

## Modelling the Role of Super-Spreaders on the Dynamic of Rumor

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### ABSTRACT

In this study a variant of the model of Borge-Holthoefer *et al* (2012) on rumor dynamics is formulated, we investigated that two equilibria of the model exist namely: rumor-free equilibrium (RFE) and endemic (rumor) equilibrium. Using the next-generation matrix, we computed the rumor reproduction potential ( $R_p$ ). The local stability analysis of rumor-free equilibrium was done in terms of  $R_p$ . Numerical simulation shows that rumors can spread if certain parameters are not reduced.

**Keywords:** Local Stability, Equilibrium, Rumor, Equilibrium, Reproduction potential

### 1.0 INTRODUCTION

The social phenomenon is part of the ecology of humans as it plays a significant role in a variety of ways in human relationships. Communication is a key parameter that drives human/non-human interaction daily from one geographical region to another. Different forms of communication such as signs, verbal, and writing account for this dispersal of information. This can happen consciously or unconsciously as the case may be (Guo *et al*, 2020). However, alongside these, not only needed or useful information is passed, but both true and untrue information that appears to function in its less pure state (non-authentic source) are also passed. When such a scenario occurs, it leads to several pieces of information based on rumor.

A rumor is a story or statement in general circulation without confirmation or certainty as facts; it can come in the form of gossip; or hearsay (Wu *et al*, 2019). It may be identified as a subject of propaganda (Bode *et al*, 2019). Sometimes, rumor may be referred to as “misinformation” and “disinformation” in terms of false and deliberately false as the case may be. It can usually induce a cloud of pressure, anxiety, and panic and can even cause death. For example, More than 6000 people were hospitalized due to COVID19 related misinformation in the first three months of 2020 (Pulido *et al*, 2020). Although rumor has varying negative effects on the lives of people in some cases it is useful in providing safety information to others (Wu *et al*, 2019). The spread of rumors can be controlled by checking the activities of media by scrutinizing the information before passing it to the public (Li *et al*, 2019). Influencing the belief system of people and the introduction of antithesis for the circulation of information are also found essential. Also, the screening of individuals on the entry and exit

borders of a community can be useful in curbing the unnecessary spread of false information Paek *et al*, 2019; Wang *et al*, 2019).

It is interesting to understand that the propagation of rumor is likened to the spread of epidemic (Moravec *et al*, 2019; Pulido *et al*, 2020). Thus, the spread of rumor is modeled through the modification of some standard epidemiological models to which many mathematical models on rumor spread have been developed. Rumor transmission is one of the most natural forms of social communication (Kim *et al*, 2019); many researchers have successfully proposed some basic models for rumor propaganda, some of which are (Daley and Kendall, 1965) investigated the spread of information.

Our model is an extension of the (Borge-Holthoefer *et al*. 2012) model of three classes, “Ignorant”, “Spreader” and “stiflers”. However, incorporating the fourth class called superspreaders; a 4-dimension and deterministic model is developed to trace the role of superspreaders on the dynamic of rumor

### 1.3 Aim and Objectives of the Study

The aim of this study is to model the role of super-spreaders on the dynamic of rumor

The specific objectives are to:

- extend the model of Borge-Holthoefer *et al*. (2012) by incorporating the fourth class called superspreaders.
- compute the effective reproduction number for the extended model using next generation method;
- investigate the existence and stability of the disease-free and endemic equilibrium states of the extended model;

- iv. carry out numerical simulation of the extended model;
- v. interpret results in (ii)-(iv) in biological terms

**2.0 METHODOLOGY**

We revisit a mathematical model of rumor transmission by Borde-Haalthoefer *et al.* (2012). We analyzed the extended model by the method of linearization.

**2.1 The Basic Model**

We present the existing model proposed by Borde-Haalthoefer *et al.* (2012) together with its assumptions and parameters. **Assumptions of the Basic Model**

- i. Rumor is constant with variable population size.
- ii. We assume that nodes in a network still have memory of their potential neighbor and that not all the links of a given node are concurrently active.
- iii. We assume that spreaders are not active in the first model whereas in the second model, the ignorant are not interested in spreading the rumor.

