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ISSN: 2488-9229
FEDERAL UNIVERSITY
GUSAU-NIGERIA

INTERNATIONAL JOURNAL OF SCIENCE FOR GLOBAL SUSTAINABILITY
Algebraic Theories of AUNU Permutation Using Group Action with Non-Prime Order

Dogondaji A.M and Ibrahim A. A.

Department of Mathematics, Usmanu Danfodiyo University, P.M.B. 2346, Sokoto -Nigeria

Corresponding Authors E-mail address: dogondajialiyu@gmail.com

Received on: January, 2023

Revised and Accepted on: February, 2023

Published on: March, 2023

ABSTRACT

Permutation groups have played important roles in method of enumerating combinatorial objects in recent times. However, the study of the applicability AUNU permutation groups using group action is challenging. In this paper, a new method of constructing group action using the subsequences of (123)- avoiding of AUNU Permutation patterns is provided. The method used the generating function of AUNU groups in the construction to obtain some geometric structures of orbits resembling to lattice. The research further established some algebraic properties of group action as well as orbit and stabilizer in permutation groups which are relevant to the study of finite group structures. The paper also provided some motivations for the study of group action in an abstract domain.

Keywords: Permutation; group; AUNU Permutation Patterns; AUNU numbers; Partially Ordered Set

1.0 INTRODUCTION

Group action plays an importance role in describing the formation of algebraic structures. It established an interface between pure mathematics and other fields (Dogondaji, Liman and Nura, 2019), (Ariffin, Abubakar, Yunos and Asbullah, 2019), and (Judson, 2010) reported that the study of group action in permutation group helped to answer many of the 20th Century challenges in terms of the formation of algebraic structures. It therefore helped in discovering some algebraic structures in an abstract domain. Ibrahim (2007) described that permutation group has been studied for a very long time. Thus, the study of group action played an important role in many fields such as automata theory, number theory, and category theory. The notions and the results about the group action helped to drive some basic theorems about a finite simple group in the formation of algebraic structures thereby given another motivation for the study of group actions (Stanley, 1997). The importance of the study of group action in permutation group to the contemporary Mathematicians can be found in (Ibrahim, 2005, 2006, 2007 and 2008) and (Chun and Magami, 2016) and (Dogondaji, Babangida and Zainab, 2020)

1.1 Permutation Group

A permutation group is a finite group G whose elements of permutation for a given set and whose group operations form a composition relation of permutation in G . Permutation groups have orders dividing n . Two permutation form a group only if one is the identity element and the other is a permutation involution that is permutation which has its own inverse. Every permutation group with more than two elements can be written as a product of its transposition (Abor & Ibrahim, 2010).

1.2 AUNU GROUP

In the original construction, Ibrahim (2004) denoted I_5 as the symmetric groups and provided an alternative way of construction the Bara'at-al-Dhimmah model using permutation mapping (α) on the symmetric groups I_5 containing the five daily prayers (Salawatul Khams). Some group theoretical interpretation of the Bara'at-al-Dhimmah models were then generated as reported by (Ibrahim and Audu, 2005).

1.3 Partially ordered set

In group theory, a partially ordered set is a set that satisfies reflexive, anti symmetric and transitive properties (Ibrahim,2007). A partially ordered set

2.0 METHOD OF CONSTRUCTION

The method employed was the used of generating function for the enumerating scheme as reported by Abor and Ibrahim (2010). The relation used

$$N(A_{n(123)}) = \frac{p_n - 1}{2}, n \geq 5 \text{ which generates primes: } 2, 3, 5, 7, 11, \dots \tag{1}$$

Thus: according to Garba and Ibrahim (2010) the generating function was later on modified by $\omega_I = (1^{mp}(1+I)^{mp}(1+2I)^{mp}(1+3I)^{mp} \dots (1+((p-1)I)^{mp})$ for $I \geq 0, i \in \mathbb{Z}$. (3.2) where mp means taking inter modulo p in each circle that is $(1+i)$ as shown in (2) above. Let ζ be a group of automorphism of Δ_s and to test the

