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A Study of Some New Statistics on the  $\Gamma_1$  - non-Deranged Permutations\*<sup>1</sup>Ibrahim M. and<sup>2</sup>A.K. Musa<sup>1</sup>Department of Mathematics, Usmanu Danfodiyo University, Sokoto State, Nigeria<sup>2</sup>Department of Mathematics, Federal University Birnin-Kebbi, Kebbi State, Nigeria\*Corresponding Author's Email: [muhammad.ibrahim@udusok.edu.ng](mailto:muhammad.ibrahim@udusok.edu.ng).

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## ABSTRACT

In this study, we looked at some novel statistics on the  $\Gamma_1$  -non deranged permutation group, a symmetric group subgroup. We analyzed and redefined some of the statistic namely Lmap, Lmal, Rmip, Rmil and Rmal on  $\Gamma_1$  -non deranged permutations. We found that in all  $\Gamma_1$  -non deranged permutations, Lmap is equivalent to Lmal. In addition, we observed that Rmal of  $\omega_1$  is empty for any  $G_p^{\Gamma_1}$  and other combinatorial features.

**Keywords:**  $\Gamma_1$  -nonderangedpermutations; left to right maximum places; right to left minimum letters; right to left maximum letters; MahonianStatistics.

## 1.0 INTRODUCTION

The study of permutation statistics has become a popular field of study among authors. This is due to its use in engineering and science, It deals with permutations whose constituent numbers are analyzed in terms of their arrangement. The term "permutation statistics" was originally used by MacMahon (1915), who described it in terms of a permutation  $\pi \in S_n$ , as well as the number of descents ( $des\pi$ ), excedance ( $exc\pi$ ), inversions ( $inv\pi$ ), and major indices ( $maj\pi$ ). He demonstrated that over the symmetric group  $S_n$ , the  $exc\pi$  is equally distributed with  $des\pi$  and the  $inv\pi$  is equally distributed with  $maj\pi$ . There are two main categories of permutation statistics: Eulerian statistics and Mahonian statistics. An Eulerian statistic is any statistic that is equally distributed among the  $des$ , while a Mahonian statistic is equally distributed among the  $inv$ . Aunu permutation pattern first arose, out of attempts to provide some combinatorial interpretations of some succession scheme and today series of results

ranging from its avoiding class to its properties which have also been studied. Garba and Ibrahim (2010) developed a strategy for the prime numbers  $p \geq 5$  and  $\Omega \subseteq N$  employing the catalan numbers as well. This scheme creates a cycle of permutation patterns that is utilized to decide the arrangements. Researchers have over time looked also at permutation group with certain properties; one that comes to mind is the permutation patterns that have any of the element fixed or the one that has no fixed element, here the idea of deranged and non-deranged permutation surface. It is in line with this understanding that Ibrahim *et al.* (2016) modified the scheme of Garba and Ibrahim (2010) to two line notation and the scheme generated a set of permutations with a fix at 1 (which generated the natural identity). This obtained set of permutations form permutation group called the  $\Gamma_1$  -non deranged permutation group and is denoted as  $G_p^{\Gamma_1}$ . Ibrahim *et al.* (2017) outlined the theoretical characteristics of the Ascent set in regard to the  $\Gamma_1$  -non deranged permutation and

