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# Amelioration of Corruption Dynamics in Nigeria: Mathematical Models and Control Strategies

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### ABSTRACT

The most pressing problems in Nigeria have to do with corruption, which can be defined as giving or receiving of bribe. It has been asserted that almost everybody above 12 years of age have been infected with corruption one way or the other. The impact of corruption on national development cannot be overemphasized. Different scholars from social sciences, psychology, political sciences, and religious studies have attempted a working definition for corruption from their various disciplines. However, all of the working definitions are interwoven. Despite various research works and the availability of both preventive and capital punishment on those involved in corruption related offences, corruption still remains incurable social disease in the world. According to Transparency International Index 2020 survey, Nigeria is the 31<sup>st</sup> most corrupt nation out of the 180 countries considered. In our research, we developed and analysed a mathematical model for the Dynamics of corruption to highlight the relationship between corruption and social stratification using ordinary differential equations, we complimented the existing mathematical models of corruption by incorporating the impacts of social stratification and vice versa. We obtain the corruption-free and endemic equilibrium points of the model, we also compute the effective basic reproduction number,  $R_C$  of the model, we employed the mathematical tools to analyse the corruption-free and endemic equilibrium states of the model for stability

**Keywords:** Corruption Model, Reproduction number, Equilibrium, Endermic, Stability, stratification.

### 1.0 INTRODUCTION

It is unconformable, profit gained by contradicting the law or rules, norms, or the action set in the community. The World Bank (WB) has defined corruption as the abuse of public office for private gain (World Bank, 1997). The Transparency International (TI) also defined corruption as the misuse of entrusted power for private benefit (Hathroubi *etal*, 2014). Corruption is colourless, shapeless, odourless, collusive, secret, stealthy, and shameless (Abdulrahman, 2014). It is a worm within the body of the society and a cancer to economic, social, and political development, corruption involves bribes, illegal gratitude, conflict of interests, embezzlement of public funds, and economic extortion (BinuyoIt, 2019). It is major cause of poverty around the world, especially in Africa (Danford *etal*.). It is one of the causes of instability and conflict which is seen in the present situation in Ethiopia (Legesse and Shiferaw, 2018).

The development of viable intervention strategies to mitigate corruption and corrupt practices requires a thorough understanding of the corrupt process and prevention and disengagement programs

Mathematical modelling is the process of using mathematical concepts such as equations, functions and graphs to represent real life phenomena. While model refers to as an abstraction that reduces a problem to its essential characteristics. Models are designed to pay attention on certain aspect of the object of study, (Olukayode & Akinsola, 2020).

(Nathan and Jakob, 2019). Mathematical modeling in epidemiology has become an effective tool for understanding and describing the dynamics of infectious disease (Hattaf, 2020). Several authors have studied the epidemiological corruption modeling approach as a disease. In

particular, (Gweryina et al, 2019). derived and analyzed an epidemiological model of corruption with immunity clause in Nigeria. The simulation result shows that treating corruption in the presence of immunity clause regime will be a hard war to be won if the clause continued to exist. (Hathroubi *et al*, 2014) proposed a model on corruption dynamics. Based on the approximation of the honest population, an epidemiological corruption threshold was determined, but the stability analysis of both corruptionfree and endemic equilibria was not carried out.

(Crokidakis and Martins, 2019) proposed and analyzed a simple model of social contagion that represents the dynamics of social influences among politicians in an artificial corrupt parliament. The results show that there is a critical density of such zealot above which the honest deputies cannot survive as honest after a long time, while zealot density below the mentioned critical density, honest deputies coexist in the parliament with corrupt ones. (Nathan and Jakob, 2019) divided the total population into three compartments, susceptible (S), corrupt (C), and political corrupt (M). Through media and campaigning from opposing parties such as anticorruption bodies, churches, and government political will to fight, the corrupt individuals  $C(t)$  becomes susceptible  $S(t)$ . The model exhibits a threshold dynamics characterized by the basic reproduction  $R_0$ . When  $R_0 < 1$ , the system has a unique equilibrium point that is asymptotically stable. For  $R_0 > 1$ , the system has additional equilibrium point known as endemic which is globally asymptotically stable. They also assess strategies to counter corruption vice. (Abdulrahman, 2018) developed a deterministic mathematical model of corruption transmission dynamics as a disease. The basic reproduction number, corruption-free and endemic equilibrium point was determined. Numerical simulation was carried out and further revealed that corruption can only be removed to a manageable level but not totally eliminated. (Legesse and Shiferaw, 2018) proposed and analyzed a mathematical model for the spread of corruption dynamics. The model is similar with characteristics to (Abdulrahman, 2018) except that an individual who loses immunity gained through council in jail does not directly join corrupt class rather susceptible because of human

behavior, and also, they are concerned with the impact of the awareness created by anticorruption. (Athithan et al. 2018) developed a deterministic model of the spread of corruption and its analysis using the differential equations. Then, the model extended to optimal control and the result shows that the level of corruption in the society can be reduced if corruption control efforts through media/punish

(Pabico, 2018) presented a model that treats corruption as an infectious disease that can infect both the public servants and the citizens through their interaction with one another. He developed three mathematical models based on the dynamics of the interactions (i) among public servants (ii) among citizens; and (iii) between public servants and citizens. The corruption is in the form of bribery, which comes in two types depending on who initiated the action: Harassment bribery if initiated by the public servant and collusive bribery (or facilitation payment) if initiated by the citizen. The model only considered that of harassment bribery which involves public servants that are honest, on a crossroads, corrupt, and dismissed, the former three of which interact with the citizens who are upright, apathetic, cooperator, and whistleblower.

