

Effects Of Viscous Dissipative Fluid on Couette Flow in A Vertical Channel Due to Newtonian Heating

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Received on: January, 2024

Revised and Accepted on: March 2024

Published on: March 2024

ABSTRACT

This article presents the numerical investigation of magnetohydrodynamic (MHD) heat transfer flow of viscous dissipation in vertical channel in the presence of Newtonian heating. The coupled nonlinear dimensional partial differential equations governing the flow are converted into non-dimensional form by using the appropriate dimensionless quantities and parameters Implicit Finite Difference Method (IFMD) was employed in finding the solutions of dimensionless partial differential equations. The expressions of velocity, temperature, skin friction and Nusselt number were obtained numerically and discussed using line graphs. From the outcome of the results, it was concluded that rising the Newtonian heating parameter (γ), Brinkman number (Br), lead to a substantial growth of velocity and temperature. Similarly, as the, Grashof number (Gr) increases, the velocity of the fluid also gets increased. The skin friction gets enlarged with the increase of Grashof number (Gr), Newtonian heating parameter (γ), Brinkman number (Br) and time at $y = 0$ and $y = 1$. The Nusselt number gets reduced with the increase in Newtonian heating parameter (γ), Brinkman number, and time parameter at $y = 0$ and increases at $y = 1$.

Keyword: Viscous dissipation, Newtonian Heating, Couette flow, Implicit finite difference method (IFDM).

1.0 INTRODUCTION

Natural convection involving Newtonian heating or cooling through various geometrical shapes and channels have been reported by several scholars. Free convection flow in a boundary layer region is a motion that results from the interaction of gravity with density differences within a fluid. These differences occur due to temperature or concentration gradients or due to their composition. Studies pertaining free convection flows of incompressible viscous fluids have received much attention in recent years both theoretically (exact or approximate solutions) and experimentally (Zullkiflee *et al.* 2019). The natural convection boundary layer flow over a vertical surface with Newtonian heating was first studied by Merkin (1994). The potential uses of free convective heat and mass transfer flows in various technological and scientific contexts have piqued the interest of numerous researchers in recent years. These include transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces, and film vaporization in combustion chamber (Suresh *et al.* 2019). With these concerns in mind, Khan *et al.* (2022) analyzed the second law to determine how Newtonian heating affects the Couette flow of a viscoelastic dusty fluid and the transfer of heat in a rotating frame. Khan *et al.* (2021) outlined the consequences of Newtonian heating condition, magnetic field, chemical reaction and heat generation on viscoelastic Casson fluid free convection flow among the upstanding plates.

Loganathan *et al.* (2021) investigated the viscoelastic Casson fluid flow over a cylinder with Newtonian heating condition and heat absorption effects. Manjula and Sekhar (2021) analyzed the effect of heat transmission and thermal expansion affected by Newtonian heating condition and Casson fluid flow of a vertical plate.

The effects of thermodynamic and hydrodynamic quantities on fluid flow and thermal behavior have been studied. Internal heat generation caused by viscous forces in molecular fluid-particle interactions is known as viscous dissipation, and the mechanical energy dissipated in this form significantly disturbs the thermal buoyancy and thermal behavior of a fluid in motion. Because fluid-particle interactions affect fluid flow and temperature, viscous dissipation is critical in most lubrication industries. The study of fluid flow has gotten a lot of attention because of its applications in science, engineering, and industrial technologies such as gas drainage, cooling of electrical appliances, geothermal energy, plasma physics, gas turbines, fossil fuel combustion, food processing industries, and so on (Ajibade *et al.* 2021). One of the earliest works that considered viscous dissipation was the work of (Gebhart 1962) which used a perturbation method to investigate the effect of viscous dissipation on natural convection. In view of these considerations, Ojemeru and Onwubuya (2023) recently discussed the theoretical treatment of heat enhancement flow for an