demonstrate that the union of the Ascent set equals the identity. They also noted that the difference between  $Asc(\omega_i)$  and  $Asc(\omega_{p-1})$  is one. Aremuet *al.*(2018) utilized the direct and skew sum operation on the components of the  $\Gamma_1$ -non deranged permutation group and showed the relationships and schemes on the structures and fixed point of the permutations generated from these operations. Additionally, if  $\pi$  is the direct sum of these  $\Gamma_1$ -non deranged permutations, then the collection of permutations in the form of  $\pi$  is an abelian group, designated as  $G_p^{\Gamma_m \otimes}$ . According to Aremuet *al.*(2019), the  $Res(\omega_i)$  and  $Res(\omega_{p-1})$  of  $\Gamma_1$ -non deranged permutations are equally distributed between the right and left embracing numbers, respectively. Additionally, it notices that the height of the weighted motzkin path of  $\omega_i$  is the same height as the height of  $\omega_{p-des(\omega_i)}$  motzkin path. Ibrahim and Garba (2019) studied ascent and descent blocks of  $\Gamma_1$  - non deranged permutations. moreover, it defined the mapping  $\Psi_{AI} : G_p^{\Gamma_m} \rightarrow \Omega_p$  which converts the permutation from the weighted Motzkin path in the  $\Omega_p$  with respect to the ascent and descent blocks from the  $\Gamma_1$  - non deranged permutations group  $G_p^{\Gamma_m}$ . Inversion number and major index are not equally distributed in  $\Gamma_1$ -non deranged permutations, as demonstrated by Garba and Ibrahim (2019), who also established that the difference between the sums of the major index and the inversion numbers is equal to the sum of the descent numbers in these permutations. Ibrahim and Muhammad (2019) produced recursion formulas for the maximum and lowest block counts in  $\Gamma_1$ -non deranged permutations. They also noted that  $arc(\omega_i)$  and  $asc(\omega_i)$  are equally distributed in  $\Gamma_1$ -non deranged permutations. Using the membership function that was built for the fuzzy set on  $G_p'$ , Ibrahimet *al.* (2021) investigated some algebraic theoretic properties of the fuzzy set on  $G_p'$  and established the results for the algebraic operators of the fuzzy set on  $G_p'$ , which are the

algebraic sum, algebraic product, bounded sum, and bounded difference. They also built a relationship between the operators and the fuzzy set on  $G_p'$ . More recently Ibrahimet *al.* (2022) proved that the right embracing sum  $Res(\omega_i)$  is equal to the right embracing sum  $Res(\omega_{p-i})$  where  $1 \leq i \leq p-1$  and  $Res(\omega_i) = \frac{(p-3)(p-1)}{8}$ , where  $i = \frac{p-1}{2}, \frac{p+1}{2}$ . It also observed that the left embracing sum  $Les(\omega_i)$  is equal to the right embracing sum  $Res(\omega_i)$  and  $Les(\omega_{\frac{p+1}{2}}) = \frac{p^2-1}{8}$ , where  $p \geq 5$ .

In this paper, we show that left to right maximum places of  $\omega_i Lmap(\omega_i)$  is equal to left to right maximum letters of  $\omega_i Lmal(\omega_i)$ . We also establish that right to left maximum letters of  $\omega_i Rmal(\omega_i)$  is empty for any  $\Gamma_1$  - non deranged permutations.

**2.0 MATERIALS AND METHODS**

In this section before we outline the main results in this research paper, we attempt to define some basic concept that will help in further understanding of this work.

**Definition 2.1**Magami and Ibrahim (2021)

Let  $\Gamma$  be a non empty set of prime cardinality  $p \geq 5$  such that  $\Gamma \subset N$  A bijection  $\omega$  on  $\Gamma$  of the form

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & p \\ 1 & (1+i)_{mop} & (1+2i)_{mop} & \dots & (1+(p-1)i)_{mop} \end{pmatrix}$$

is called a  $\Gamma_1$ -non deranged permutation. We denoted  $G_p$  to be the set of all  $\Gamma_1$ -non deranged permutations.

$G_p = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$  is the set of all  $\Gamma_1$ -non deranged permutations where  $p = 7$

By definition 2.1,  $G_7$  is generated as follows

$$\omega_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$$

$$\omega_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 5 & 7 & 2 & 4 & 6 \end{pmatrix}$$

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 7 & 3 & 6 & 2 & 5 \end{pmatrix}$$

$$\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 5 & 2 & 6 & 3 & 7 & 4 \end{pmatrix}$$

$$\omega_5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 6 & 4 & 2 & 7 & 5 & 3 \end{pmatrix}$$

$$\omega_6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 7 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}$$

**Definition 2.2** Ibrahimet al. (2017)

The pair  $G_p$  and the natural permutation composition forms a group which is denoted as  $G_p^{\Gamma_1}$ . This is a special permutation group which fixes the first element of  $\Gamma$ .