Eguda *et al.* (2018) developed and analysed a mathematical model with standard incidence that investigates the dynamics of corruption as a disease. The total population is categorized into five compartments namely: Susceptible ( $S(t)$ ), Immune ( $V(t)$ ), Corrupt, ( $C(t)$ ), Prosecuted ( $P(t)$ ) and Removed ( $R(t)$ ). The immune class is generated from daily recruitment of individuals born into homes with good moral standards and can never become corrupt at a rate  $(1 - \theta)\beta$  while those prone to become corrupt progress to susceptible class at a rate  $\lambda$ . Susceptible individuals acquire infection from corrupt individuals and become corrupt at a rate  $\alpha$  while corrupt individuals are prosecuted at a rate  $\delta$ . Corrupt and prosecuted individuals become reformed while serving their jail term at rates  $\gamma$  and  $\rho$  respectively. Individuals in the removed class become susceptible after a while at a rate  $\omega$  while susceptible individuals prone to corruption become immune due to moral and religious beliefs as well as public

enlightenment campaign. All classes are subjected to natural death at a rate  $\mu$ .

**1.1 Formulation and description of the model**

A mathematical model of corruption dynamics incorporating the social stratification was developed. Using a compartmental approach, the population at time  $t$ , denoted by  $N(t)$ , is divided

into eight (8) mutually exclusive compartments, namely: Susceptible low-profile ( $S_L$ ) and high-profile ( $S_H$ ) individuals; low-profile ( $C_L$ ); high-profile ( $C_H$ ) Corrupt individuals; high-profile ( $J_L$ ) and high-profile ( $J_H$ ) Jailed individuals; low-profile ( $R_L$ ) and high-profile ( $R_H$ ) rehabilitated individuals.

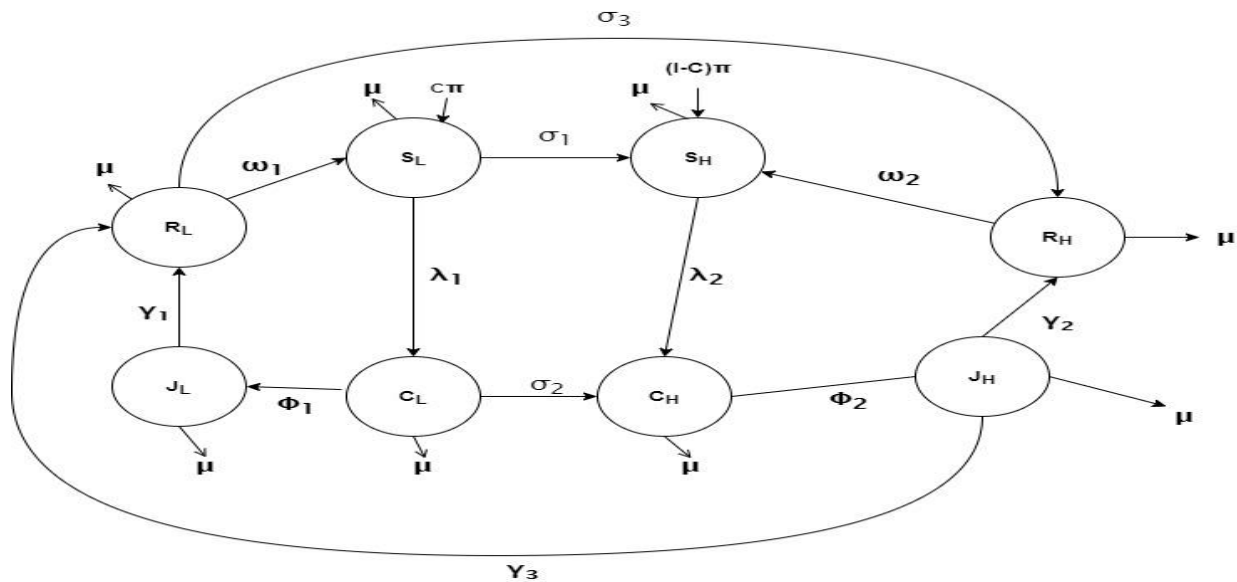


Figure 1: Schematic Diagram of the Model

The susceptible low profile  $S_L$  and high profile  $S_H$  population are generated from constant recruitment of individuals through births and immigration at the rate  $c\Lambda$  and  $(1 - c)\Lambda$  respectively. They become corrupt and move to the  $C_L$  and  $C_H$  classes respectively via effective contact with individuals in the  $C_L$  and  $C_H$  compartments, given by the term  $\lambda_1 = \frac{\beta(C_L + \eta_1 C_H)}{N}$  and  $\lambda_2 = \frac{\beta(C_H + \eta_2 C_L)}{N}$  respectively. The parameter  $\beta$  is the effective contact rate.  $\eta$  is the modification parameter associated with reduced infectiousness of  $C_h$  as compared to  $C_m$ . Individuals in the  $S_L$  and  $S_H$  compartments are increasing due to waning of rehabilitation  $R_L$  and  $R_H$  individuals at the rate  $\omega_1$  and  $\omega_2$  respectively.

