

Mathematical Modelling and Transmission on the Dynamic of Syphilis with Exposed, Early and Late Latent Syphilis

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ABSTRACT

This research presents a Mathematical Modelling and Transmission on the Dynamic of Syphilis with Exposed, Early and Late Latent Syphilis. The model is governed by a system of six ordinary differential equations namely: Susceptible (S), Exposed (E), Early latent (L_E), Late latent (L_L), Infected (I) and Recovery (R). An analysis was performed on the model to determine the disease-free equilibrium, endemic equilibrium, reproduction number, local stability, and global stability. The results of the analysis indicate that the disease-free equilibrium equation is locally asymptotically stable if the reproduction number is less than one, and unstable if the reproduction number is greater than one. The system has met all the requirements to be considered globally asymptotically stable. We have conducted numerical simulations of the model, and the results show that the number of people who are exposed will keep increasing. It will only decrease if people become aware of the syphilis problem in the community. If a person is mildly infected with a virus but shows no symptoms, they can still spread the virus and cause harm. However, if the infected person gets treated during the latency period, which is the time between infection and onset of symptoms, they can recover within the first five weeks of the pandemic. This information is supported by a graph. In general, the number of syphilis cases will decline over time if they are properly treated. On the other hand, if left untreated, out of every 300 individuals, 280 will be affected by syphilis. However, those who have recovered from syphilis suggest that with appropriate care, treatment, and safe sexual behaviour, it is possible to avoid contracting syphilis.

Keywords: Syphilis, Risky Sexual Behaviour, Prevalence, Global Stability, Incubation period.

1.0 INTRODUCTION

Syphilis is a highly contagious disease that's mostly spread through sexual activity, including oral and anal sex. The world is experiencing a resurgence in the number of syphilis infection cases in many countries around the world (Bonyah *et al.*, 2021). Syphilis infections are endemic in low and middle-income countries, while high-income countries are experiencing high syphilis burdens for groups of individuals who practice risky sexual behaviour (Bonyah *et al.*, 2021, Chico *et al.*, 2022, Lasagabaster & Guerra, (2019), Gasper, 2022). For example, the prevalence of syphilis infections among Men who have Sex with Men (MSM) across 77 countries is 7.5% compared to 0.5% among men in the general population during the years 2000 to 2020 (Milner & Zhao, 2010). According to WHO, it is estimated that there are about 376 million new infection cases each year with one of four STIs: chlamydia (127 million), gonorrhoea (87 million), syphilis (6.3 million) and trichomoniasis (156 million). In the year 2006, an estimated 98,800 pregnant women were infected with syphilis, resulting in over 350,000 stillbirths, giving birth to infants with congenital syphilis and deaths (Kersh, & Workowski, 2020, WHO, 2023). Tiantian *et al.*, (2023) Syphilis, identified by Phillippe Ricord in 1838, is a

sexually transmitted infection. Known as the "great imitator," its symptoms can mimic those of other diseases. While treatable with antibiotics, it remains a public health concern, especially in developing countries. The spirochete *Treponema pallidum* subspecies *pallidum* is the cause of the infectious disease syphilis, which is primarily spread through sexual contact. It was identified in Western Europe in the years after Columbus' return from the Americas in 1500. The spiral-shaped bacteria known as *T. pallidum* subspecies *pallidum* are members of the Spirochaetaceae family, which also includes other pathogenic treponemes that cause non-venereal diseases, such as *T. pallidum* subspecies *endemicum* (bejel), *T. pallidum* subspecies *pertenue* (yaws), and *T. carateum* (pinta). These treponemes can be distinguished from *T. pallidum* subspecies *pallidum* by their respective clinical manifestations and genetic variations. *T. pallidum* is 0.2 μm in diameter and ranges in length from 6 to 15 μm (Omamea, *et al.*, 2020). If left untreated, syphilis is a multistage illness that advances from primary to secondary to tertiary infection. In both cases of treated and untreated syphilis, partial immunity develops. Because untreated cases can result in a variety of neurological and cardiovascular conditions as well as unfavourable pregnancy outcomes like stillbirth and congenital syphilis, it is a disease of significant public