**2.1.1 Variables and parameters of the basic model.**

The variable and parameters of the existing model are represented in the **Table 1** as follows:

**Table 1: The variables and parameters of the basic model.**

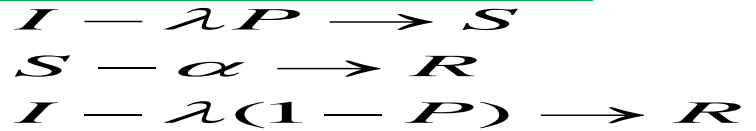
Variables/Parameters	Description
$\rho(t)$ :	Density of spreaders size at time t
$r(t)$ :	Density of ignorant size at time t
$I(t)$ :	Density of stiflers size at time t
$r_\infty$ :	Final density of stiflers in the network
$\alpha$ :	Varying probability
$\lambda$ :	Fixed probability
$k$ :	Number of contact each spreader has per unit time
N	Number of nodes
S	Number of simulations

**2.2 Basic Model Equations**

The model is described by the following system of partial differential equation.

$$\begin{aligned} \frac{\partial r(t)}{\partial t} &= -\lambda p k p(t) i(t) - (1 - p) \lambda k p(t) i(t) \\ \frac{\partial p(t)}{\partial t} &= \lambda p k p(t) i(t) - \alpha \lambda k p(t) + r(t) \\ \frac{\partial i(t)}{\partial t} &= \alpha \lambda k p(t) i - r(t) + (1 - p) \lambda k p(t) i(t) \end{aligned}$$

The transitions allowed by the model are:



**2.2.1 The New Model**

The new model presented in this section is based on the following assumptions and parameters as given below:

**2.2.2 Assumptions of the New Model**

The following assumptions are made about the new model:

- i. The influx of ignorant population is constant
- ii. Immigration and emigration take place at a constant rate.
- iii. We assume that no transition of rumor class happens unless a spreader and super-spreader contact someone.
- iv. The natural death rate takes place at every class.

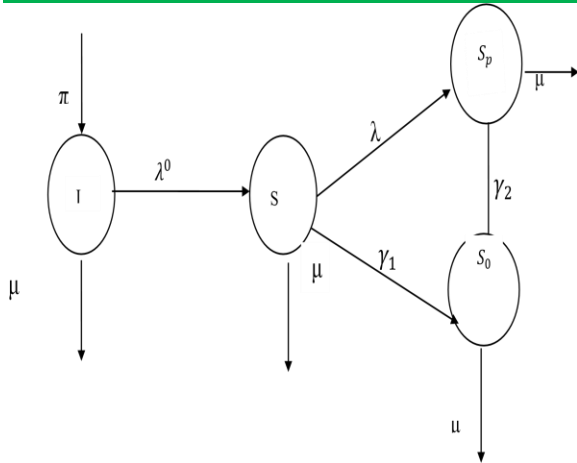
Variable and Parameters of New Model The variable and parameters of the existing model are represented in the **Table 2** as follows;

**Table 2: The variable and parameters of new model**

Variable/parameter	Description
$I(t)$	Number of ignorant at time t
$S(t)$	Number of spreaders at time t
$S_0(t)$	Number of stiflers at time t
$S_p(t)$	Number of super-spreaders at time t
$N$	Total individual population
$\beta$	The rate of spreading the rumor
$\lambda$	Progressive rate of spreaders to super-spreaders
$\gamma_1$	Rate by which spreaders become stiflers
$\gamma_2$	Rate which super-spreaders become tired of rumor spread
$\mu$	Natural death rate
$\Psi$	The transmission rate of super-spreaders
$\pi$	Influx of ignorant population

**2.2.3 The Equations of the New Model**

The formulation of the model below is based on our earlier assumptions and the model flow diagram is described in figure (1).