The  $C_L$  and  $C_H$  compartment are generated via effective contact between  $S_L$  and  $S_H$  with individuals in the  $C_L$  and  $C_H$  compartments,

given by the term  $\frac{\beta(C_L + \eta_1 C_H)}{N}$  and  $\frac{\beta(C_L + \eta_2 C_H)}{N}$  as earlier explained. The classes  $C_L$  and  $C_H$  decrease due prosecution at the rates  $\phi_L$  and  $\phi_H$  respectively.

The  $C_H$  compartment increases as result of progression also from individuals in  $C_L$  whose social status improved due to wealth they gotten or position attained in the society at the rate  $\sigma_2$ . They decrease due to prosecution at the rates  $\phi_1$  and  $\phi_2$  respectively.

Individuals in  $J_L$  and  $J_H$  classes are similarly generated from  $C_L$  and  $C_H$  compartments due to prosecution as earlier noted at the rate  $\phi_1$  and  $\phi_H$  respectively. The  $J_L$  and  $J_H$  individuals

decreases due to adequate rehabilitation at the correctional centres at the rates  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$ . The  $R_L$  and  $R_H$  compartments is generated from  $J_L$  and  $J_H$  compartments due to adequate rehabilitation at the correctional centres  $J_L$  and  $J_H$  individuals at the rates the  $\gamma_1$  and  $\gamma_2$  respectively. Some proportion of  $J_H$  after prosecution will lose their social status or position and move to  $R_L$ . The classes  $R_L$  and  $R_H$  decreases due to waning of the discipline impacted at the correctional centres at the rate  $\omega_1$  and  $\omega_2$  to  $S_L$  and  $S_H$  compartments respectively. And finally, natural death occurs in all classes at the rate  $\mu$ .

The following assumptions are taken into account in the construction of the model.

- i. There is homogeneous mixing of the population, where all susceptible individuals are equally likely to become corrupted by the corrupt individuals in case of effective contact;
- ii. Appropriate punishment is given without fear nor favour to any (group of) person(s), found of corruption.
- iii. No death due to corruption.

Applying the assumptions, definition of parameters and variables, description of terms and the inter-relation (changes) between the variables and parameters (over-time) as described in the flow diagram of fig. 1, the corresponding mathematical equation can be described by a system of ordinary differential equations given as:

$$\frac{dS_L}{dt} = c\Lambda - \lambda_1 S_L - (\sigma_1 + \mu)S_L + \omega_1 R_1 \quad (1)$$

$$\frac{dS_H}{dt} = (1-c)\Lambda + \sigma_1 S_L - \lambda_2 S_H - \mu S_H + \omega_2 R_2 \quad (2)$$

$$\frac{dC_L}{dt} = \lambda_1 S_L - (\sigma_2 + \phi_1 + \mu)C_L \quad (3)$$

$$\frac{dC_H}{dt} = \lambda_2 S_H + \sigma_2 C_L - (\phi_2 + \mu)C_H \quad (4)$$

$$\frac{dJ_L}{dt} = \phi_1 C_L - (\gamma_1 + \mu)J_L \quad (5)$$

$$\frac{dJ_H}{dt} = \phi_2 C_H - (\gamma_2 + \gamma_3 + \mu)J_H \quad (6)$$

$$\frac{dR_L}{dt} = \gamma_1 J_L + \gamma_3 J_H - (\omega_1 + \sigma_3 + \mu)R_1 \quad (7)$$

$$\frac{dR_H}{dt} = \gamma_2 J_H + \sigma_3 R_L - (\omega_2 + \mu)R \quad (8)$$

where

$$N(t) = S_L(t) + S_H(t) + C_L(t) + C_K(t) + J_L(t) + J_H(t) + R_L(t) + R_H(t) \quad (9)$$

So that

$$\frac{dN}{dt} = \Lambda - \mu N \quad (8)$$

## 2.0 BASIC PROPERTIES OF THE MODEL

For the model to be epidemiological meaningful, it is important to prove that if a solution of the model differential equations starts on a given space, surface or curve (manifold or set) then it will remain within

for all time. Thus, a positively invariant set or manifold will have solutions that are positive for all time. In other words, the solutions of the model equations with positive initial data will remain positive for all  $t > 0$ . This is done below.

**Theorem 1.** The closed set

$$\Omega_1 = \left\{ (S_1, S_2, S_3, C_1, C_2, C_3 \in \mathbb{R}_+^6 : N = \frac{\Lambda}{\mu}) \right\} \quad (9)$$

for humans and

$$\Omega_2 = \left\{ L \in \mathbb{R}_+^1 : L = \frac{m_1 C_1 + m_2 C_2 + m_3 C_3}{\delta} \right\} \quad (10)$$

for concentration of corruption in the society, is positively-invariant and attract all positive solutions of model.