electrically conducting and viscous dissipative fluid traveling vertically through a thermodynamic system where the parallel plates are constantly heated in a slit microchannel filled with porous material. Yale *et al.* (2023) inspected the analytical solution of heat transfer and hydromagnetic flow for an incompressible, electrically-conducting, and viscous dissipative fluid traveling vertically through an isothermally heated parallel plate in a superhydrophobic micro-channel. Narender *et al.* (2021) examined the viscous dissipation and thermal effects on magneto hydrodynamic mixed convection stagnation point flow of Maxwell nanofluid passing over a stretching surface. The governing partial differential equations are transformed into a system of ordinary differential equations by utilizing similarity transformations. Swain *et al.* (2020) evaluated the implications of viscous dissipation, MHD flow and gradient heat transfer of a Newtonian fluid along a stretching sheet embedded in a porous medium. Uka *et al.* (2023) analytically and numerically solved the problem of fluid flow in the presence of viscous dissipation and permeability effect as a result of a wedge in motion using a well-established regular perturbation and mathematica V.10 scheme. Amar *et al.* (2023) investigated the problem of study is the impacts of viscous dissipation, thermal radiation on the MHD heat transfer flow of Casson fluid across a moving wedge with convective boundary condition in the existence of and internal heat generation/absorption. Prakash and Sivakumar (2018) carried out a study to determine the combined effects of viscous and Ohmic heating in the steady, mixed convective flow of a viscous, incompressible, and electrically conducting nanofluid flow over an inclined nonlinear stretching plate embedded in a porous medium. In the work of Ajibade *et al.* (2018), the effect of viscous dissipation on steady natural convection Couette flow of a heat-generating fluid in a vertical channel was investigated. They concluded that the effect of viscous dissipation is very important in the study of natural convection problem and should not be overlooked in future studies. Other studies that considered the effect of viscous dissipation on fluid flow are the works of Ajibade and Umar (2020a, 2020b).

This work is aimed at extending the work of Zulkifree *et al.* (2019) by providing the numerical solution of unsteady Couette flow of viscous dissipative fluid in a vertical channel with Newtonian heating.

2.0 FORMULATION OF THE PROBLEM

Consider an unsteady free convective Couette flow of an incompressible fluid in a vertical channel with Newtonian heating. The motion of the fluid is caused by the one of the moving vertical plate as well as the viscous dissipation of the fluid.

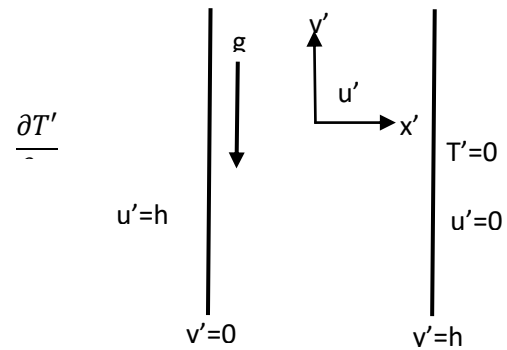


Figure 1: Geometry of the Problem

where y' and x' are the dimensional

distances along and perpendicular to the plate. u' and T' are the dimensional velocity and temperature ν , k , ρ , c_p , β and g are the dimensional heat source/sink parameter, kinematic viscosity, thermal conductivity, density, specific heat at constant pressure, thermal expansion coefficient, and acceleration due to gravity of the fluid, respectively. The problem is governed by the following differential equations:

$$\frac{\partial u'}{\partial t'} = \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_0) \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 \tag{2}$$

The corresponding initial and boundary conditions are:

$$\left. \begin{aligned} U = 1, \frac{\partial \theta}{\partial y} &= -\gamma(\theta + 1) \text{ at } y = 0 \\ U = 0, \theta &= 0 \text{ at } y = h \end{aligned} \right\} \tag{3}$$

To solve equation (1) and (2), we apply the dimensionless quantities and parameters:

$$\left. \begin{aligned} u &= \frac{u'}{U}, y = \frac{y'}{h}, \theta = \frac{T' - T_0}{T_w - T_0}, x = \frac{x'v}{Uh^2} \\ B_r &= \frac{u^2}{h^2(T_w - T_0)}, Gr = \frac{g\beta(T_w - T_0)h^3}{\nu^2} \end{aligned} \right\} \tag{4}$$

Applying the dimensionless quantities in equation (4) into equation (1), (2) and (3) then the following non-dimensional equations and boundary conditions are formed:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} + Gr\theta + Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + Br \left(\frac{\partial u}{\partial y}\right)^2 \tag{5}$$