health relevance. Terlu & Terefe (2023) Furthermore, in 1900, when syphilis was still incurable, 15% of blindness cases were thought to be related to the disease. These days, early syphilis is easily treated with an antibiotic dose, such as azithromycin or penicillin (Panel, Saad-Roy, Zhisheng & van den Driessche, 2016). According to estimates from the World Health Organization, 12 million new cases of syphilis are reported worldwide each year, with the majority of cases affecting residents of underdeveloped nations. On the other hand, since the 1970s, there have been multiple reports of syphilis comeback in various countries. Many factors, including the 1970s gay liberation movement, the 1980s crack cocaine epidemic, changes in the syphilis control program's intensity, changes in sexual behaviour, the HIV epidemic in the 1990s, and sporadic variations in the demographic composition of the two populations, can be attributed to these more recent outbreaks. A consistent thread emerges from an analysis of these outbreaks: the existence of a "core group" of high-risk, high-activity individuals that drives and controls the dynamics of the epidemics (Terlu & Terefe, 2023).

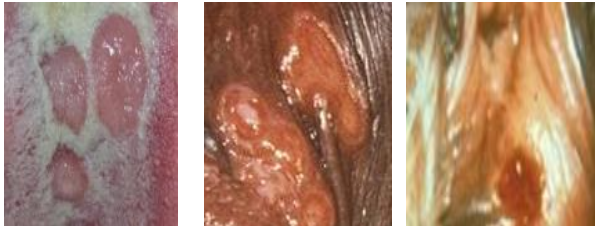


Figure 1. The primary stage of Syphilis Bacteria in both men and female reproductive
(Centres for Disease Control and Prevention, 2024)

Mbachu (2020) demonstrated that when reinfection results in an increased rate of resistance loss, backward bifurcations may happen if resistance is behaviourally created. In contrast, when resistance is produced by immunology, backward bifurcations do not develop. A recent study (Roth, 2020) calculated an epidemic period of 8–11 years by fitting real-world data to a SIRS model. But while examining the same model, (Ashleigh 2015, Cheng-Long, *etal.*, 2023) discovered no indication of cyclic recurrence, ostensibly supporting the CDC's effort to end syphilis in the United States.

1.1 Objectives of the Study

The Objectives of The Study Are to:

- i. Modify models by Milner and Zhao (2010) by including exposed (E), early latent (L_E), and late latent (L_L).

- ii. Obtain both the disease-free and endemic equilibrium of the syphilis.
- iii. Derive the reproduction number for the syphilis.
- iv. Carrying out the local stability of the Syphilis using the Ruth Hurwitz criterion.
- v. Carried out the global stability of the syphilis.
- vi. Carried out the numerical simulation of the syphilis.

2.0 MATHEMATICAL FORMULATION

In this paper we divided the model into Six compartments namely: Susceptible $S(t)$, expose $E(t)$, Early latent syphilis $L_E(t)$ (Early latent syphilis is latent syphilis where infection occurs within the past 12 months), late latent syphilis infection $L_L(t)$ (Late latent syphilis is latent syphilis where infection occurs more than 12 months ago) and the recovery $R(t)$. It is assumed that the bacteria do not kill the vector (human), but they destroy the reproductive organ, It is also assumed that the infection varies in different classes (Primary, secondary, latent and tertiary), lastly we that syphilis is transmissible by sexual contact with infected lesions, from mother to foetus in utero, via blood product transfusion, and occasionally through breaks in the skin that comes into contact with infectious lesions.

The recruitment of individuals into the susceptible class is at the rate Λ , and the susceptible class increases by ε , and decreases at the incident rate $\frac{\beta SI}{N}$ and each compartment decreases due to natural death μ . The exposed compartment $E(t)$, can decrease at γ , σ and δ due to the movement out of the compartment into the early latent stage $L_E(t)$, late latent stage $L_L(t)$, and infected stage $I(t)$ of the disease and increase at the rate $\frac{\beta SI}{N}$. Early latent syphilis is latent syphilis where the infection increases at σ within the past 12 months of infection from the exposed class and decreases at ψ and η , while late latent syphilis is latent syphilis where the infection increases at the rate γ and decreases at the rate π and $(\delta + \mu)$ for more than 12 months ago. The infected stage increases due to the progressions from exposed, early and late latent syphilis at the rates $(\psi + \theta + \pi)$ and decreases at the rate $(\mu + \theta + \tau)$. The recovered compartment increases at η , τ , δ and decreases at the rates $(\varepsilon + \mu)$.