**Figure 1:** The flow diagram of rumor transmission dynamics

The new model is described by the following system of differential equation

$$\begin{aligned} \frac{dI}{dt} &= \pi - \lambda^0 I - \mu I \\ \frac{dS}{dt} &= \lambda^0 I - (\lambda + \gamma_1 + \mu) S \\ \frac{dS_p}{dt} &= \lambda S - (\gamma_2 + \mu) S_p \\ \frac{dS_o}{dt} &= \gamma_1 S + \gamma_2 S_p - \mu S_o \end{aligned} \tag{3.1}$$

where;

$$\lambda^0 = \beta \frac{S_p + \psi S}{N}$$

(Force of rumor spread)

### 2.3 Model Analysis

We intend to investigate the existence and local stability of Rumor-free equilibrium (RFE) state of the model by linearized approach.

#### 2.3.1. Existence of Model Equilibrium State

At equilibrium, equations (3.1) are independent of time, this means

$$0 = \pi - \lambda^0 I - \mu I \tag{3.2}$$

$$0 = \lambda^0 I - (\lambda + \gamma_1 + \mu) S \tag{3.3}$$

$$0 = \lambda S - (\gamma_2 + \mu) S_p \tag{3.4}$$

$$0 = \gamma_1 S + \gamma_2 S_p - \mu S_o \tag{3.5}$$

Given,

$$\frac{dI}{dt} = \frac{dS}{dt} = \frac{dS_p}{dt} = \frac{dS_o}{dt} = 0,$$

At rumor-free equilibrium, we assume that there is no spread. Hence, we have

$$S = S_p = S_o = 0$$

from equation (3.2), we have

$$I = \frac{\pi}{(\lambda^0 + \mu)} \tag{3.6}$$

$$S = \frac{\lambda^0 I}{\mu + \gamma_1 + \lambda} = \frac{\lambda^0 \pi}{(\mu + \gamma_1 + \lambda)(\lambda^0 + \mu)} = \tau_1 \frac{\lambda^0}{(\lambda^0 + \mu)} \tag{3.7}$$

$$S_p = \frac{\lambda S}{\mu + \gamma_2} = \frac{\lambda \pi}{\mu + \gamma_2(\mu + \gamma_1 + \lambda)} \cdot \frac{\lambda^0}{(\lambda^0 + \mu)} = \tau_2 \frac{\lambda^0}{(\lambda^0 + \mu)}$$

$$S_o = \frac{\gamma_1 S}{\mu} + \frac{\gamma_2 S_p}{\mu} = \frac{\gamma_1}{\mu} \cdot \tau_1 \frac{\lambda^0}{(\lambda^0 + \mu)} + \frac{\gamma_2}{\mu} \cdot \tau_2 \frac{\lambda^0}{(\lambda^0 + \mu)} = \tau_3 \frac{\lambda^0}{(\lambda^0 + \mu)} \tag{3.8}$$

$$\lambda^0 = \beta \frac{S_p + \psi S}{N} \tag{3.9}$$

but the sum of  $S_p$  and  $S$  is given as

$$S_p + S = \psi \tau_1 + \tau_2 \frac{\lambda^0}{(\lambda^0 + \mu)}$$

And

$$N = I + S + S_p + S_o$$

$$\begin{aligned}
 &= \frac{\pi}{\lambda^0 + \mu} + \tau_1 \frac{\lambda^0}{\lambda^0 + \mu} + \tau_2 \frac{\lambda^0}{\lambda^0 + \mu} + \tau_3 \frac{\lambda^0}{\lambda^0 + \mu} \\
 &= \frac{\pi}{\lambda^0 + \mu} + \tau_1 + \tau_2 + \tau_3 \frac{\lambda^0}{\lambda^0 + \mu} \\
 &= \frac{1}{\lambda^0 + \mu} \pi + \tau_1 + \tau_2 + \tau_3 \lambda^0
 \end{aligned}$$