Boundary conditions in dimensionless forms are:

$$\left. \begin{aligned} \frac{\partial \theta}{\partial y} &= -\gamma(\theta + 1) \\ U &= 1 \\ \theta &= 0 \\ U &= 0 \end{aligned} \right\} \text{at } y = 0 \tag{7}$$

$$\left. \begin{aligned} U &= 1 \\ \theta &= 0 \\ U &= 0 \end{aligned} \right\} \text{at } y = 1$$

3.0 METHOD OF SOLUTION

In order to get the solution of (5) and (6) subject to the boundary condition (7) finite difference method (IFDM) was applied as follows:

$$\frac{U_i^{j+1} - U_i^j}{Dt} = \frac{u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}}{(Dy)^2} + \frac{Gr}{Pr} \frac{(\theta_i^{j+1} - \theta_i^j)}{Dt} = \frac{\theta_{i+1}^{j+1} - 2\theta_i^{j+1} + \theta_{i-1}^{j+1}}{(Dy)^2} + Bii \frac{(U_{i+1}^j - U_i^j)^2}{4(Dy)^2} \tag{8}$$

The skin friction at the plate is given by in non-dimensional form as:

$$\tau = \left[\frac{\partial u}{\partial y} \right]_{y=0, y=1} \tag{10}$$

The rate of heat transfer coefficient can be calculated in terms of Nusselt number is given in dimensionless form as:

$$Nu = - \left[\frac{\partial \theta}{\partial y} \right]_{y=0, y=1} \tag{11}$$

4.0 RESULTS AND DISCUSSION

This section is committed to presenting the impacts of viscous dissipative effects on unsteady free convection Couette flow of an incompressible electrically-conducting fluid in the presence of Newtonian heating. The implicit finite difference method (IFDM) was used to evaluate the unsteady governing partial differential equations. Several graphs were drawn to showcase the influences of various settings on the flow configuration and energy profile. The default values chosen for this study are ($\gamma = 0.1, Br = 0.1$ and $Gr=0.1, Pr = 0.71$), except otherwise stated, as it relates to real life situation

Velocity and Temperature Profile

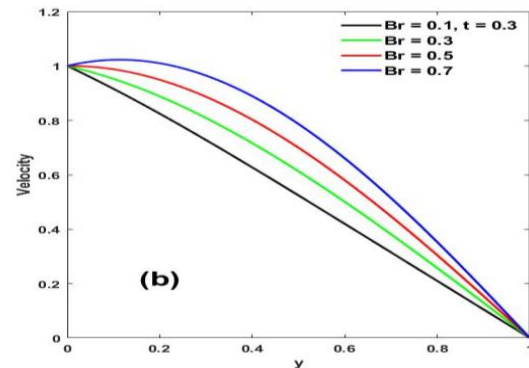
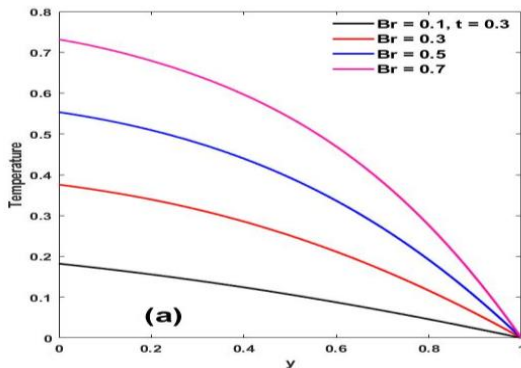


Figure 2: Effect of Br on Velocity and Temperature Profile

Figures 2a and 2b show the impact of Brinkman Number (Br) on temperature and velocity profile respectively. It is clearly seen, from the Figures that temperature and velocity of the fluid rises by increasing values of Brinkman number.

