

MHD Inclination Effects on Unsteady Basic Gaseous Micro-Flow Past on Sinusoidally Moving Horizontal Plate

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ABSTRACT

This study investigates the impact of magnetohydrodynamic (MHD) inclination on unsteady gaseous micro-flow past a sinusoidally moving horizontal plate in the slip flow regime. The exact solution of the momentum equation with the corresponding boundary conditions and the expression for skin friction are obtained and solved analytically. The effects of varying magnetic field inclinations, phase angle, Hartman number, driving force frequency, and Knudsen number have been considered. Numerical and graphical simulations show that MHD inclination significantly influences flow behavior, leading to velocity reductions and fluid friction enhancements. The findings of this research have important implications for designing and optimizing micro-scale devices, such as micro-pumps, micro-heat exchangers, and micro-valves, in engineering applications.

Keywords: MHD Inclination, Frequency of the Driving Force, Stoke's Second Problem Flow, Sinusoidal Velocity, Basic Gaseous Micro-Flows.

NOMENCLATURE

Kn	Knudsen number ($= \lambda / L$)
L	Reference length ($= \nu / u_o$)
S	Dimensionless suction/injection (S^+ for injection, while S^- for suction)
t	Time
t_r	Reference time ($= L^2 / \nu$)
U	Dimensionless axial velocity ($= u / u_o$)
u	Axial velocity (in x -direction)
u_o	Velocity of the moving wall
V	Complex solution function for velocity
W	Complex solution function for temperature
X	Dimensionless axial coordinate ($= x / L$)
x	Axial coordinate
Y	Dimensionless transverse coordinate ($= y / L$)

y Transverse coordinate

M Magnetic force

Greek symbols

λ Mean-free-path length

ρ Density

μ Dynamic viscosity

ν Kinematic viscosity

σ_v Tangential momentum accommodation coefficient ($= 0.7$)

ϖ Frequency

ω Dimensionless frequency ($= \varpi / \omega_r$)

ω_r Reference frequency ($= \nu / L^2$)

τ Dimensionless time ($= t / t_r$)

$\omega\tau$ Dimensionless phase angle

α MHD inclination angle

1.0 INTRODUCTION

Micro-flows' dynamics have earned significant attention in recent years due to their relevance in various industrial applications, including micro-electro-mechanical systems (MEMS), biomedical devices, and nanotechnology. Magnetohydrodynamic (MHD) effects can significantly influence the behavior of gaseous micro-flows, which are crucial in various applications such as micro-scale heat transfer, chemical processing, and biomedical devices. MHD inclination effects on unsteady gaseous micro-flow past a sinusoidally moving horizontal plate is a configuration that simulates the oscillatory motion of micro-mechanical components, and understanding

these effects is vital for designing and optimizing micro-scale devices such as micro-pumps, micro-sensors, and micro-actuators.

Some of the related literature includes the work of Haddad and Al-Nimr (2005) who analyzed the effect of frequency of fluctuating driven force on basic gaseous micro-flows in slip flow regime, where they considered four flow cases, namely; the transient Couette flow, the pulsating Poiseuille flow, the Stokes second problem, and natural convection. In their analysis, they found the slip in velocity and jump in temperature to increase with the increase in Knudsen number and/or frequency of the driving forces, though

the increase/decrease is found to be negligible for sufficiently small frequency of the driving forces and/or the Knudsen number. Moyi and Sani (2024) researched pulsating Poiseuille micro-gas flow with an MHD inclination effect. It is found from their study that velocity increases with small values of angle of inclination & magnetism and then decreases with higher values. Increasing values of phase angle and the Knudsen number are found to increase the velocity. Suction/injection effects of Pulsating Poiseuille fluctuating basic gaseous micro-flows were considered by Sani & Jibril (2021). The Effect of MHD on unsteady oscillatory Couette flow through porous media is analyzed by Sharma et al. (2022). Rajput & Kumar (2018) have worked on rotation and radiation effects on MHD flow past an inclined plate with variable wall temperature and mass diffusion in the

presence of Hall current. Numerical simulation of Stoke's second problem affected by magnetic field was considered by Ennayar et al. (2021).

Chamuah and Ahmed (2024) analyzed MHD flow past an impulsively started inclined plate with parabolic plate velocity in thermal diffusion and thermal radiation. Osman et al. (2022) have considered MHD free convective flow past an infinite inclined plate. MHD micropolar fluid flow and mixed convection features using entropy production analysis of an inclined porous sheet were considered by Sadighi et al (2022). Uddin & Murad (2021) considered analytical solutions to Stoke's second problem and oscillatory Couette flow for a two-layer fluid. The present work is intended to investigate the effect of MHD inclination on the fluctuating Stoke's second problem flow.