Table 1: Variables and Parameters of the Model

VARIABLE/PARAMETERS	DESCRIPTION	Values	Ref
$S(t)$	Susceptible compartment	2000	Mbachu (2020)
$E(t)$	Exposed compartment	200	Momoh, etal., (2021)
$L_E(t)$	Early latent compartment	200	Kersh, & Workowski (2020)
$L_L(t)$	Late latent compartment	200	Terlu & Terefe (2023)
$I(t)$	Infected compartment	200	Momoh, etal., (2021)
$R(t)$	Recovery Compartment	10	Assume
γ	Progression from exposure to late latent	0.125	Mbachu (2020)
μ	Natural death	0.0000508/day	Oamea,etal., (2020)
Λ	Recruitment	$1.020 \cdot 10^3/\text{day}$	Assume
N	Total population	2000	Momoh, etal., (2021)
β	Contact rate	0.3	Assume
σ	Progression from exposure to Early latent	0.01	Mbachu (2020)
θ	Progression from exposure to infected	0.062	Washachi, Odekunle & Momoh . (2021)
ψ	Progression from early latent to infected	0.618	Lasagabaster& Guerra (2019)
π	Progression from late latent to infected	0.01	Tiantian <i>etal.</i> , (2023)
τ	Recovery from infected	0.1	Milner & Zhao (2010)
η	Recovery from early latent	0.04762/day	Milner and R. Zhao (2010).
δ	Recovery from late latent	0.9	Assume
ε	Loss of immunity	0.1	Washachi, Odekunle & Momoh . (2021)
α	Induced death rate	0.06849/day	Mbachu (2020)