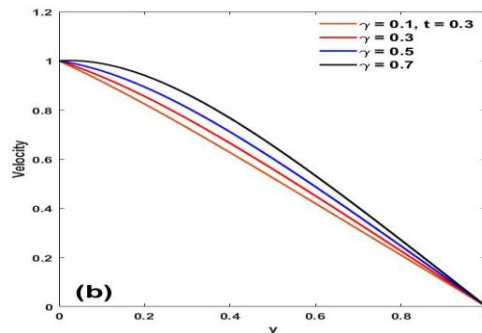
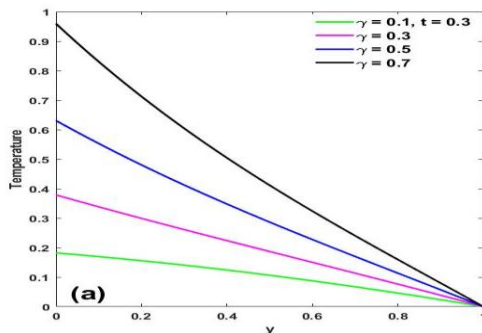


Figure 3: Effect of γ on Velocity and Temperature Profile

Figures 3a and 3b display the effect of Newtonian heating parameter (γ) on temperature and velocity profile respectively. From the Figures it is observed that both temperature and velocity of the fluid enhanced by increasing values Newtonian heating parameter (γ). Higher temperature is recorded at the lower channel of the fluid due to viscous heating of the lower channel wall.

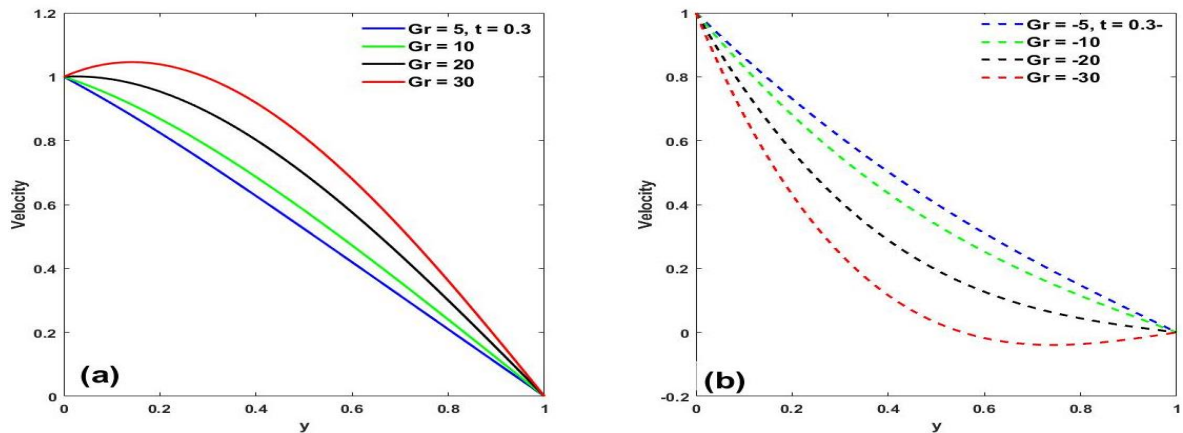


Figure 4: Effect of Gr on Velocity and Profile

Figure 4a and 4 show the impact of Grashof Number on velocity profile. Figure 4a represent positive values of Gr while Figure 4b represents negative values of Gr. It is noticed that increasing values of $Gr > 0$ enhances fluid flow significantly while similar trend in opposite direction is observed for $Gr < 0$. Physically Gr represents the effect of free convection current and therefore increase in it magnifies the fluid velocity profile and vice versa.