(or Poset) formalizes and generalizes the intuitive concept of an ordering, sequencing, or arrangement of the elements of a set (Garba and Abubakar,2015)

2.1 GROUP ACTIONS WITH NON-PRIME ORDER

Ibrahim *et al*, (2012) by creating identity I in the relation and thus represented as:

$$\omega_I = (1^{mp}(1+I)^{mp}(1+2I)^{mp}(1+3I)^{mp} \dots (1+((p-1)I)^{mp}) \text{ for } I \geq 0, i \in \mathbb{Z}. \tag{2}$$

associative group action of these Posets. We then begin as follows:

2.2 GROUP ACTION WITH TEN (10) ORDERED SAMPLE SIZE

Let the permutation patterns of the generated function of ten elements sample as in accordance with (3) be:

$$\begin{aligned}
 i.e \quad \xi_{10} = & \{(1); (1234567891011) \overset{\delta_1}{\downarrow} (1357911246810) \overset{\delta_2}{\downarrow} (1471025811369) \overset{\delta_3}{\downarrow} \\
 & (15292610371148) \overset{\delta_4}{\downarrow} (1611510493827) \overset{\delta_5}{\downarrow} (1728394105116) \overset{\delta_6}{\downarrow} \\
 & (1841173106295) \overset{\delta_7}{\downarrow} (1963118521074) \overset{\delta_8}{\downarrow} (1108642119753) \overset{\delta_9}{\downarrow} \\
 & (1111098765432)\} \overset{\delta_{10}}{\downarrow}
 \end{aligned}$$

If we can associate those structures with arbitrary subset $\Delta \subset \mathbb{R}$ using the sample of orders 10 12, 14. That is $\Delta_{10} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ then we can

compute the action ξ_{10} as a group action on each set of elements in Δ_{10} that is (when ξ_{10} acts on sets Δ_{10}) and obtained as follows:

Action Δ_i on ξ_{10}

$$\begin{aligned} \Delta_1^{\delta_1} &= 1^1 = 1 \\ \Delta_2^{\delta_1} &= 2^{(1)} = 2 \\ \Delta_3^{\delta_1} &= 3^{(1)} = 3 \quad (12345678910) \\ \Delta_4^{\delta_1} &= 4^{(1)} = 4 \\ \Delta_5^{\delta_1} &= 5^{(1)} = 5 \\ \Delta_6^{\delta_1} &= 6^{(1)} = 6 \\ \Delta_7^{\delta_1} &= 7^{(1)} = 7 \\ \Delta_8^{\delta_1} &= 8^{(1)} = 8 \\ \Delta_9^{\delta_1} &= 9^{(1)} = 9 \\ \Delta_{10}^{\delta_1} &= 10^{(1)} = 10 \end{aligned}$$

FOR

$$\begin{aligned} \Delta_1^{\delta_3} &= (1)^{(1471025811369)} = 3 \\ \Delta_2^{\delta_3} &= (2)^{(1471025811369)} = 4 \\ \Delta_3^{\delta_3} &= (3)^{(1471025811369)} = 5 \quad (4567891011123) \\ \Delta_4^{\delta_3} &= (4)^{(1471025811369)} = 6 \\ \Delta_5^{\delta_3} &= (5)^{(1471025811369)} = 7 \\ \Delta_6^{\delta_3} &= (6)^{(1471025811369)} = 8 \\ \Delta_7^{\delta_3} &= (7)^{(1471025811369)} = 9 \\ \Delta_8^{\delta_3} &= (8)^{(1471025811369)} = 10 \\ \Delta_9^{\delta_3} &= (9)^{(1471025811369)} = 1 \\ \Delta_{10}^{\delta_3} &= (10)^{(1471025811369)} = 2 \end{aligned}$$

