Fever in a society.
- ii. The effort of preventing Lassa Fever in human population is a very vital optimal control parameter which leads to fast eradication of Lassa Fever in a society,
- iii. Reducing rodents population helps in Lassa fever mitigation in a society.

The following advancements in knowledge were made:

- i. The research has enhanced the models already in use on the dynamics of transmission and management of Lassa fever.
- ii. The research has demonstrated effective methods for managing Lassa fever in any community, and
- iv.

REFERENCES

- Abubakar, S. F. and Ibrahim, M. O. (2022). Optimal Control Analysis of Treatment Strategies of the Dynamics of Cholera. *Journal of Optimisation*, Hindawi. (2022), 1-26.
- Adewale, S.O., Olopade, I.A., Ajao, S.O., Adeniran, G.A. and Oyedemi, O.T. (2016). Mathematical Analysis of Lassa Fever model with Isolation. *Asian Journal of Natural & Applied Sciences*. Vol. 5(3), 47-57.
- Anorue, O.F. and Okeke, A.A (2020). Mathematical model for lassa fever transmission and control. *J. of mathematics and computer science*, 2020, 5(6);110-118.<http://www.sciencepublishinggroup.com/j/mcs>, doi:10.11648/j.mcs.20200506.13.
- Center for Disease Control and prevention (2004). Imported Lassa fever. *MMWR Morbidity and Mortality Wkly Rep.* 894-897.
- Center for Disease Control and prevention (2020). CDC methods for the establishment and management of public health rapid response teams for disease outbreaks. Atlanta: Centers for Disease Control and prevention. <https://www.cdc.gov/globalhealth/heal508.pdf>
- Onah I.S. and Collins O.C. (2020), Dynamical system analysis of a lassa fever model with varying socio-economic classes. *J. appl. Math.* 2020.
- James, T.O., Abdulrahman, S., Akinyemi, S. and Akinwande, N.I. (2015). Dynamics Transmission of Lassa Fever Disease. *International Journal of Innovation and Research in Educational Sciences*. Vol. 2, Issue 1, ISSN (online):2349-5219.
- Mahmoud, A.I. and Attila, D. (2021). A mathematical model for lassa fever transmission dynamics in a seasonal environment with a view to the 2017-20 epidemic in Nigeria. *J. of nonlinear analysis: Real world application* 60(2021)103310.www.elsevier.com/locate/nonrwa.<http://doi.org/10.1016/j.nonrwa.2021.103310>
- Marien J., Borremans B., Kourouma F., Baforday J., Rieger T., Gunther S., Magassouba N., Leirs H., Fichet-calvet E. (2019), Evaluation of rodent control to fight lassa fever based on field data and mathematical modelling. *Emerg. Microbes infections* 8(1), 640-649.
- Musa S.S., Zhao S., Gao D., Lin Q., Chowell G., He D. (2020), Mechanistic modelling of the large scale lassa fever epidemics in Nigeria from 2016 to 2019. *J. Theor. Biol.* 493, 110209 (2020).
- National Bureau of Statistics (NBS) 2019: An estimation made by the bureau in the yearly program.
- Obasi, C. and Mbah, G.C.E. (2019). On the stability analysis of a mathematical model of lassa fever disease dynamics. *Journal of the Nigerian society for mathematical biology*. Vol. 2 (2019), Pp. 135-144.
- Panja, P. (2019). "Optimal control analysis of a cholera epidemic model," *Biophysical Reviews and Letters*, Vol. 14, no. 01, Pp. 27 – 48.
- Pontryagin, L.S., Boltyanskii, V.G., et al. (1962). *The mathematical theory of optimal processes*. Interscience publishers john wiley & sons, inc., new York-london, translated from the Russian by K.N., Tirogoff.

- iii. The research has demonstrated the potential for a disease-free equilibrium that can be globally stable.

4.2 Recommendations

A preventive measure must be taken, and the fight against the pandemic must be intensified, as the introduction of the virus into a population poses a clear health risk. Therefore, our society would find the following recommendations helpful.

- i. Healthcare providers ought to follow infection prevention and control measures like washing their hands, donning gloves, and promptly reporting needle injuries.
- ii. People who live in high-risk areas ought to be informed about how to keep rodent populations in their homes to a minimum and steer clear of feeding them.
- iii. The government should create quicker diagnostic tools and expand access to ribavirin, the only effective treatment.



- Tara, K.H. (2004). Virology notes in lassa fever.
Retrieved on March 10, 2012 from
www.taraharper.com/vlass.html
- Wordometers (2019). Elaboration of data by United Nations, Department of Economics and Social Affairs, Population Division.
World Population Prospects.
www.worldometers.info.
- Center for Disease Control and prevention (2020). CDC methods for the establishment and management of public health rapid response teams for disease outbreaks. Atlanta: Centers for Disease Control and prevention.
<https://www.cdc.gov/globalhealth/health508.pdf>
- World Health Organization (21 feb, 2022), Disease outbreak News; Lassa fever united kingdom of Great Britain and Northern Ireland.
<https://www.who.int/emergencies/disease-outbreak-news/item/lassa-fever-united-kingdom-of-great-britain-and-northern-ireland>