**Definition2.3**

Given a permutation  $\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}$

We define left to right maximum places as

$$Lmap(\omega_i) = \{i : \sigma(j) < \sigma(i), j < i\} \text{ and}$$

$$lmap = |Lmap(\omega_i)|.$$

**Definition2.4.**

Given a permutation  $\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}$

We define left to right maximum letters as

$$Lmal(\omega_i) = \{\sigma(i) : \sigma(j) < \sigma(i), j < i\} \text{ and}$$

$$lmal = |Lmal(\omega_i)|.$$

**Definition2.5**

Given a permutation

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}$$

We define right to left minimum positions as

$$Rmip(\omega_i) = \{i : \sigma(j) > \sigma(i), j > i\} \text{ and}$$

$$rmip = |Rmip(\omega_i)|$$

**Definition2.6**

Given a permutation

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}$$

We define right to left minimum letters as

$$Rmil(\omega_i) = \{\sigma(i) : \sigma(j) > \sigma(i), j > i\} \text{ and}$$

$$rmil = |Rmil(\omega_i)|$$

**Definition2.7**

Given a permutation

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}$$

We define right to left maximum letters as

$$Rmal(\omega_i) = \{\sigma(i) : \sigma(j) < \sigma(i), j > i\} \text{ and}$$

$$rmal = |Rmal(\omega_i)|$$

### 3.0 RESULTS ANDDISCUSSION

In this section, we presented some results on left to right maximum places, left to right maximum letters, right to left minimum positions, right to left minimum letters and right to left maximum letters of subgroup  $G_p^{\Gamma_1}$  of  $S_p$  (Symmetry group of prime order with  $p \geq 5$ ).

**Proposition 3.1** Let  $\omega_i \in G_p^{\Gamma_1}$  Then,

$$Lmap(\omega_i) = \{2, 3, \dots, p\}$$

**Proof.**

Let  $\omega_i \in G_p^{\Gamma_1}$  be represented by

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & p \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}, \text{ where}$$

$\sigma(1) = 1$ . Thus,  $\sigma(1) = 1$  is at the first position and is the least letter in the permutation. Therefore, for any  $i \in \{1, 2, 3, \dots, p\}$  with  $1 < i$ , we have that

$\sigma(1) = 1 < \sigma(i)$ , then

$$Lmap(\omega_i) = \{i : \sigma(j) < \sigma(i), j < i\} = \{2, 3, \dots, p\}$$

**Proposition 3.2** Let  $\omega_i \in G_p^{\Gamma_1}$  Then

$$Lmal(\omega_i) = \{2, 3, \dots, p\}$$

**Proof.**

Let  $\omega_i \in G_p^{\Gamma_1}$  be represented by

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & p \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}, \text{ where}$$

$\sigma(1) = 1$ . Thus,  $\sigma(1) = 1$  is at the first position and is the least letter in the permutation. Therefore for any  $i \in \{1, 2, 3, \dots, p\}$  with  $1 < i$  we have that  $\sigma(1) = 1 < \sigma(i)$  then

$$Lmal(\omega_i) = \{\sigma(i) : \sigma(j) < \sigma(i), j < i\} = \{2, 3, \dots, p\}$$

**Proposition 3.3** Let  $\omega_i \in G_p^{\Gamma_1}$  Then,

$$Lmap(\omega_i) = Lmal(\omega_i)$$

**Proof.**

From proposition 3.1, we have that any  $\omega_i \in G_p^{\Gamma_1}$

$$Lmap(\omega_i) = \{2, 3, \dots, p\}.$$

Similarly, from proposition 3.2, we have that for any  $\omega_i \in G_p^{\Gamma_1}$ ,  $Lmal(\omega_i) = \{2, 3, \dots, p\}$ .

Therefore,  $Lmap(\omega_i) = Lmal(\omega_i)$ .

**Proposition 3.4** Let  $\omega_i \in G_p^{\Gamma_1}$  Then,

$$lmap(\omega_i) = p - 1$$

**Proof.**

From proposition 3.1, we have that any  $\omega_i \in G_p^{\Gamma_1}$

$Lmap(\omega_i) = \{2, 3, \dots, p\}$  Hence, it has  $p - 1$  elements.