**Proof.** Adding the equations (1) – (6) for human population gives

$$\frac{dN}{dt} = \Lambda - \mu N \quad (11)$$

Thus, whenever  $N(t) > \frac{\Lambda}{\mu}$ , then  $\frac{dN}{dt} < 0$ . Solving equation (3.27) gives

$$N(t) = N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t}) \quad (12)$$

i.e.

$$N(t) = \frac{\Lambda}{\mu} + \left( N(0) - \frac{\Lambda}{\mu} \right) e^{-\mu t} \quad (13)$$

If  $N(0) < \frac{\Lambda}{\mu}$  then  $N(t) \leq \frac{\Lambda}{\mu}$  for all  $t > 0$ .  $N(0) > \frac{\Lambda}{\mu}$ , then either the solution enters  $\Omega_1$  in finite time or  $N(t)$  approaches to  $\frac{\Lambda}{\mu}$  as  $t \rightarrow \infty$ . Thus, the region  $\Omega_1$  is positively-invariant set under the flow described by equations (1) – (6) so that no solution path leaves through any boundary of  $\Omega_1$

And similarly, for the concentration of corruption within the society, we have

$$\frac{dL}{dt} = m_1 C_1 + m_2 C_2 + m_3 C_3 - \delta L \quad (14)$$

Thus, whenever  $L(t) > \frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta}$ , then  $\frac{dL}{dt} < 0$ . Solving equation (14) gives

$$L(t) = L(0)e^{-\delta t} + \frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta}(1 - e^{-\delta t}) \quad (15)$$

i.e.

$$L(t) = \frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta} + \left( L(0) - \frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta} \right) e^{-\delta t} \quad (16)$$

If  $L(0) < \frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta}$  then  $L(t) \leq \frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta}$  for all  $t > 0$ .  $L(0) > \frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta}$ , then either the solution enters  $\Omega_2$  in finite time or  $L(t)$  approaches to  $\frac{m_1 C_1(t) + m_2 C_2(t) + m_3 C_3(t)}{\delta}$  as  $t \rightarrow \infty$ . Thus, the region  $\Omega_2$  is positively-invariant set under the flow described by equations (7) so that no solution path leaves through any boundary of  $\Omega_2$

Hence, it is sufficient to consider the dynamics of the model in  $\Omega_1$  and  $\Omega_2$ . Thus, the model is mathematically and epidemiological well posed.

### 3.0 EXISTENCE OF CORRUPTION-ENDEMIC EQUILIBRIUM, $E^*$

At the corruption-endemic equilibrium we have persistence of infection. Thus, at least one of the infected classes is greater than zero. In order to find the positive endemic equilibrium of the system (5), denoted by

$$E^* = (S^{**}, C^{**}, J^{**}, H^{**}) \quad (17)$$

the equations in the system (5) are solved as explained in [32] in terms of the associated force of infection at steady-state, given by

$$\lambda^{**} = \frac{\beta C^{**}}{N^{**}} \quad (18)$$

Solving the equations of the model (5) at steady-state gives

$$\begin{aligned} S^{**} &= \frac{\Lambda}{K_1 + \lambda^{**}}, \\ C^{**} &= \frac{\lambda^{**} \Lambda K_3}{(K_1 + \lambda^{**})(K_2 K_3 - \phi \psi (1 - \theta))} \\ J^{**} &= \frac{\lambda^{**} \Lambda \phi}{(K_1 + \lambda^{**})(K_2 K_3 - \phi \psi (1 - \theta))} \\ H^{**} &= \frac{\Lambda \theta ((K_2 K_3 - \phi \psi (1 - \theta)) + (K_3 + \phi \psi K_3) \lambda^{**})}{(K_1 + \lambda^{**})(K_2 K_3 - \phi \psi (1 - \theta)) \mu} \end{aligned} \quad (19)$$

Using the second equation of system (19) in (18) and simplifying gives

$$\begin{aligned} \lambda^{**} &= \frac{\beta \lambda^{**} \Lambda K_3}{N^{**} (K_1 + \lambda^{**})(K_2 K_3 - \phi \psi (1 - \theta))} \\ N^{**} K_1 (K_2 K_3 - \phi \psi (1 - \theta)) + N^{**} \lambda^{**} (K_2 K_3 - \phi \psi (1 - \theta)) &= \beta \Lambda K_3 \\ N^{**} \lambda^{**} (K_2 K_3 - \phi \psi (1 - \theta)) &= \beta \Lambda K_3 - N^{**} K_1 (K_2 K_3 - \phi \psi (1 - \theta)) \\ \lambda^{**} &= \frac{\beta \Lambda K_3}{N^{**} (K_2 K_3 - \phi \psi (1 - \theta))} - K_1 \end{aligned} \quad \{***\}$$

$$\lambda^{**} = K_1 \left( \frac{K_3 \beta \Lambda}{K_1 (K_2 K_3 - \phi \psi (1 - \theta)) N^{**}} - 1 \right)$$

i.e.