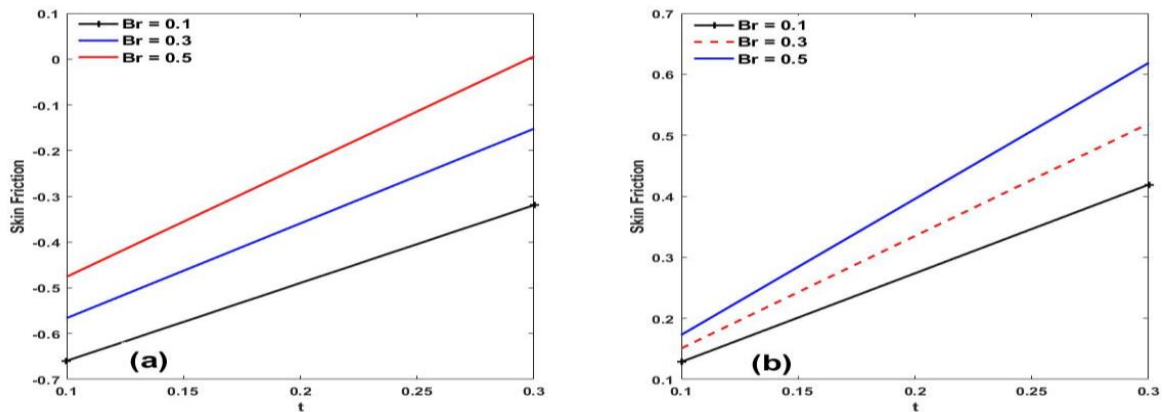


Figure 5: Effect of Br on Skin Friction at $y=0$ and $y=1$

The impact of skin friction against t for different values of Br at $y = 0$ and $y = 1$ is displayed in Figure 5a and 5b respectively. It is seen that at both plate growing values of Br and t enhances skin friction at $y = 0$ and $y = 1$.

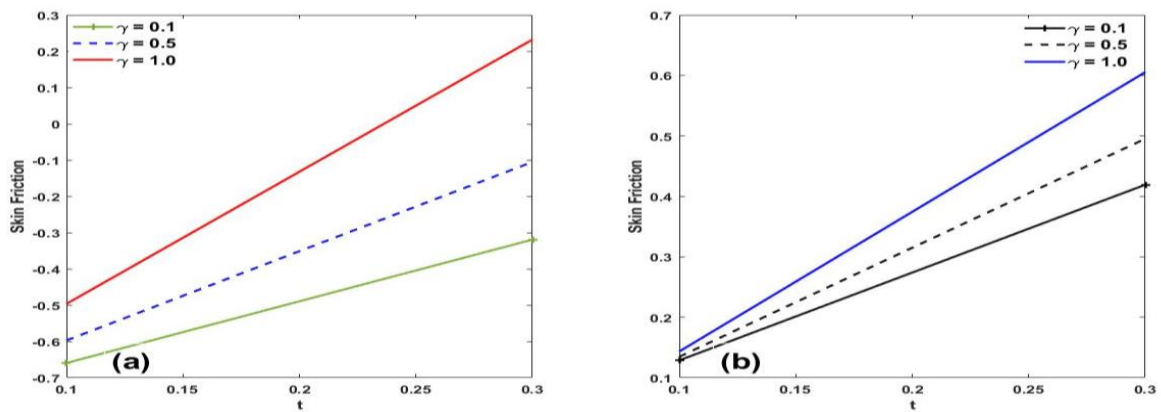


Figure 6: Effect of γ and t on Skin Friction at $y=0$ and $y=1$

Figure 6a and 6b displayed the influence of time parameter (t) and Newtonian heating parameter on skin friction at $y=0$ and $y=1$ respectively. From the Figures both Newtonian heating parameter and time parameter (t) enlarges skin friction $y=0$ and $y=1$.