FOR

$$\begin{aligned} \Delta_1^{\delta_2} &= (1)^{(1357911246810)} = 2 \\ \Delta_2^{\delta_2} &= (2)^{(1357911246810)} = 3 \\ \Delta_3^{\delta_2} &= (3)^{(1357911246810)} = 4 \quad (23456789101) \\ \Delta_4^{\delta_2} &= (4)^{(1357911246810)} = 5 \\ \Delta_5^{\delta_2} &= (5)^{(1357911246810)} = 6 \\ \Delta_6^{\delta_2} &= (6)^{(1357911246810)} = 7 \\ \Delta_7^{\delta_2} &= (7)^{(1357911246810)} = 8 \\ \Delta_8^{\delta_2} &= (8)^{(1357911246810)} = 9 \\ \Delta_9^{\delta_2} &= (9)^{(1357911246810)} = 10 \\ \Delta_{10}^{\delta_2} &= (10)^{(1357911246810)} = 1 \end{aligned}$$

AG

$$\begin{aligned} \Delta_1^{\delta_4} &= (1)^{(1592610371148)} = 4 \\ \Delta_2^{\delta_4} &= (2)^{(1592610371148)} = 5 \\ \Delta_3^{\delta_4} &= (3)^{(1592610371148)} = 6 \quad (56789101112311) \\ \Delta_4^{\delta_4} &= (4)^{(1592610371148)} = 7 \\ \Delta_5^{\delta_4} &= (5)^{(1592610371148)} = 8 \\ \Delta_6^{\delta_4} &= (6)^{(1592610371148)} = 9 \\ \Delta_7^{\delta_4} &= (7)^{(1592610371148)} = 10 \\ \Delta_8^{\delta_4} &= (8)^{(1592610371148)} = 1 \\ \Delta_9^{\delta_4} &= (9)^{(1592610371148)} = 2 \\ \Delta_{10}^{\delta_4} &= (10)^{(1592610371148)} = 3 \end{aligned}$$