$$\lambda^{**} = K_1 (R_0 - 1) \quad (20)$$

Since all model parameters are assumed nonnegative with  $\mu > 0$ , it follows that  $\lambda^{**} > 0$ , whenever  $R_0 > 1$ . The components of  $E^*$  are then determined by substituting (20) into (19), given by

$$\begin{aligned}
 S^{**} &= \frac{\Lambda}{K_1 R_0} \\
 C^{**} &= \frac{\Lambda K_3 (R_0 - 1)}{(K_2 K_3 - \phi \psi (1 - \theta)) R_0} \\
 J^{**} &= \frac{\Lambda \phi (R_0 - 1)}{(K_2 K_3 - \phi \psi (1 - \theta)) R_0} \\
 H^{**} &= \frac{\Lambda \theta ((K_2 K_3 - \phi \psi (1 - \theta)) + (K_3 + \phi \psi K_3) (R_0 - 1))}{\mu (K_2 K_3 - \phi \psi (1 - \theta)) R_0}
 \end{aligned}
 \tag{21}$$

Noting that  $R_0 < 1$  implies that the force of infection at steady state ( $\lambda^{**}$ ) is negative (which is biologically meaningless). And hence, the model has no positive equilibria in this case. Thus, we established the following result.

**Lemma1.** The system has a unique endemic (positive) equilibrium whenever  $R_0 > 1$ , and no positive equilibrium otherwise.

### 3.1 Global stability of corruption-endemic equilibrium, $E^*$

**Theorem 3:** If  $R_0 > 1$ , then the corruption-endemic equilibrium,  $E^*$  of (5) is globally asymptotically stable (GAS) in  $\Omega$  when

$$\begin{aligned}
 \frac{S}{S^{**}} &= \frac{C}{C^{**}} \\
 \psi &= 1
 \end{aligned}
 \tag{22}$$

**Proof:** Consider the Lyapunov function:

$$\begin{aligned}
 U &= \left( S - S^{**} - S^{**} \ln \frac{S}{S^{**}} \right) + \left( C - C^{**} - C^{**} \ln \frac{C}{C^{**}} \right) \\
 &+ \frac{\psi (1 - \theta) J^{**}}{\phi C^{**}} \left( J - J^{**} - J^{**} \ln \frac{J}{J^{**}} \right) + \frac{H^{**}}{\theta} + \left( H - H^{**} - H^{**} \ln \frac{H}{H^{**}} \right) \\
 &+ \frac{1}{2\mu} \left[ (S - S^{**}) + (C - C^{**}) + (J - J^{**}) + (H - H^{**}) \right]^2
 \end{aligned}
 \tag{23}$$

its derivatives along the solutions of the model equations is

$$\begin{aligned}
 \dot{U} &= \left( 1 - \frac{S^{**}}{S} \right) \dot{S} + \left( 1 - \frac{C^{**}}{C} \right) \dot{C} + \frac{\psi (1 - \theta) J^{**}}{\phi C^{**}} \left( 1 - \frac{J^{**}}{J} \right) \dot{J} + \frac{H^{**}}{\theta} \left( 1 - \frac{H^{**}}{H} \right) \dot{H} \\
 &+ \frac{1}{\mu} \left[ \begin{aligned} &(S - S^{**}) + (C - C^{**}) \\ &+ (J - J^{**}) + (H - H^{**}) \end{aligned} \right] (\dot{S} + \dot{C} + \dot{J} + \dot{H}) \\
 \dot{U} &= \left( 1 - \frac{S^{**}}{S} \right) \left( \Lambda - \frac{\beta CS}{N} - K_1 S \right) + \left( 1 - \frac{C^{**}}{C} \right) \left( \frac{\beta CS}{N} + \psi (1 - \theta) J - K_2 C \right) \\
 &+ \frac{\psi (1 - \theta) J^{**}}{\phi C^{**}} \left( 1 - \frac{J^{**}}{J} \right) (\phi C - K_3 J) \\
 &+ \frac{H^{**}}{\theta} \left( 1 - \frac{H^{**}}{H} \right) (\theta S + \theta C + \psi \theta J - \mu H) \\
 &+ \frac{1}{\mu} \left[ \begin{aligned} &(S - S^{**}) + (C - C^{**}) \\ &+ (J - J^{**}) + (H - H^{**}) \end{aligned} \right] (\Lambda - \mu (S + C + J + H))
 \end{aligned}
 \tag{24}$$

Considering (8) at  $E^*$ , we can deduced that

$$\Lambda = \mu(S^{**} + C^{**} + J^{**} + H^{**}) \quad (25)$$

Then, from (22) and (24), (25) becomes

$$\begin{aligned} \dot{U} = & \left(1 - \frac{S^{**}}{S}\right) \left(\frac{\beta C^{**} S^{**}}{N^{**}} + K_1 S^{**} - \frac{\beta C S}{N} - K_1 S\right) \\ & + \left(1 - \frac{C^{**}}{C}\right) \left(\frac{\beta C S}{N} + \psi(1-\theta)J - \frac{\beta C S^{**}}{N^{**}} - \frac{\psi(1-\theta)CJ^{**}}{C^{**}}\right) \\ & + \frac{\psi(1-\theta)J^{**}}{\phi C^{**}} \left(1 - \frac{J^{**}}{J}\right) \left(\phi C - \frac{\phi C^{**} J}{J^{**}}\right) \\ & + \frac{H^{**}}{\theta} \left(1 - \frac{H^{**}}{H}\right) \left(\theta(S + C + \psi J) - \theta\left(\frac{S^{**} H}{H^{**}} - \frac{C^{**} H}{H^{**}} - \frac{\psi J^{**} H}{H^{**}}\right)\right) \\ & + \frac{1}{\mu} \left[ \begin{array}{l} (S - S^{**}) + (C - C^{**}) \\ + (J - J^{**}) + (H - H^{**}) \end{array} \right] \left[ \begin{array}{l} \mu(S^{**} + C^{**} + J^{**} + H^{**}) \\ - \mu(S + C + J + H) \end{array} \right] \end{aligned}$$