Reputing into the rumor spread force,

$$\lambda^0 = \frac{\beta \psi \tau_1 + \tau_2 \frac{\lambda^0}{\lambda^0 + \mu}}{\pi + \tau_1 + \tau_2 + \tau_3 \frac{\lambda^0}{\lambda^0 + \mu}}$$

$$\lambda^0 = \frac{\beta \psi \tau_1 + \tau_2 \lambda^0}{\pi + \tau_1 + \tau_2 + \tau_3 \lambda^0}$$

$$\lambda^0 - \frac{\beta \psi \tau_1 + \tau_2 \lambda^0}{\pi + \tau_1 + \tau_2 + \tau_3 \lambda^0} = 0$$

$$1 - \frac{\beta \psi \tau_1 + \tau_2 \lambda^0}{\pi + \tau_1 + \tau_2 + \tau_3 \lambda^0} \lambda^0 = 0 \quad (3.10)$$

either

$$\lambda^0 = 0 \quad (3.11)$$

or

$$1 - \frac{\beta \psi \tau_1 + \tau_2 \lambda^0}{\pi + \tau_1 + \tau_2 + \tau_3 \lambda^0} = 0 \quad (3.12)$$

Substituting  $\lambda^0 = 0$  into (3.6) to (3.9) gives the rumor-free equilibrium (RFE) denoted as

$$E_0 = \left( \frac{\pi}{\mu}, 0, 0, 0 \right)$$

### Calculation of the Rumor Reproduction Potential ( $R_p$ )

We will compute  $R_p$  using the next generation matrix as applied in Vanden Driessche and Watnough (2002). We will first of all re-arrange the system in (3.1).

$$\frac{ds}{dt} = \lambda^0 I - (\lambda + \gamma_1 + \mu) \quad (3.13)$$

$$\frac{ds_p}{dt} = \lambda S - (\gamma_2 + \mu) S_p \quad (3.14)$$

$$\frac{dI}{dt} = \pi - \lambda^0 I - \mu I \quad (3.15)$$

$$\frac{dS_0}{dt} = \gamma_1 S + \gamma_2 S_p - \mu S_0 \quad (3.16)$$

Since there are two class of individuals that enforce the spread of rumor,

$$f = \begin{pmatrix} \beta \frac{S_p + \psi S}{N} \\ 0 \end{pmatrix} \quad \text{and} \quad u = \begin{pmatrix} (\mu + \gamma_1 + \lambda) S \\ -\lambda S + (\mu + \gamma_2) S_p \end{pmatrix}$$

from which we get

$$F = \begin{pmatrix} \beta \psi & \beta \\ 0 & 0 \end{pmatrix} \quad U = \begin{pmatrix} \mu + \gamma_1 + \lambda & 0 \\ -\lambda & \mu + \gamma_2 \end{pmatrix}$$

with

$$U^{-1} = \begin{pmatrix} \frac{1}{\lambda + \gamma_1 + \mu} & 0 \\ \frac{1}{\lambda} & \frac{1}{\mu + \gamma_2} \end{pmatrix}$$

Therefore, the next generation matrix  $FU^{-1}$  is given by

$$\begin{aligned} FU^{-1} &= \begin{pmatrix} \beta\psi & \beta & \frac{1}{\lambda + \gamma_1 + \mu} & 0 \\ 0 & 0 & \frac{1}{\lambda} & \frac{1}{\mu + \gamma_2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\beta\psi}{\lambda + \gamma_1 + \mu} & \frac{\beta\lambda}{\mu + \gamma_2 (\lambda + \gamma_1 + \mu)} & \frac{\beta}{\mu + \gamma_2} & 0 \end{pmatrix} \end{aligned} \quad (3.17)$$

The eigenvalues of matrix (3.17) are obtained from

$$\begin{aligned} \det FU^{-1} - \omega I &= 0 \\ &= \begin{vmatrix} \frac{\beta\psi}{\lambda + \gamma_1 + \mu} - \omega & \frac{\beta\lambda}{\mu + \gamma_2} \\ 0 & \frac{\beta}{\mu + \gamma_2} - \omega \end{vmatrix} = 0 \end{aligned}$$

or

$$\omega^2 - \frac{\beta\psi}{\lambda + \gamma_1 + \mu} \omega + \frac{\beta\lambda}{\mu + \gamma_2} \omega = 0 \quad (3.18)$$

from which we have

$$\omega = 0 \quad \text{or} \quad \omega = \frac{\beta\psi}{\lambda + \gamma_1 + \mu} + \frac{\beta\lambda}{\mu + \gamma_2 (\lambda + \gamma_1 + \mu)}$$