2.0 FORMULATION OF THE PROBLEM

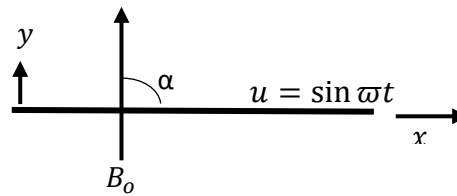


Figure 1: Schematic diagram

Consider an unsteady hydro-dynamically fully developed MHD flow of a basic micro-gas on a horizontally moving plate of an infinite length with MHD inclination effects, in the slip flow regime. A coordinate system is chosen such that the x -axis is parallel to the plate while the y -axis orthogonal to both. The plate is moving with sinusoidal velocity (*i.e.* $u = \sin(\omega t)$). The governing equation and its boundary conditions for this flow problem are given as follows:

Governing equation:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \beta_0^2 u \sin^2 \alpha}{\rho} \quad (1)$$

Boundary conditions :

$$u(t, \infty) = 0 \quad (2)$$

$$u(t, 0) - \sin(\omega t) = \frac{2 - \sigma_v}{\sigma_v} \lambda \frac{\partial u}{\partial y} \Big|_{y=0} \quad (3)$$

Introducing the following dimensionless variables:

$$\tau = \frac{\nu t}{L^2}, \omega = \frac{\omega L^2}{\nu}, Y = \frac{y}{L}, Kn = \frac{\lambda}{L}, U = \frac{u}{u_o}, \quad (4)$$

$$M^2 = \frac{L^2 \sigma \beta_0^2}{\mu},$$

We transform the above governing equation and boundary conditions to dimensionless form as follows:

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial Y^2} - M^2 U \sin^2 \alpha \quad (5)$$

$$U(\tau, \infty) = 0 \quad (6)$$

$$U(\tau, 0) - \sin(\omega \tau) = \frac{2 - \sigma_v}{\sigma_v} Kn \frac{\partial U}{\partial Y} \Big|_{Y=0} \quad (7)$$

3.0 ANALYTICAL SOLUTION

An exact solution for this problem is possible by assuming the following complex solution:

$$U(\tau, Y) = \text{Im}\{\exp(i\omega\tau)V(Y)\} \quad (8)$$

Where 'Im' represents the imaginary part of the complex solution and $i = \sqrt{-1}$. Differentiating Eq. (8) and substituting into the governing equation and boundary conditions, they are transformed into ordinary differential equations whose solutions are:

$$V(Y) = \frac{1}{1+a} \left(\exp(-Y\sqrt{K}) \right) \quad (9)$$

$$U(\tau, Y) = \text{Im}\left\{ \frac{1}{1+a} \exp(i\omega\tau - Y\sqrt{K}) \right\} \quad (10)$$

Where $K = M^2 \sin^2 \alpha + i\omega$,

$$a = \frac{2 - \sigma_v}{\sigma_v} Kn \sqrt{K} \quad (11)$$

From Eq. (11), the skin friction on the wall is obtained as follows:

$$\tau_0 = \frac{\partial U}{\partial Y} \Big|_{Y=0} = \text{Im} \left\{ \frac{-\sqrt{K}}{1+a} \exp(i\omega\tau) \right\} \quad (12)$$

4.0 DISCUSSION OF RESULTS

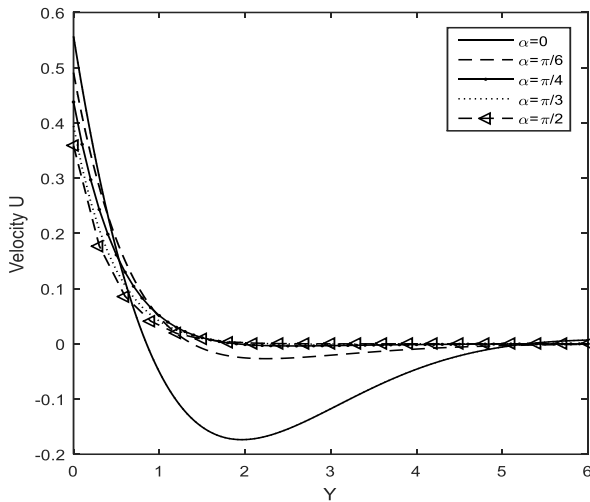


Fig. 2: Velocity profile when varying α .