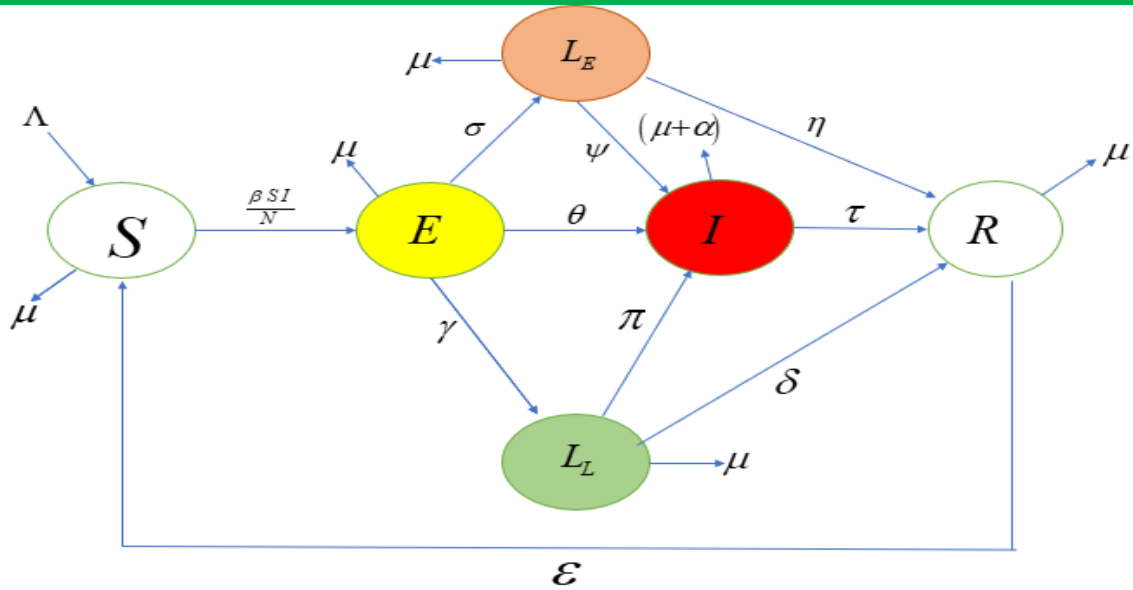


Figure 2 Schematic Diagram of the Model

2.1 Equations of The Model

$$\left. \begin{aligned} \frac{dS}{dt} &= \varepsilon R + \Lambda - \mu S - \frac{\beta SI}{N} \\ \frac{dE}{dt} &= \frac{\beta SI}{N} - A_1 E \\ \frac{dL_E}{dt} &= \sigma E - A_2 L_E \\ \frac{dL_L}{dt} &= E\gamma - A_3 L_L \\ \frac{dI}{dt} &= \psi L_E + \pi L_L + \theta E - A_4 I \\ \frac{dR}{dt} &= \tau I + \delta L_L + \eta L_E - A_5 R \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} A_1 &= -(\mu + \gamma + \sigma + \theta) \\ A_2 &= -(\mu + \psi + \eta) \\ A_3 &= -(\mu + \pi + \delta) \\ A_4 &= -(\mu + \alpha + \tau) \\ A_5 &= -(\mu + \varepsilon) \end{aligned} \right\} \quad (2)$$

With initial conditions; $S(0) = S_0, E(0) = E_0, L_E(0) = L_{E_0}, L_L(0) = L_{L_0}, I(0) = I_0, R(0) = R_0$.

3.0 MODEL ANALYSIS

In this section, we obtain the following results, which guarantee that the Syphilis disease is governed by the system (1) is epidemiologically and mathematically well-posed in a feasible Region given by:

$$S_0 = \left\{ \begin{array}{l} \{S, E, L_E, L_L, I, R_H\} \in R_+^6, S > 0, E \geq 0, L_E \geq 0, L_L \geq 0, I \geq 0, \\ R \geq 0, N_{(t)} \leq \frac{\Lambda}{\mu} + \left(\mu_0 - \frac{\Lambda}{N} \right) e^{\mu t} \end{array} \right\}$$

3.1 Disease-Free And Endemic Equilibrium Points Of The Syphilis Model

We find the equilibrium points by setting the right-hand side of the system (1.0) to zero as follows:

$$\left. \begin{array}{l} \varepsilon R + \Lambda - \mu S - \frac{\beta SI}{N} = 0 \\ \frac{\beta SI}{N} - A_1 E = 0 \\ \sigma E - A_2 L_E = 0 \\ E\gamma - A_3 L_L = 0 \\ \psi L_E + \pi L_L + \theta E - A_4 I = 0 \\ \tau I + \delta L_L + \eta L_E - A_5 R = 0 \end{array} \right\} \quad (3)$$

When there is no disease in the population

($E = 0, L_E = 0, L_L = 0, I = 0$) the disease-free equilibrium (DFE) point of the Syphilis model (1.0) exists and is

$$\text{given by } E_0 = (S_0, E_0, L_{E_0}, L_{L_0}, I_0, R_0) = \left\{ \frac{\Lambda}{\mu}, 0, 0, 0, 0, 0 \right\}$$

To compute the endemic equilibrium point (EEP), from equations (1) and (2) we have:

3.2 Basic Reproduction Number

The basic reproduction number denoted by R_0 is the expected number of secondary cases produced in a completely susceptible population by a typical infective individual.

Let $F(x)$ be the rate of appearance of new infections in the compartment V_j^* , (x) be the rate of transfer of individuals out of the compartment van den Driessche & Watmoug (2002). It can be calculated as $R_0 = \eta(FV^{-1})$ where

$$F = \begin{pmatrix} 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} A_1 & 0 & 0 & 0 \\ -\sigma & A_2 & 0 & 0 \\ -\gamma & 0 & A_3 & 0 \\ -\theta & -\psi & -\pi & A_4 \end{pmatrix}$$

$$\text{and } FV^{-1} = \begin{pmatrix} \frac{\beta(\pi\gamma A_2 + \sigma\psi A_3 + \theta A_2 A_3)}{A_1 A_2 A_3 A_4} & \beta \frac{\psi}{A_2 A_4} & \pi \frac{\beta}{A_3 A_4} & \frac{\beta}{A_4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence the reproduction number for Syphilis is the dominant eigenvalue, thus $R_{0M} = \eta(FV^{-1})$, where ρ is the spectral radius such that

$$\therefore R_0 = \frac{(\pi\beta\gamma A_2 + \sigma\beta\psi A_3 + \theta\beta A_2 A_3)}{A_1 A_2 A_3 A_4} \quad (5)$$