Since rumor Reproductive Potential ( $R_p$ ) is the largest eigenvalue of  $FU^{-1}$  we get

$$R_p = \rho FU^{-1} = \max_{1 \leq i \leq 2} w_i$$

Hence

$$R_p = \frac{\beta\psi}{\lambda + \gamma_1 + \mu} + \frac{\beta\lambda}{\mu + \gamma_2 (\lambda + \gamma_1 + \mu)} = R_{p1} + R_{p2},$$

where;

$$R_{p1} = \frac{\beta\psi}{\lambda + \gamma_1 + \mu} \text{ is the rumor reproduction potential generated by spreaders}$$

$$R_{p2} = \frac{\beta\lambda}{\mu + \gamma_2 (\lambda + \gamma_1 + \mu)} \text{ is the rumor reproduction potential generated by superspreaders}$$

### Lemma 1

The rumor reproduction potential for the rumor-free equilibrium state is given to be

$$R_p = \frac{\beta\psi}{\lambda + \gamma_1 + \mu} + \frac{\beta\lambda}{\mu + \gamma_2 (\lambda + \gamma_1 + \mu)}$$

Local Stability of Rumor-free Equilibrium State

In order to determine the local stability of Rumor-free equilibrium state (RFE), the following theorem may be found useful

**Theorem:** Stability criteria (Colwall and Hua, 1994)

Let  $x$  be a solution of  $\dot{x} = Ax$  and also let  $\det A \neq 0$  and  $\xi_i$  be the eight value of  $A$ .  $x$  is asymptotically stable if all  $\xi_i$  are real and negative. To achieve this, we linearize our system of equations (3.1).

Let

$$f_1 = \pi - \beta \frac{sp + \varphi S}{N} I - \mu I$$

$$f_2 = \beta \frac{sp + \varphi S}{N} I - \lambda + \gamma_1 + \mu S$$

$$f_3 = \lambda S - (\gamma_2 + \mu) Sp$$

$$f_4 = \gamma_1 S + \gamma_2 Sp - \mu S_0$$

Then,

The partial derivatives are given as follows

$$\frac{\partial f_1}{\partial I} = -\beta \frac{sp + \varphi S}{N} - \mu, \frac{\partial f_1}{\partial S} = -\beta \psi \frac{1}{N}, \frac{\partial f_1}{\partial sp} = -\beta \frac{1}{N}, \frac{\partial f_1}{\partial S_0} = 0$$

$$\frac{\partial f_2}{\partial I} = \beta \frac{sp + \varphi S}{N}, \frac{\partial f_2}{\partial S} = \beta \psi \frac{1}{N} - \gamma_1 - \mu - \lambda, \frac{\partial f_2}{\partial sp} = -\beta \frac{1}{N}, \frac{\partial f_2}{\partial S_0} = 0$$

$$\frac{\partial f_3}{\partial I} = 0, \frac{\partial f_3}{\partial S} = \lambda, \frac{\partial f_3}{\partial sp} = \gamma_2 - \mu, \frac{\partial f_3}{\partial S_0} = 0$$

$$\frac{\partial f_4}{\partial I} = 0, \frac{\partial f_4}{\partial S} = \gamma_1, \frac{\partial f_4}{\partial sp} = \gamma_2, \frac{\partial f_4}{\partial S_0} = -\mu$$

Using the above partial derivations, is given by

$$J = \begin{pmatrix} -\beta \frac{sp + \varphi S}{N} - \mu & -\beta \psi \frac{1}{N} & -\beta \frac{1}{N} & 0 \\ -\beta \frac{sp + \varphi S}{N} & \beta \psi \frac{1}{N} - (\lambda + \gamma_1 + \mu) & \beta \frac{1}{N} & 0 \\ 0 & \lambda & -(\gamma_2 + \mu) & 0 \\ 0 & \gamma_1 & \gamma_2 & -\mu \end{pmatrix} \quad (3.19)$$