Let  $\alpha = \frac{\beta}{N}$ ,  $\alpha^* = \frac{\beta}{N^{**}}$ , then, we have

$$\begin{aligned} \dot{U} = & \left(1 - \frac{S^{**}}{S}\right) \left(\alpha^* C^{**} S^{**} \left(1 - \frac{\alpha C S}{\alpha^* C^{**} S^{**}}\right) - K_1(S - S^{**})\right) \\ & + \left(1 - \frac{C^{**}}{C}\right) \left(\alpha^* C^{**} S^{**} \left(\frac{\alpha C S}{\alpha^* C^{**} S^{**}} - \frac{C}{C^{**}}\right) + \psi(1-\theta)J^{**} \left(\frac{J}{J^{**}} - \frac{C}{C^{**}}\right)\right) \\ & + \frac{\psi(1-\theta)J^{**}}{\phi C^{**}} \left(1 - \frac{J^{**}}{J}\right) \left(\phi C^{**} \left(\frac{C}{C^{**}} - \frac{J}{J^{**}}\right)\right) \\ & + H^{**} \left(1 - \frac{H^{**}}{H}\right) \left\{ \begin{array}{l} \left(\frac{S H^{**} - S^{**} H}{H^{**}}\right) \\ + \left(\frac{C H^{**} - C^{**} H}{H^{**}}\right) + \psi \left(\frac{J H^{**} - J^{**} H}{H^{**}}\right) \end{array} \right\} \\ & + \frac{1}{\mu} \left[ \begin{array}{l} (S - S^{**}) + (C - C^{**}) \\ + (J - J^{**}) + (H - H^{**}) \end{array} \right] \left[ \begin{array}{l} -\mu \left( (S - S^{**}) + (C - C^{**}) \right) \\ + (J - J^{**}) + (H - H^{**}) \end{array} \right] \end{aligned}$$

$$\begin{aligned} \dot{U} = & \frac{-K_1(S - S^{**})^2}{S} \\ & + \alpha^* C^{**} S^{**} \left[ \left( 1 - \frac{\alpha CS}{\alpha^* C^{**} S^{**}} \right) \left( 1 - \frac{S}{S} \right) + \left( \frac{\alpha CS}{\alpha^* C^{**} S^{**}} - \frac{C}{C^{**}} \right) \left( 1 - \frac{C}{C} \right) \right] \\ & + \psi(1 - \theta) J^{**} \left\{ \left( 1 - \frac{C}{C} \right) \left( \frac{J}{J^{**}} - \frac{C}{C^{**}} \right) + \left( 1 - \frac{J}{J} \right) \left( \frac{C}{C^{**}} - \frac{J}{J^{**}} \right) \right\} \\ & + H^{**} \left( 1 - \frac{H}{H} \right) \left\{ \left( \frac{SH^{**} - S^{**}H}{H^{**}} \right) \right. \\ & \left. + \left( \frac{CH^{**} - C^{**}H}{H^{**}} \right) + \psi \left( \frac{JH^{**} - J^{**}H}{H^{**}} \right) \right\} \\ & - \left( \frac{(S - S^{**}) + (C - C^{**})}{+(J - J^{**}) + (H - H^{**})} \right) \left( \frac{(S - S^{**}) + (C - C^{**})}{+(J - J^{**}) + (H - H^{**})} \right) \end{aligned}$$

$$\begin{aligned} \dot{U} = & \frac{-K_1(S - S^{**})^2}{S} \\ & + \alpha^* C^{**} S^{**} \left[ \left( 1 - \frac{\alpha CS}{\alpha^* C^{**} S^{**}} \right) \left( 1 - \frac{S}{S} \right) + \left( \frac{\alpha CS}{\alpha^* C^{**} S^{**}} - \frac{C}{C^{**}} \right) \left( 1 - \frac{C}{C} \right) \right] \\ & + \psi(1 - \theta) J^{**} \left\{ \left( 1 - \frac{C}{C} \right) \left( \frac{J}{J^{**}} - \frac{C}{C^{**}} \right) + \left( 1 - \frac{J}{J} \right) \left( \frac{C}{C^{**}} - \frac{J}{J^{**}} \right) \right\} \\ & + \left( \frac{H - H^*}{H} \right) \left\{ \left( \frac{SH^* - S^*H}{H} \right) \right. \\ & \left. + \left( \frac{CH^* - C^{**}H}{H} \right) + \psi \left( \frac{JH^* - J^{**}H}{H} \right) \right\} \\ & - \left( \frac{(S - S^{**}) + (C - C^{**})}{+(J - J^{**}) + (H - H^*)} \right) \left( \frac{(S - S^{**}) + (C - C^{**})}{+(J - J^{**}) + (H - H^*)} \right) \end{aligned}$$