3.3 Local Stability of the Disease-Free Equilibrium of the Syphilis Model

Proposition 1: The disease-free equilibrium E_0 point of the syphilis model is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$. This proposition is achieved by $|J(E_0) - \lambda I| = 0$. From (1) we have

$$\begin{pmatrix} -\mu - \lambda_1 & 0 & 0 & 0 & -\beta & \varepsilon \\ 0 & A_1 - \lambda_2 & 0 & 0 & \beta & 0 \\ 0 & \sigma & A_2 - \lambda_3 & 0 & 0 & 0 \\ 0 & \gamma & 0 & A_3 - \lambda_4 & 0 & 0 \\ 0 & \theta & \psi & \pi & A_4 - \lambda_5 & 0 \\ 0 & 0 & \eta & \delta & \tau & A_5 - \lambda_6 \end{pmatrix} \quad (6)$$

Using the reduction method, we have

$$\begin{pmatrix} A_1 - \lambda_1 & 0 & 0 & \beta \\ \sigma & A_2 - \lambda_1 & 0 & 0 \\ \gamma & 0 & A_3 - \lambda_1 & 0 \\ \theta & \psi & \pi & A_4 - \lambda_1 \end{pmatrix} \quad (7)$$

$$\lambda_1 = -\mu, \quad \lambda_6 = -A_5$$

We applied the Routh-Hurwitz criterion which stated that all roots of the polynomial (7) have negative real parts if and only if the coefficient, A_i are positive and the determinant of the matrices $H_i > 0$ for $i = 1, 2, \dots, n_i$

$$\begin{pmatrix} A_1 & 0 & 0 & 0 \\ \sigma & A_2 & 0 & 0 \\ \gamma & 0 & A_3 & 0 \\ \theta & \psi & \pi & A_4 \end{pmatrix} \quad (8)$$

The characteristic polynomial of (8), we have

$$X^4 + A_{10}X^3 + A_{11}X^2 + A_{12}X + A_{13}$$

$$B_1 = 1$$

$$B_2 = (-A_1 - A_2 - A_3 - A_4)$$

$$B_3 = (A_3(A_1 + A_2) - \theta\beta + A_1A_2 + A_4(A_1 + A_2 + A_3))$$

$$B_4 = (\theta\beta(A_1 + A_2 + A_3) - \pi\beta\gamma - \sigma\beta\psi - \theta\beta A_1 - A_4(A_3(A_1 + A_2) - A_1A_2A_3))$$

$$B_5 = ((\pi\beta\gamma + \sigma\beta\psi + \theta\beta A_1)(A_1 + A_2 + A_3) - \pi(\beta\gamma A_1 + \beta\gamma A_3)$$

$$- \psi(\sigma\beta A_1 + \sigma\beta A_2) - \theta\beta A_1^2 - \theta\beta A_3(A_1 + A_2) + A_1A_2A_3A_4)$$

$$H = \begin{bmatrix} B_1 & B_3 & B_5 & 0 & 0 \\ 1 & B_2 & B_4 & 0 & 0 \\ 0 & B_1 & B_3 & B_5 & 0 \\ 0 & 1 & B_2 & B_4 & 0 \\ 0 & 0 & B_1 & B_3 & B_5 \end{bmatrix} \quad (9)$$

$$H_1 = B_1 > 0$$

$$H_2 = \begin{bmatrix} B_1 & B_3 \\ 1 & B_2 \end{bmatrix} = B_1 B_2 > B_3$$

$$H_3 = \begin{bmatrix} B_1 & B_2 & B_5 \\ 1 & B_2 & B_4 \\ 0 & B_1 & B_3 \end{bmatrix} = B_2 B_1 B_3 + B_5 B_1 > B_3^2 + B_4 B_1^2$$

$$H_4 = \begin{bmatrix} B_1 & B_3 & B_5 & 0 \\ 1 & B_2 & B_4 & 0 \\ 0 & B_1 & B_3 & B_5 \\ 0 & 1 & B_2 & B_4 \end{bmatrix} = B_1 B_2 B_3 B_4 + 2B_1 B_4 B_5 + B_2 B_3 B_5 > B_1^2 B_4^2 + B_1 B_2^2 B_5 + B_3^2 B_4 + B_5^2$$

$$H_5 = \begin{bmatrix} B_1 & B_3 & B_5 & 0 & 0 \\ 1 & B_2 & B_4 & 0 & 0 \\ 0 & B_1 & B_3 & B_5 & 0 \\ 0 & 1 & B_2 & B_4 & 0 \\ 0 & 0 & B_1 & B_3 & B_5 \end{bmatrix} = B_1 B_2 B_3 B_4 B_5 + 2B_1 B_4 B_5^2 + B_2 B_3 B_5^2 > B_1^2 B_4^2 B_5 + B_1 B_2^2 B_5^2 + B_3^2 B_4 B_5 + B_5^3$$

Since $H_1 = H_2 = H_3 = H_4 = H_5 > 0$, hence, the Syphilis is locally asymptotically stable.