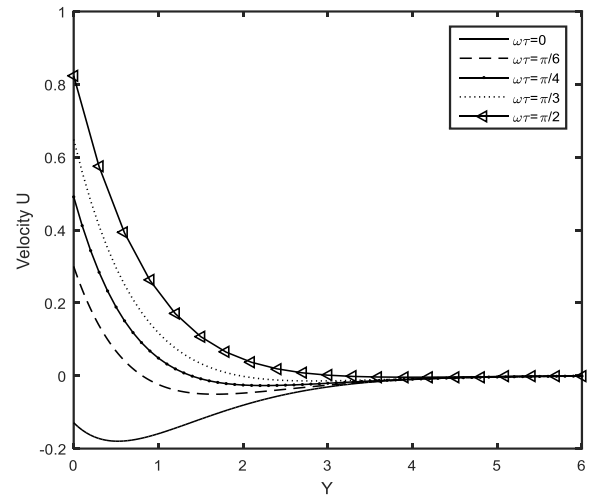


Fig. 4: Velocity profile when varying $\omega\tau$.

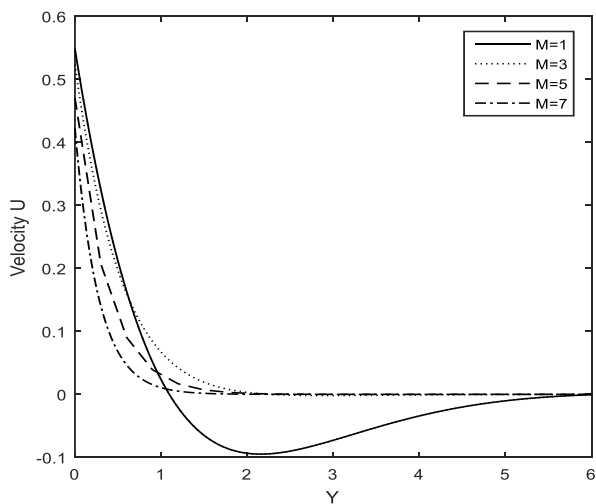


Fig. 3: Velocity profile when varying M

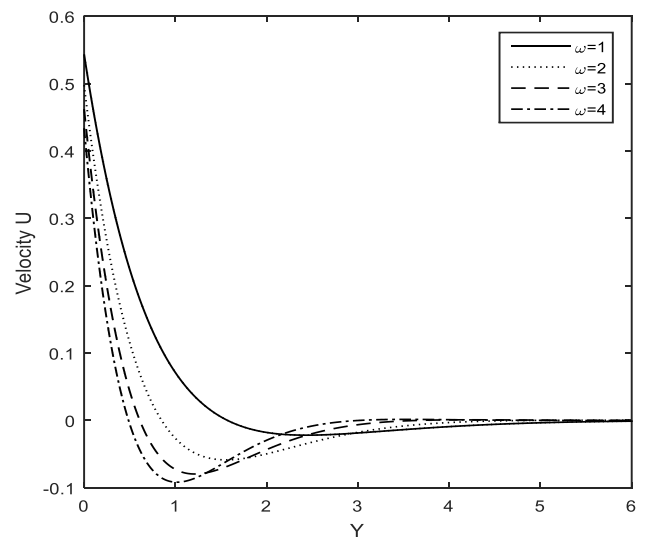


Fig. 5: Velocity profile when varying ω .

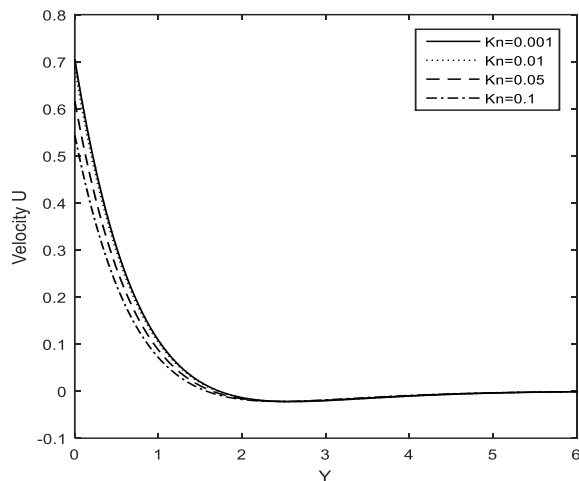


Fig. 6: Velocity profile when varying Kn

Table 1: Numerical values for velocity and skin friction

α	τ_0	U
0	-0.99868717	0.705475573
$\pi/18$	-1.00036143	0.705472463
$\pi/9$	-1.02386826	0.705428808
$\pi/6$	-1.09632350	0.705294248
$\pi/3$	-1.43794690	0.704659805
$\pi/2$	-1.59433894	0.704369362
M	τ_0	U
0	-0.99868717	0.705475573
1	-1.00607477	0.705461853
2	-1.09632350	0.705294248
3	-1.31201364	0.704893681
4	-1.59477808	0.704368547