At Rumor-Free Equilibrium the Jacobian matrix (3.19) becomes

$$\begin{bmatrix} -\mu & -\beta \Psi & -\beta & 0 \\ 0 & -\beta \Psi - (\lambda + \gamma_1 + \mu) & -\beta & 0 \\ 0 & \lambda & -(\gamma_2 + \mu) & 0 \\ 0 & \gamma_1 & \gamma_2 & -\mu \end{bmatrix}$$

With the corresponding characteristic equation defined by  $\det(J_0 - \xi I) = 0$

$$-\mu - \xi^2 \det B = 0 \quad (3.20)$$

where;

$$B = \begin{bmatrix} \beta \psi - (\lambda + \gamma_1 + \mu) & \beta \\ \lambda & -\gamma_2 + \mu \end{bmatrix}$$

Is obvious from (3.20) that  $\xi_1 = \xi_2 = -\mu$  is a negative eigenvalue. Therefore, the nature of the remaining eigenvalues is determined by considering matrix B using trace determinant approach.

$$\begin{aligned} \text{Trace } B &= \beta \psi - \lambda + \gamma_1 + \mu - (\mu + \gamma_2) \\ &= \lambda + \gamma_1 + \mu \frac{\beta \psi}{\lambda + \gamma_1 + \mu} - 1 - (\mu + \gamma_2) \\ &= \lambda + \gamma_1 + \mu R_{p1} - 1 - \mu + \gamma_2 < 0 \text{ if } R_{p1} < 1 \end{aligned}$$

and

$$\begin{aligned} \det B &= -\mu + \gamma_2 + \mu + \lambda + \gamma_1 + \mu - \beta \lambda \\ &= -\beta \lambda - \mu + \gamma_2 + \mu + \lambda + \gamma_1 + \mu \\ &= \mu + \gamma_2 \frac{\lambda + \gamma_1 + \mu}{\mu + \gamma_2} - \beta \lambda + 1 - \frac{\beta \psi}{\lambda + \gamma_1 + \mu} \\ &= \mu + \gamma_2 \frac{\lambda + \gamma_1 + \mu}{\lambda + \gamma_1 + \mu} 1 - R_{p1} > 0 \text{ if } R_{p1} > 1 \end{aligned}$$

**Lemma 2**

The Rumor-Free Equilibrium (RFE) is locally asymptotically stable if  $R_{p1} < R_{p2} < 1$  and unstable if  $R_{p2} > R_{p1} > 1$

**Existence of endemic equilibrium**

In this section we shall prove the following result.

**Lemma 3**

The endemic equilibrium of the model (3.6) - (3.9) exist if  $R_p > 1$

**Proof**

In order to determine the existence of endemic equilibrium at the point  $\lambda^0 \neq 0$ , we can recall from (3.10) that

$$\pi + \tau_1 + \tau_2 + \tau_3 \lambda^0 - \beta \psi \tau_1 + \tau_2 = 0$$

$$\beta \psi \tau_1 + \tau_2 - \tau_1 + \tau_2 + \tau_3 \lambda^0 = \pi$$

from which we get

$$\lambda^0 = \frac{\pi}{\beta \psi \tau_1 + \tau_2 - \tau_1 + \tau_2 + \tau_3} = \frac{\pi}{R_p - \tau_1 + \tau_2 + \tau_3}$$

this follows that

$$\lambda^0 = \frac{\pi}{R_p} \frac{1}{1 - \frac{\tau_1 + \tau_2 + \tau_3}{R_p}} > 0 \text{ if } R_p > 1$$

Thus, we have the following rumor equilibrium given by

$$I^{**} = \frac{\pi}{\lambda^0 + \mu}$$

$$S^{**} = \tau_1 \frac{\lambda^0}{\lambda^0 + \mu}$$

$$S_p^{**} = \tau_2 \frac{\lambda^0}{\lambda^0 + \mu} \text{ and}$$

$$S_0^{**} = \tau_3 \frac{\lambda^0}{\lambda^0 + \mu}$$

Where;

$$\tau_1 = \frac{\pi}{\mu + \gamma_1 + \lambda}, \tau_2 = \frac{\lambda \pi}{\mu + \gamma_2}, \text{ and } \tau_3 = \frac{\gamma_1}{\mu} \frac{1}{\mu + \gamma_1 + \lambda} + \frac{\gamma_2}{\mu} \frac{\lambda}{\mu + \gamma_2} \pi$$

**2.4 Numerical Simulation**

In this section, we carry out the numerical simulation of the model using Runge-Kutta method. This is coded in MATLAB. (see the appendix). The parameter symbols coded in the program are given in the table below.