3.4 Global Stability of the Disease-Free Equilibrium Point

For global stability of the DFE, the technique used by Castillo-Chavez et al. Castillo-Chavez, Zhilag & Huang (2002) was employed. The model is rewritten as follows:

$$\left. \begin{aligned} \frac{dX}{dt} &= K(X, Z) \\ \frac{dZ}{dt} &= G(X, Z), \quad G(X, 0) = 0 \end{aligned} \right\}$$

(10)

where $X \in \mathbb{R}^2$ and $X = \{S, R\}$ denotes the number of uninfected individuals and $Z \in \mathbb{R}^3$ where $Z = \{E, L_E, L_L, I\}$ denotes the number of infected individuals. $E^0 = \left\{ \frac{A}{\mu}, 0, 0, 0, 0 \right\}$ denotes the disease-free equilibrium point of this system where $x^* = \left\{ \frac{A}{\mu} \right\}$

(11)

Condition (10) may be met to guarantee global asymptotic stability

(H_1): For $\frac{dx}{dt} = k(x, 0)$, x^* is globally asymptotic stable.

gamma is (H_2): For $G(X, Z) = AZ - \hat{G}(X, Z)$, $\hat{G}(X, Z) \geq 0 \forall (x, z) \in \Gamma$ where $A = D^G(x^*, 0)$ is an m matrix and Γ is the region where the model has biological meaning.

Theorem 1. If the system (1) satisfies condition (10), then the fixed point $E^0 = \left\{ \frac{A}{\mu}, 0, 0, 0, 0 \right\}$ is a globally asymptotically stable equilibrium of the system (1) provided that $R_0 < 1$ and the conditions (H_1) and (H_2) are satisfied:

Proof

Consider $K(X, 0) = [A - \mu S]$ and $G(X, Z)$ and $\hat{G}(X, Z) = AZ - \hat{G}(X, Z)$

$$A = \begin{bmatrix} -A_1 & 0 & 0 & \frac{\beta S}{N} \\ \sigma & -A_2 & 0 & 0 \\ \gamma & 0 & -A_3 & 0 \\ \theta & \psi & \pi & -A_4 \end{bmatrix}$$

(12)

And

$$Z = \begin{bmatrix} E \\ L_E \\ L_L \\ I \end{bmatrix}$$

(13)

$$\left[\begin{aligned} \frac{\beta SI}{N} - A_1 E &= 0 \\ \sigma E - A_2 L_E &= 0 \\ E \gamma - A_3 L_L &= 0 \\ \psi L_E + \pi L_L + \theta E - A_4 I &= 0 \end{aligned} \right]$$

(14)

Given

$$\hat{G}(X, Z) = AZ - G(X, Z)$$

(15)

Substituting equations (12), (13), and (14) into (15) we have

$$G(\hat{X}, Z) = \begin{bmatrix} -A_1 & 0 & 0 & \frac{\beta S}{N} \\ \sigma & -A_2 & 0 & 0 \\ \gamma & 0 & -A_3 & 0 \\ \theta & \psi & \pi & -A_4 \end{bmatrix} \begin{bmatrix} E \\ L_E \\ L_L \\ I \end{bmatrix} - \begin{bmatrix} \frac{\beta SI}{N} - A_1 E \\ \sigma E - A_2 L_E \\ E\gamma - A_3 L_L \\ \psi L_E + \pi L_L + \theta E - A_4 I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

Since all the conditions are satisfied, $\hat{G}(X, Z) \geq 0$, the DFE, E^0 is globally asymptotically stable.