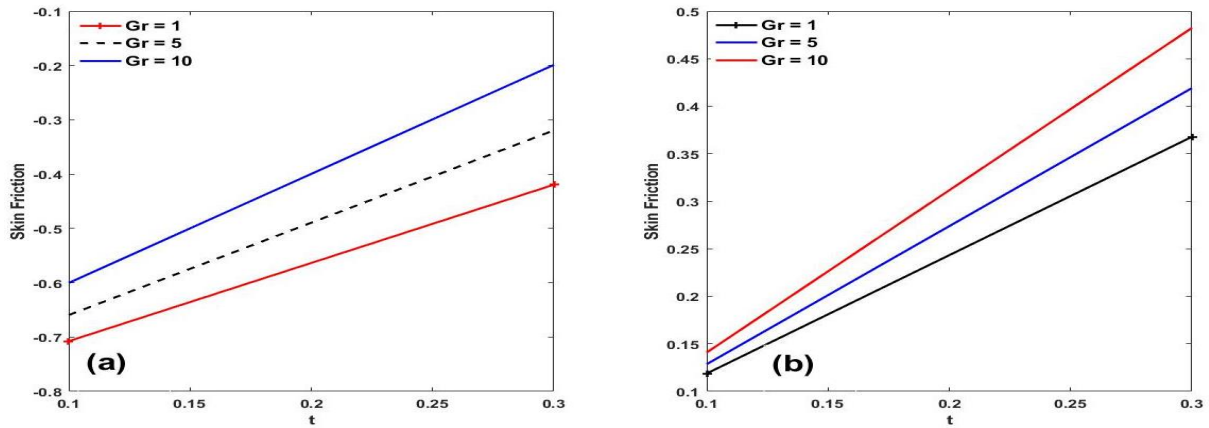


Figure 7: Effect of Gr and t on Skin Friction at $y=0$ and $y=1$

Figure 7a and 7b represent the effect of skin friction against t for different values of Gr at the plate $y=0$ and $y=1$ respectively. From these Figures, it is observed that higher value of Gr lead to intensification of the skin friction at both plates.

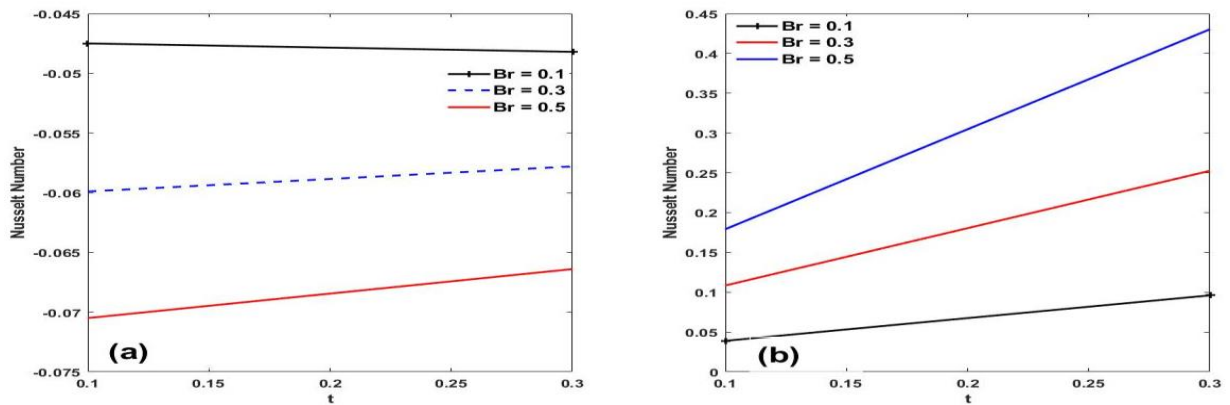


Figure 8: Effect of Br and t on Nusselt Number at $y=0$ and $y=1$

Figures 8a and 8b show the impact of time parameter (t) and Brinkman Number (Br) on Nusselt number at $y=0$ and $y=1$ respectively. From the Figure8a, it is seen that, increasing values of Brinkman Number decreases Nusselt number while a similar phenomenon in reverse direction is recorded at $y=1$ (see Fig.8b).