Let  $(x, y, z) = \left( \frac{S}{S^{**}}, \frac{C}{C^{**}}, \frac{J}{J^{**}} \right)$  and  $g(N) = \frac{\alpha}{\alpha^*}$ , then, we get

$$\begin{aligned} \dot{U} = & \frac{-K_1(S - S^*)^2}{S} \\ & + \alpha^* C^{**} S^{**} \left[ \left( 1 - g(N)xy \right) \left( 1 - \frac{1}{x} \right) + \left( g(N)xy - y \right) \left( 1 - \frac{1}{y} \right) \right] \\ & + \psi(1 - \theta) J^{**} \left\{ \left( 1 - \frac{1}{y} \right) (z - y) + \left( 1 - \frac{1}{z} \right) (y - z) \right\} \\ & + \left( \frac{H - H^{**}}{H} \right) \left\{ \left( \frac{SH^{**} - S^{**}H}{H} \right) + \left( \frac{CH^{**} - C^{**}H}{H} \right) \right. \\ & \left. + \psi \left( \frac{JH^{**} - J^{**}H}{H} \right) \right\} \\ & - \left[ (S - S^{**}) + (C - C^{**}) + (J - J^{**}) \right] \left[ \frac{(S - S^{**}) + (C - C^{**})}{+(J - J^{**}) + (H - H^{**})} \right] \\ & - (H - H^{**}) \left[ (S - S^{**}) + (C - C^{**}) + (J - J^{**}) \right] \\ & - (H - H^{**})(H - H^{**}) \end{aligned}$$

$$\begin{aligned}
 \dot{U} = & - \left\{ \begin{aligned} & \frac{K_1 C^{**} (S - S^{**})^2}{S} \\ & + \left[ (S - S^{**}) + (C - C^{**}) + (J - J^{**}) \right] \left[ \left( \frac{(S - S^{**}) + (C - C^{**})}{+(J - J^{**}) + (H - H^{**})} \right) \right] \\ & + (H - H^{**})^2 \end{aligned} \right\} \\
 & + \alpha^* C^{**} S^{**} \left[ \left(1 - g(N)xy\right) \left(1 - \frac{1}{x}\right) + (g(N)xy - y) \left(1 - \frac{1}{y}\right) \right] \\
 & + \psi(1 - \theta) J^{**} \left\{ \left(1 - \frac{1}{y}\right)(z - y) + \left(1 - \frac{1}{z}\right)(y - z) \right\} \\
 & + \left( \frac{H - H^{**}}{H} \right) \left\{ \begin{aligned} & (SH^{**} - S^{**}H) \\ & + (CH^{**} - C^{**}H) + \psi(JH^{**}J^{**}H) \end{aligned} \right\} \\
 & - (H - H^{**}) \left( (S - S^{**}) + (C - C^{**}) + (J - J^{**}) \right) \\
 \dot{U} = & - \left\{ \begin{aligned} & \frac{K_1 C^{**} (S - S^{**})^2}{S} + (H - H^{**})^2 \\ & + \left[ (S - S^{**}) + (C - C^{**}) + (J - J^{**}) \right] \left[ \left( \frac{(S - S^{**}) + (C - C^{**})}{+(J - J^{**}) + (H - H^{**})} \right) \right] \end{aligned} \right\} \tag{34} \\
 & + \alpha^* C^{**} S^{**} f_1 + \psi(1 - \theta) J^{**} f_2 + f_3
 \end{aligned}$$

where

$$\begin{aligned}
 f_1 &= \left[ \left(1 - g(N)xy\right) \left(1 - \frac{1}{x}\right) + (g(N)xy - y) \left(1 - \frac{1}{y}\right) \right] \\
 f_2 &= \left\{ \left(1 - \frac{1}{y}\right)(z - y) + \left(1 - \frac{1}{z}\right)(y - z) \right\} \\
 f_3 &= \left( \frac{H - H^{**}}{H} \right) \left\{ \begin{aligned} & (SH^{**} - S^{**}H) \\ & + (CH^{**} - C^{**}H) + \psi(JH^{**}J^{**}H) \end{aligned} \right\} \\
 & - (H - H^{**}) \left( (S - S^{**}) + (C - C^{**}) + (J - J^{**}) \right)
 \end{aligned}$$

Considering  $f_1$ , we have

$$\begin{aligned}
 f_1 &= \left[ \left(1 - g(N)xy\right) \left(1 - \frac{1}{x}\right) + (g(N)xy - y) \left(1 - \frac{1}{y}\right) \right] \\
 &= g(N)xy \left( \frac{1}{x} - 1 + 1 - \frac{1}{y} \right) + 1 - \frac{1}{x} - y + 1 \\
 &= 2 - \frac{1}{y} - y \quad (\text{with } x = y)
 \end{aligned}$$

i.e.