Fig. 2 shows the MHD inclination effect on the velocity distribution in the flow. Velocity on the plate is found to be maximum as a result of the sinusoidal movement of the plate and diminishes to zero when the free stream is approached. Fig. 3 indicates magnetic effects on the velocity of the fluid. Increasing Magnetism reduces the velocity of the flow. Fig. 4 indicates the significance of varying phase angles. Increasing the phase angle is found to decrease the velocity of the fluid. Fig. 5 and Fig. 6 respectively indicate the effects of the frequency of the driving force and the Knudsen number, where both are found to decrease the velocity of the moving fluid.

Table 1 indicates the numerical values for velocity and skin friction. It is observed that skin friction increases with an increase in Magnetism and MHD inclination angle.

5.0 CONCLUSION

The effects of magnetohydrodynamic (MHD) inclination on unsteady gaseous micro-flow past a sinusoidally moving horizontal plate has been analyzed. It follows from the analysis that increasing the Magnetism, Angle of Inclination, Knudsen number, the frequency of the fluctuation, and the phase angle lead to a decrease in the velocity of flow; while increasing Magnetism and the inclination angle leads to an increase in skin friction.

6.0 Acknowledgment

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REFERENCES

- Chamuah, K., & Ahmed, N. (2024). MHD Flow Past an Impulsively Started Inclined Plate with Parabolic Plate Velocity in the Presence of Thermal Diffusion and Thermal Radiation. *Biointerface Research in Applied Chemistry*, 13(3), 1 - 18. <https://doi.org/10.33263/BRIAC143.060>
- Ennayar, H., Christian, K., & Boeck, T. (2021). Numerical simulation of Stokes' second problem affected by magnetic field. 10th International Conference on Electromagnetic Processing of Materials. Riga, Latvia.
- Haddad, O., & Al-Nimr, M. (2005). The effect of frequency of fluctuating driven force on basic gaseous micro-flows. *Acta-Mechanica*(179), 249-259.
- Moyi, A. U., & Sani, A. (2024). Pulsating Poiseuille micro-gas flow with MHD inclination effects. *Journal of Mathematical Problems, Equations and Statistics*, 5(1), 19-23. <https://doi.org/10.22271/math.2024.v5.i2a.144>
- Osman, H. I., Omar, N. F., Vieru, D., & Ismail, Z. (2022). A Study of MHD Free Convection Flow Past an Infinite Inclined Plate. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 99(1), 18 - 27. <https://doi.org/10.37934/arfm.92.1.1827>

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| <p>Rajput, U. S., & Kumar, G. (2018, June). Rotation and radiation effects on MHD flow past an inclined plate with variable wall temperature and mass inclined plate with variable wall temperature and mass. <i>Applications and Applied Mathematics: An International Journal</i>, 13(I), 484 - 495. Retrieved from http://pvamu.edu/aam</p> <p>Sadighi, S., Afshar, H., Jabbari, M., & Ashtiani, H. A. (2022). An analytical approach to entropy production in MHD mixed convection micropolar fluid flow over an inclined porous stretching sheet. <i>Frontiers in Mechanical Engineering</i>, 1 -12. https://doi.org/10.3389/fmech.2022.900316</p> | <p>Sani, A., & Jibril, H. M. (2021). Suction/Injection Effects on Pulsating Poiseuille Basic Gaseous Fluctuating-Micro-Flows. <i>International Journal of Science for Global Sustainability</i>, 7(4), 31-38.</p> <p>Sharma, B. K., Sharma, P. K., & Chauhan, S. K. (2022). Effect of MHD on Unsteady Oscillatory Couette Flow Through Porous Media. <i>Int. J. of Applied Mechanics and Engineering</i>, 27(1), 188 - 202. https://doi.org/10.2478/ijame-2022-0012</p> <p>Uddin, M., & Murad, A. (2022). Stokes' Second Problem and Oscillatory Couette Flow for a Two-Layer Fluid: Analytical Solutions. <i>Alexandria Engineering Journal</i>, 61(12), 10197 - 10218. https://doi.org/10.1016/j.aej.2022.03.023</p> |
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