**Proposition 3.5** Let  $\omega_i \in G_p^{\Gamma_1}$  Then,

$$lmal(\omega_i) = p - 1$$

**Proof.**

From proposition 3.2, we have that any

$\omega_i \in G_p^{\Gamma_1}$   $Lmal(\omega_i) = \{2, 3, \dots, p\}$ . Hence, it has  $p - 1$  elements.

**Proposition 3.6** Let  $\omega_i \in G_p^{\Gamma_1}$  Then

$$lmap(\omega_i) = lmal(\omega_i)$$

**Proof.**

From proposition 3.3. we have that any  $\omega_i \in G_p^{\Gamma_1}$

$Lmap(\omega_i) = Lmal(\omega_i)$ . Also, from proposition

3.4 for any  $\omega_i \in G_p^{\Gamma_1}$   $lmap(\omega_i) = p - 1$  and

similarly from proposition 3.5 for any  $\omega_i \in G_p^{\Gamma_1}$

$lmal(\omega_i) = p - 1$  Therefore,

$$lmap(\omega_i) = lmal(\omega_i) = p - 1$$

**Proposition 3.7** Let  $\omega_i \in G_p^{\Gamma_1}$  Then,

$$Rmip(\omega_i) = \{1, 2, \dots, p - 1\}$$

**Proof.**

Let  $\omega_i \in G_p^{\Gamma_1}$  be represented by

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & p \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}, \text{ where}$$

$\sigma(1) = 1, i \in \{1, 2, 3, \dots, p\}$  Thus,  $\sigma(p) = p$  is at the least position and is the biggest letter in the permutation. Therefore, for any  $i \in \{1, 2, 3, \dots, p\}$  with  $p > 1$ , we have that  $\sigma(p) = p > \sigma(i)$ . Hence,

$$Rmip(\omega_i) = \{i : \sigma(j) > \sigma(i), j > i\} = \{1, 2, \dots, p - 1\}$$

**Proposition 3.8** Let  $\omega_i \in G_p^{\Gamma_1}$ . Then,

$$Rmil(\omega_i) = \{1, 2, \dots, p - 1\}$$

**Proof.**

Let  $\omega_i \in G_p^{\Gamma_1}$  be represented by

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \dots & p \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}, \text{ where}$$

$\sigma(1) = 1, i \in \{1, 2, 3, \dots, p\}$  Thus,  $\sigma(p) = p$  is at the least position and is the biggest letter in the permutation. Therefore for any  $i \in \{1, 2, 3, \dots, p\}$  with  $p > 1$ , we have that  $\sigma(p) = p > \sigma(i)$ .

Hence,

$$Rmil(\omega_1) = \{ \sigma(i) : \sigma(j) > \sigma(i), j > i \}$$

$$= \{1, 2, \dots, p-1\}$$

**Proposition 3.9** Let  $\omega_1 \in G_p^{\Gamma_1}$ . Then,

$$Rmip(\omega_1) = Rmil(\omega_1)$$

**Proof.**

From proposition 3.7, we have that any  $\omega_1 \in G_p^{\Gamma_1}$   $Rmip(\omega_1) = \{1, 2, \dots, p-1\}$ . Similarly, from proposition 3.8 we have that for any  $\omega_1 \in G_p^{\Gamma_1}$ ,  $Rmil(\omega_1) = \{1, 2, \dots, p-1\}$ . Therefore,  $Rmip(\omega_1) = Rmil(\omega_1)$ .

**Proposition 3.10** Let  $\omega_1 \in G_p^{\Gamma_1}$ . Then,

$$rmip(\omega_1) = p-1.$$

**Proof.**

From proposition 3.7, we have that any  $\omega_1 \in G_p^{\Gamma_1}$   $Rmip(\omega_1) = \{1, 2, \dots, p-1\}$ . Hence, it has  $p-1$ .