**Table 3: Parameters in MATLAB code**

Parameter	MATLAB code
$\beta$	Beta
$\gamma_1$	Gam1
$\gamma_2$	Gam2
$\mu$	Mu
$\Psi$	Phi
$\pi$	Pi
$I(0)$	n(1)

$S(0)$	n(2)
$S_p(0)$	n(3)
$S_0(0)$	n(4)

The corresponding model parameters are given in the next table

**Table 4: Parameters for the numerical solution**

Experiment/Parameter	1	2	3	4
$\beta$	0.0	0.2	0.5	0.8
$\gamma_1$	0.0833 3	0.0833 3	0.0833 3	0.0833 3
$\gamma_2$	0.0833 3	0.0833 3	0.0833 3	0.0833 3
$\Psi$	0.2	0.2	0.2	0.2
$\pi$	10,000	10,000	10,000	10,000

$\lambda$	0.3	0.3	0.3	0.3
$I(0)$	500	500	500	500
$S(0)$	500	500	500	500
$S_p(0)$	500	500	500	500
$S_0(0)$	500	500	500	500

**3.0 RESULTS AND DISCUSSION**

Here we will present both some analytical and numerical results of our findings; the numerical computations were performed using computer programs written in MATLAB based on the model parameters in the Table 4.1. We shall discuss our results based on our findings.

**3.1 Results Analytical Results**

From the linearized stability analysis, the following results were obtained.

**Lemma 1**

The rumor reproduction potential for the rumor-free equilibrium state is given to be

$$R_p = \frac{\beta\psi}{\lambda + \gamma_1 + \mu} + \frac{\beta\lambda}{\mu + \gamma_2 (\lambda + \gamma_1 + \mu)}$$

**Lemma 2**

The Rumor-Free Equilibrium (RFE) is locally asymptotically stable if  $R_{p1} < R_{p2} < 1$  and unstable if  $R_{p2} > R_{p1} > 1$

**Lemma 3**

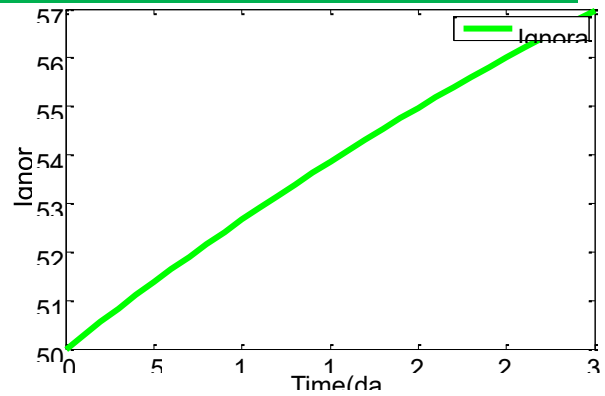
The endemic equilibrium of the model (3.6) - (3.9) exist if  $R_p > 1$ .

**3.2 Numerical Results**

Based on the numerical simulation done, the following graphs and discussion of numerical results of the model were presented.