4.1 Numerical Simulation

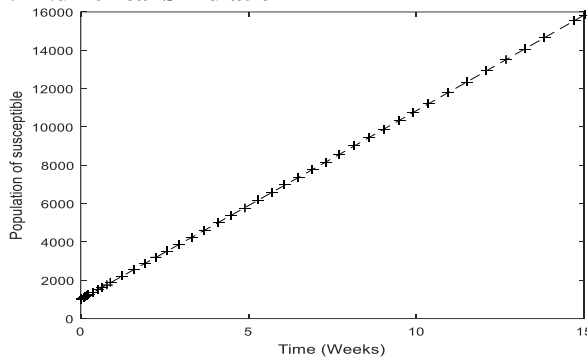


Figure 3: Diagram showing the dynamics of Susceptible Humans

As seen by the straight line connecting the first week to the last week of the graph, the susceptible populations at this time demonstrate that there is no breach in the population and that there is no syphilis in the population.

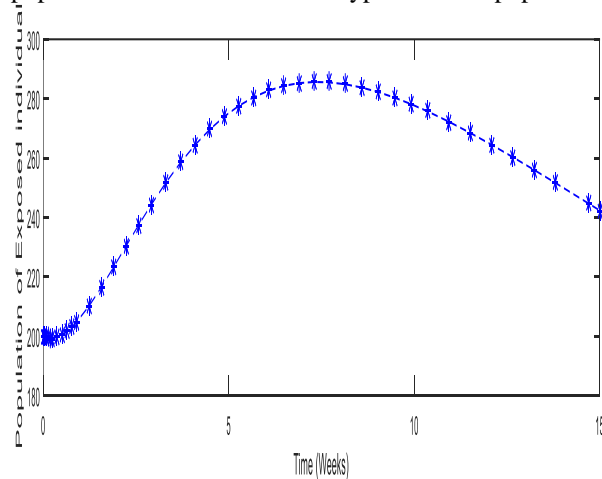


Figure 4: Diagram showing the dynamics of Exposed Humans

The zigzag line in Figure 4 indicates that there is a breach in the population, which is why 200–280 people have been exposed to the diseases. This means that the number of exposed people increased until 280 people

were exposed and then dropped to 260, indicating that people were aware of the diseases and were beginning to take the necessary precautions.

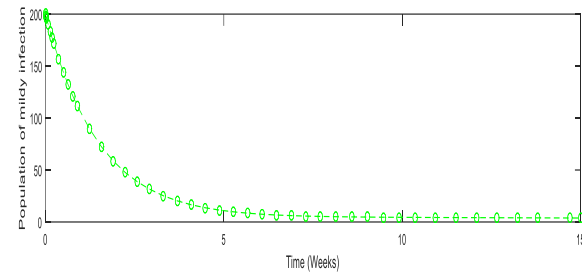


Figure 5: Diagram showing the dynamics of mild infection in Humans Population.

The movement of mildly infected individuals against time (weeks) is depicted in Figure 5, which demonstrates how 200 mildly infected individuals nearly disappeared from the population over fifteen weeks. This suggests that mildly infected individuals may eventually disappear from the population because, starting in the fifth week, the disease is nearly stable.

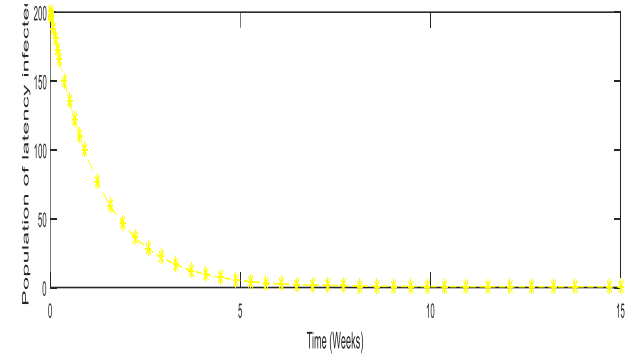


Figure 6: Diagram showing the dynamics of latency infection in Humans Population

Figure 6 illustrates the impact of latency infection. It can be seen that mildly infected individuals do not return to their place of origin, indicating a low prevalence of the disease in the population. In contrast, latency-infected individuals return home after the fifth week. From a biological perspective, this indicates that there are few symptoms and that treatment produces the ideal outcome.