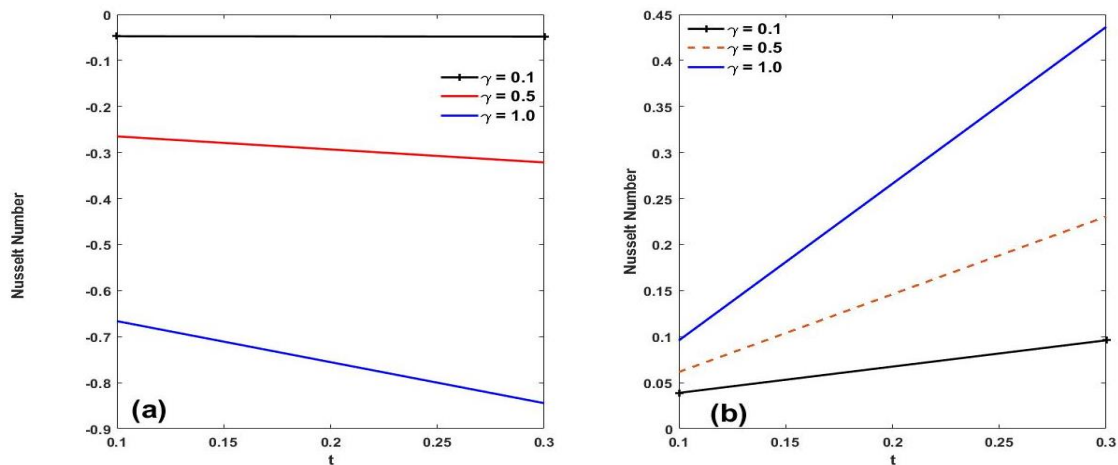


Figure 9: Effect of γ and t on Nusselt Number at $y=0$ and $y=1$

Figures 9a and 9b show the impact of time parameter (t) and Newtonian heating parameter (γ) on fluid Nusselt number at $y=0$ and $y=1$ respectively. From Figure 8a the Nusselt number gets reduced with the increase in both Newtonian heating and time parameter, while in Figure 8b the Nusselt number gets enhanced with the increase in both Newtonian heating and time parameter.

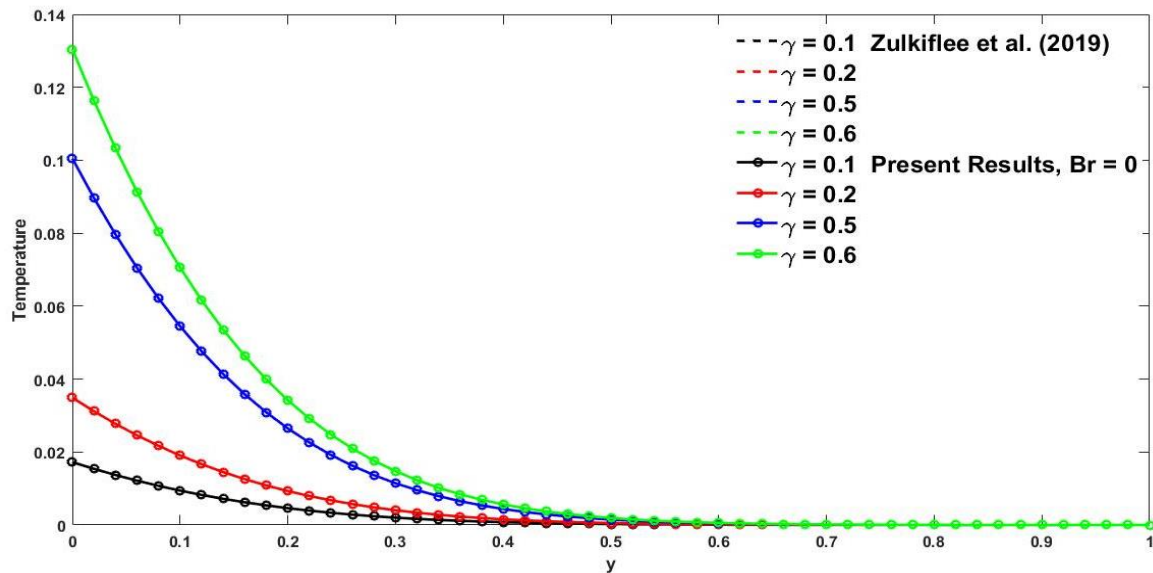


Figure 10: Comparison between the work of Zulkiflee et al. (2019) and the present work when $Br = 0$.

Figure 10 represents a comparison between the present model in the absence of Br and the work of Zulkiflee et al. (2019). The figure indicates an excellent agreement between the two.

5.0 CONCLUSION

In the present study, numerical study was carried out to investigate the influence of viscous dissipation (Br) on Couette flow in the presence of Newtonian heating. The study applied implicit finite difference method (IFDM) in obtaining the numerical solution of governing partial differential equations. From the study the following conclusions were made:

- Raising the Newtonian heating parameter, Brinkman number, lead to a substantial growth of velocity and temperature.
- As the, Grashof number increases, the temperature of the fluid also gets increased.
- The skin friction gets enlarged with the increase of Grashof number, Newtonian heating parameter, Brinkman number and time parameter at $y = 0$ and $y = 1$.
- The Nusselt number gets reduced with the increase in Newtonian heating parameter, Brinkman number, and time parameter at $y = 0$ and increases at $y = 1$

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