**Figure 2**

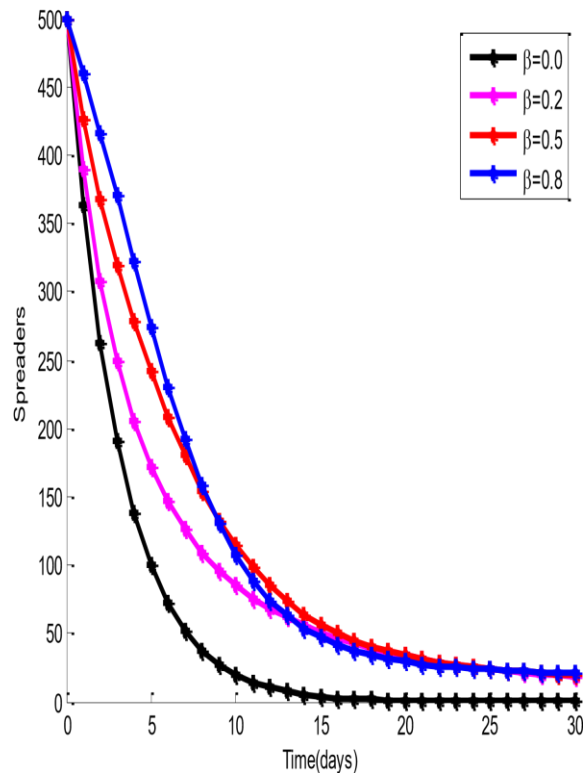
In figure 2 we seek to know the number of ignorant people against time when the passage of rumor stands at zero level. As observed above, the numbers of ignorant increases as people are unaware of the rumor. This shows that at rumor free environment people are not exposed to any rumor, or it is because before information goes, they must be total scrutiny to avoid leak of untrue information.



**Figure 2: The graph of Ignorants against time in Rumor- Free State**

**Figure 3**

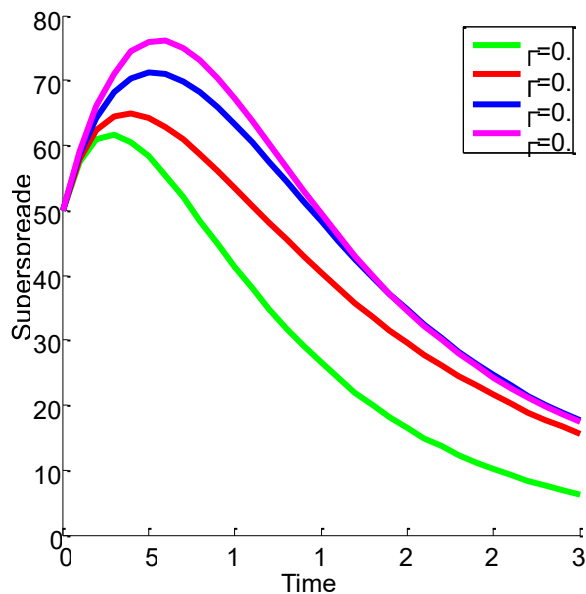
Here it can be observed that as the rate of rumor spreaders increases, the population of exposed individuals shows rapid decrease as  $\beta$  changes from 0.0 to 0.2, 0.5 and 0.8. The decreases in the exposed population could be due to early detection as spreaders enter the spreader class. The decrease can also be because the individual confirms the falsehood of the rumor transmission as a result only few individuals go out infected.



**Figure 3: The graph of spreaders against time in rumor dynamics.**

**Figure 4**

We realized that the number of super-spreaders increases steadily as rate increases. This is due to early detection of the rumor transmission. It can also be observed that as  $\beta$  increases from 0.0 to 0.2, 0.5, and 0.8, the rumor rises up steadily for some number of days and then drops and remain nearly stable. This could be due to the greater number of awareness people have about the rumor as days pass by.



**Figure 4: The graph of super-spreaders against time in rumor dynamic**

#### 4.0 CONCLUSION

In this research we studied a variant of the mathematical model of rumor by Borge-Hoalhoefter *et al.* (2012). The model has two equilibria - the rumor-free equilibrium and the rumor equilibrium. Both equilibria have been studied to be locally asymptotically stable in terms of the rumor reproduction potential. The model equations were solved using MATLAB and graphical solutions were similar in pattern to those obtained from previous work. We have shown that we can control the spread of rumor by reducing contact with spreaders or increasing stifter rate and by scrutinizing the information before passing it to the public. We recommend that government and media agent as well social networks device mean of controlling the flow of information to avoid multiplication of rumors which can generate panic in the society. This can be achieved through quality public enlightenment as well as proper and timely transmission of information.

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