**Proposition 3.11** Let  $\omega_1 \in G_p^{\Gamma_1}$ . Then,

$$rmil(\omega_1) = p-1.$$

**Proof.**

From proposition 3.8, we have that any  $\omega_1 \in G_p^{\Gamma_1}$   $Rmil(\omega_1) = \{1, 2, \dots, p-1\}$ . Hence it has  $p-1$ .

**Proposition 3.12** Let  $\omega_1 \in G_p^{\Gamma_1}$ . Then,

$$rmip(\omega_1) = rmil(\omega_1).$$

**Proof.**

From proposition 3.9, we have that any  $\omega_1 \in G_p^{\Gamma_1}$   $Rmip(\omega_1) = Rmil(\omega_1)$ . Also, from proposition 3.10, we have that for any  $\omega_1 \in G_p^{\Gamma_1}$ ,  $rmip(\omega_1) = p-1$ . and similarly from proposition 3.11 we have that for any  $\omega_1 \in G_p^{\Gamma_1}$ ,  $rmil(\omega_1) = p-1$ . Therefore,  $rmip(\omega_1) = rmil(\omega_1) = p-1$ .

**Proposition 3.13** Let  $\omega_1 \in G_p^{\Gamma_1}$ . Then,

$$Rmal(\omega_1) = \emptyset.$$

**Proof.**

Let  $\omega_1 \in G_p^{\Gamma_1}$  be represented by

$$\omega_1 = \begin{pmatrix} 1 & 2 & 3 & \dots & p \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix}, \text{ where}$$

$\sigma(1) = 1, i \in \{1, 2, 3, \dots, p\}$ . Thus, then for any  $i < j$ , we have that  $\sigma(i) < \sigma(j)$ . Therefore,

$$Rmal(\omega_1) = \{ \sigma(i) : \sigma(j) < \sigma(i), j > i \} = \emptyset$$

**Proposition 3.14** For any  $\omega_{p-1} \in G_p^{\Gamma_1}$ . Then,

$$Rmip(\omega_{p-1}) = Rmil(\omega_{p-1}).$$

**Proof.**

For any  $\omega_{p-1} \in G_p^{\Gamma_1}$ , we have the hypothesis  $j < i$  such that  $\sigma(j) < \sigma(i)$  holds only for  $i = 1$ . Hence, since  $\sigma(i) = 1$ , we have

$$Rmip(\omega_{p-1}) = Rmil(\omega_{p-1}).$$

**Proposition 3.15** For any  $\omega_{p-1} \in G_p^{\Gamma_1}$ . Then,

$$rmip(\omega_{p-1}) = rmil(\omega_{p-1}) = 1.$$

**Proof.**

This follows directly from proposition 3.14.

**Proposition 3.16** For any  $\omega_{p-1} \in G_p^{\Gamma_1}$ . Then,

$$Rmal(\omega_{p-1}) = \{3, 4, \dots, p\}.$$

**Proof.**

Let  $\omega_{p-1} \in G_p^{\Gamma_1}$ . Then, for any  $j > i$ , we have that  $\sigma(j) > \sigma(i)$  except for  $i = 1$ . Therefore, since  $\sigma(i) = 1$ , we have that

$$Rmal(\omega_1) = \{ \sigma(i) : \sigma(j) < \sigma(i), j > i \} = \{3, 4, \dots, p\}$$

**Proposition 3.16** For any  $\omega_{p-1} \in G_p^{\Gamma_1}$ . Then,

$$rmal(\omega_{p-1}) = p-2.$$

**Proof.**

This follows immediately from proposition 3.16.

#### 4.0 CONCLUSION

In this paper, we computed the statistics viz.  $Lmap$ ,  $Lmal$ ,  $Rmip$ ,  $Rmil$ ,  $Rmal$  on  $\Gamma_1$ -non deranged permutations as well as redefined some of these statistics. We established that left to right maximum places of  $\omega_i$   $Lmap(\omega_i)$  is equal to left to right maximum letters of  $\omega_i$   $Lmal(\omega_i)$ . We also observed that right to left maximum letters of  $\omega_i$   $Rmal(\omega_i)$  is empty for any  $\Gamma_1$  - non deranged permutations.

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