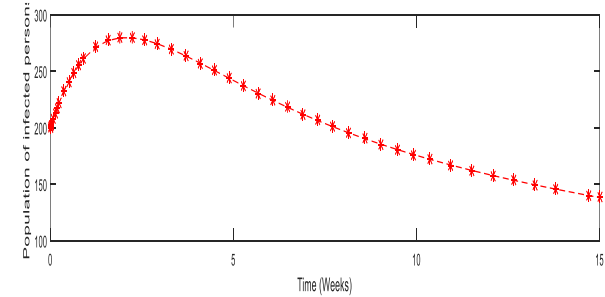


Figure 7: Diagram showing the dynamics of infected Humans Population.

Figure 7 shows that there are between 200 and 280 infected people in the population, but during the second week of the break, syphilis was shown to be causing a lot of devastation. According to biology, this indicates that during the first week of the disease, individuals did not take preventative measures since they were unaware of it. However, after the disease was discovered in the population and treated, the number of cases decreased to 140 in just fifteen weeks. It is assumed that, if a disease is treated, it will eventually go extinct due to the graph's nature.

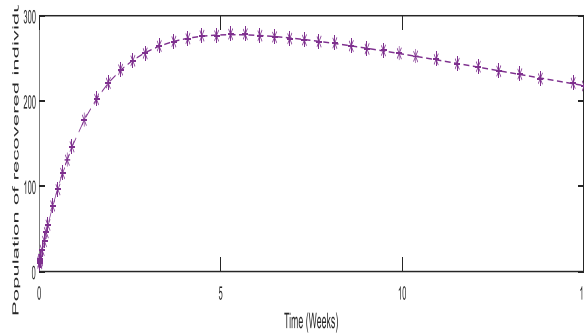


Figure 8: Diagram showing the dynamics of Recovered of Humans Population.

The population that has recovered is depicted in Figure 8, and it can be seen that nearly everyone who is infected will get better with treatment. We are advising everyone reading this to visit a hospital for testing and treatment if they begin to exhibit symptoms of syphilis.

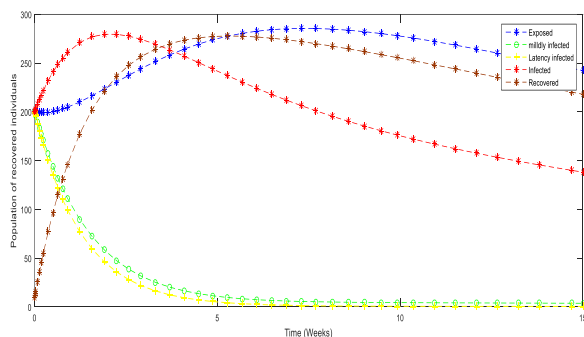


Figure 9: Diagram showing the dynamics of Exposed, mildly, latency infected, infected and Recovered Human Population.

The group of people with diseases from the exposed, mildly infected, latency infected, infected, and recovered populations is shown in Figure 7. The number of people who are exposed has increased; this will only decrease if people become aware of it. If someone is mildly infected and is not tested, the disease will continue to spread and cause havoc because there is no chance that it will reach

its origin. After all, it has no symptoms and only causes a small number of deaths. If someone is latency and receives treatment, the disease will reach its origin within the first five weeks of the pandemic. If treated, the number of syphilis cases will decline over time; in the absence of therapy, there are 280 cases out of 300 populations. The group that has recovered suggests that, with proper care, treatment, and safe sexual behaviour, they can avoid contracting syphilis.

5.0 CONCLUSION

This study introduces a novel deterministic model that sheds light on the transmission dynamics of syphilis within a population. The model not only facilitates the evaluation of treatment strategies' impact on the entire population but also accounts for critical disease features, such as the various untreated infection stages (i.e., "early" and "late") and disallows re-infection and relapse in recovered individuals.

If the re-infection of recovered individuals is low, then implementing a control strategy such as treating confirmed cases can effectively control or even eliminate the disease from the community. This can be achieved by bringing and maintaining the associated reproduction number of the model to a value that is less than one. According to this study, it seems that treating syphilis can be an effective way to control the disease in a population. After conducting a thorough analysis of the model, it was found that the two most significant parameters that influence the reproduction number of the model, and thereby the disease burden, are: the proportion of individuals who were treated in the early stages and recovered successfully, and the modification parameter that is linked to the decrease in the recovery rate for those individuals who did not respond to the treatment. This study suggests that treating syphilis effectively in a population is promising. It is necessary to take more measures to increase the speed of diagnosis and treatment of confirmed syphilis cases, along with educating the public about safe sex practices.

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