$$f_1 = 2 - \frac{C^{**}}{C} - \frac{C}{C^{**}}$$

Similarly, for  $f_2$ , we have

$$\begin{aligned}
 f_2 &= \left\{ \left( 1 - \frac{1}{y} \right) (z - y) + \left( 1 - \frac{1}{z} \right) (y - z) \right\} \\
 &= (z - y) \left( 1 - \frac{1}{y} - 1 + \frac{1}{z} \right) \\
 &= (z - y) \left( \frac{1}{z} - \frac{1}{y} \right) \\
 &= 1 - \frac{z}{y} - \frac{y}{z} + 1 \\
 &= 2 - \frac{z}{y} - \frac{y}{z}
 \end{aligned}$$

i.e.

$$f_2 = 2 - \frac{C^{**}J}{CJ^{**}} - \frac{CJ^{**}}{C^{**}J}$$

And for  $f_3$ , with  $\psi = 1$ , we have

$$\begin{aligned}
 f_3 &= \left( \frac{H - H^{**}}{H} \right) (SH^{**} - S^{**}H) - (H - H^{**})(S - S^{**}) \\
 &\quad + \left( \frac{H - H^{**}}{H} \right) (CH^{**} - C^{**}H) - (H - H^{**})(C - C^{**}) \\
 &\quad + \left( \frac{H - H^{**}}{H} \right) (JH^{**} - J^{**}H) - (H - H^{**})(J - J^{**}) \\
 &= (SHH^{**} - S^{**}HH - SH^{**}H^{**} + S^{**}HH^{**}) - (SHH - SHH^{**} - S^{**}HH + S^{**}HH^{**}) \\
 &\quad + (CHH^{**} - C^{**}HH - CH^{**}H^{**} + C^{**}HH^{**}) - (CHH - CHH^{**} - C^{**}HH + C^{**}HH^{**}) \\
 &\quad + (JHH^{**} - J^{**}HH - JH^{**}H^{**} + J^{**}HH^{**}) - (JHH - JHH^{**} - J^{**}HH + J^{**}HH^{**}) \\
 &= (2SHH^{**} - SH^{**}H^{**} - SHH) \\
 &\quad + (2CHH^{**} - CH^{**}H^{**} - CHH) + (2JHH^{**} - JH^{**}H^{**} - JHH) \\
 &= SHH^{**} \left( 2 - \frac{H^{**}}{H} - \frac{H}{H^{**}} \right) + CHH^{**} \left( 2 - \frac{H^{**}}{H} - \frac{H}{H^{**}} \right) + JHH^{**} \left( 2 - \frac{H^{**}}{H} - \frac{H}{H^{**}} \right)
 \end{aligned}$$

i.e.

$$f_3 = HH^{**} (S + C + J) \left( 2 - \frac{H^{**}}{H} - \frac{H}{H^{**}} \right)$$

Thus, equation (34), becomes

$$\begin{aligned}
 \dot{U} &= - \left\{ \frac{K_1 C^{**} (S - S^{**})^2}{S} + (H - H^{**})^2 \right. \\
 &\quad \left. + \left[ \begin{array}{l} (S - S^{**}) \\ + (J - J^{**}) + (C - C^{**}) \end{array} \right] \left[ \begin{array}{l} ((S - S^{**}) + (C - C^{**})) \\ + (J - J^{**}) + (H - H^{**}) \end{array} \right] \right\} \\
 &\quad + \alpha^* C^{**} S^{**} \left( 2 - \frac{C^{**}}{C} - \frac{C}{C^{**}} \right) + \psi (1 - \theta) J^{**} \left( 2 - \frac{C^{**}J}{CJ^{**}} - \frac{CJ^{**}}{C^{**}J} \right) \\
 &\quad + HH^{**} (S + C + J) \left( 2 - \frac{H^{**}}{H} - \frac{H}{H^{**}} \right)
 \end{aligned}$$

And by the arithmetic-geometric mean inequality, the following inequalities hold

$$\left(2 - \frac{C^{**}}{C} - \frac{C}{C^{**}}\right), \left(2 - \frac{C^{**}J}{CJ^{**}} - \frac{CJ^{**}}{C^{**}J}\right), \left(2 - \frac{H^{**}}{H} - \frac{H}{H^{**}}\right) \leq 0$$

Thus,  $\dot{U}(S, C, J, H)$  is less or equal to zero with equality only if  $\frac{C}{C^{**}} = \frac{J}{J^{**}} = \frac{H}{H^{**}} = 1$ . Then, the largest invariant set of system (5) on the set  $\{(S, C, J, H) \in \mathbb{R}_+^4 : \dot{U}(S, C, J, H) = 0\}$  is the endemic equilibrium point  $E^*$ . Thus, by LaSalle's invariance principle, see (J. P. Lasalle, 1976) it follows that the endemic equilibrium  $E^*$  is globally asymptotically stable when the conditions in (22) are satisfied.

#### 4.0 CONCLUSION

We have developed and rigorously analyzed a compartmental model of (Kermack-Mckenderick) type to monitor the transmission dynamics of Corruption. It is confirmed that the local asymptotic stability of the disease-free equilibrium and the global asymptotic stability of the unique endemic equilibrium is guaranteed whenever the computed basic reproduction number  $R_0$  is below and  $R_0$  greater than unity respectively.

Furthermore, it should also be emphasized that the present study will be strengthened in future research work by analyzing and investigating the modern fractional operators and optimal control system. The control strategies are indeed a significant step in drastically curtailing the unknown characteristics and other features of corruption epidemic.

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