<p>AG 4</p> $\Delta_1^{\delta_5} = (1)^{(1611\ 510\ 493827)} = 5$ $\Delta_2^{\delta_5} = (2)^{(1611\ 510\ 493827)} = 6$ $\Delta_3^{\delta_5} = (3)^{(1611\ 510\ 493827)} = 7 \quad (6789\ 10\ 11\ 1\ 2345)$ $\Delta_4^{\delta_5} = (4)^{(1611\ 510\ 493827)} = 8$ $\Delta_5^{\delta_5} = (5)^{(1611\ 510\ 493827)} = 9$ $\Delta_6^{\delta_5} = (6)^{(1611\ 510\ 493827)} = 10$ $\Delta_7^{\delta_5} = (7)^{(1611\ 510\ 493827)} = 1$ $\Delta_8^{\delta_5} = (8)^{(1611\ 510\ 493827)} = 2$ $\Delta_9^{\delta_5} = (9)^{(1611\ 510\ 493827)} = 3$ $\Delta_{10}^{\delta_5} = (10)^{(1611\ 510\ 493827)} = 4$	<p>AG 7</p> $\Delta_1^{\delta_8} = (1)^{(1728394105116)} = 8$ $\Delta_2^{\delta_8} = (2)^{(1728394105116)} = 9$ $\Delta_3^{\delta_8} = (3)^{(1728394105116)} = 10 \quad (7891011\ 1\ 23456)$ $\Delta_4^{\delta_8} = (4)^{(1728394105116)} = 1$ $\Delta_5^{\delta_8} = (5)^{(1728394105116)} = 2$ $\Delta_6^{\delta_8} = (6)^{(1728394105116)} = 3$ $\Delta_7^{\delta_8} = (7)^{(1728394105116)} = 4$ $\Delta_8^{\delta_8} = (8)^{(1728394105116)} = 5$ $\Delta_9^{\delta_8} = (9)^{(1728394105116)} = 6$ $\Delta_{10}^{\delta_8} = (10)^{(1728394105116)} = 7$
<p>AG 5</p> $\Delta_1^{\delta_6} = (1)^{(1728394105116)} = 6$ $\Delta_2^{\delta_6} = (2)^{(1728394105116)} = 7$ $\Delta_3^{\delta_6} = (3)^{(1728394105116)} = 8 \quad (7891011\ 1\ 2345678)$ $\Delta_4^{\delta_6} = (4)^{(1728394105116)} = 9$ $\Delta_5^{\delta_6} = (5)^{(1728394105116)} = 10$ $\Delta_6^{\delta_6} = (6)^{(1728394105116)} = 1$ $\Delta_7^{\delta_6} = (7)^{(1728394105116)} = 2$ $\Delta_8^{\delta_6} = (8)^{(1728394105116)} = 3$ $\Delta_9^{\delta_6} = (9)^{(1728394105116)} = 4$ $\Delta_{10}^{\delta_6} = (10)^{(1728394105116)} = 5$	<p>AG 8</p> $\Delta_1^{\delta_8} = (1)^{(1728394105116)} = 9$ $\Delta_2^{\delta_8} = (2)^{(1728394105116)} = 10$ $\Delta_3^{\delta_8} = (3)^{(1728394105116)} = 1 \quad (7891011\ 1\ 23456)$ $\Delta_4^{\delta_8} = (4)^{(1728394105116)} = 2$ $\Delta_5^{\delta_8} = (5)^{(1728394105116)} = 3$ $\Delta_6^{\delta_8} = (6)^{(1728394105116)} = 4$ $\Delta_7^{\delta_8} = (7)^{(1728394105116)} = 5$ $\Delta_8^{\delta_8} = (8)^{(1728394105116)} = 6$ $\Delta_9^{\delta_8} = (9)^{(1728394105116)} = 7$ $\Delta_{10}^{\delta_8} = (10)^{(1728394105116)} = 8$
<p>AG 6</p> $\Delta_1^{\delta_7} = (1)^{(1728394105116)} = 7$ $\Delta_2^{\delta_7} = (2)^{(1728394105116)} = 8$ $\Delta_3^{\delta_7} = (3)^{(1728394105116)} = 9 \quad (7891011\ 1\ 23456)$ $\Delta_4^{\delta_7} = (4)^{(1728394105116)} = 10$ $\Delta_5^{\delta_7} = (5)^{(1728394105116)} = 1$ $\Delta_6^{\delta_7} = (6)^{(1728394105116)} = 2$ $\Delta_7^{\delta_7} = (7)^{(1728394105116)} = 3$ $\Delta_8^{\delta_7} = (8)^{(1728394105116)} = 4$ $\Delta_9^{\delta_7} = (9)^{(1728394105116)} = 5$ $\Delta_{10}^{\delta_7} = (10)^{(1728394105116)} = 6$	<p>AG 9</p> $\Delta_1^{\delta_9} = (1)^{(110\ 8642\ 119753)} = 10$ $\Delta_2^{\delta_9} = (2)^{(110\ 8642\ 119753)} = 1$ $\Delta_3^{\delta_9} = (3)^{(110\ 8642\ 119753)} = 2 \quad (10\ 11\ 1\ 23456789)$ $\Delta_4^{\delta_9} = (4)^{(110\ 8642\ 119753)} = 3$ $\Delta_5^{\delta_9} = (5)^{(110\ 8642\ 119753)} = 4$ $\Delta_6^{\delta_9} = (6)^{(110\ 8642\ 119753)} = 5$ $\Delta_7^{\delta_9} = (7)^{(110\ 8642\ 119753)} = 6$ $\Delta_8^{\delta_9} = (8)^{(110\ 8642\ 119753)} = 7$ $\Delta_9^{\delta_9} = (9)^{(110\ 8642\ 119753)} = 8$ $\Delta_{10}^{\delta_9} = (10)^{(110\ 8642\ 119753)} = 9$

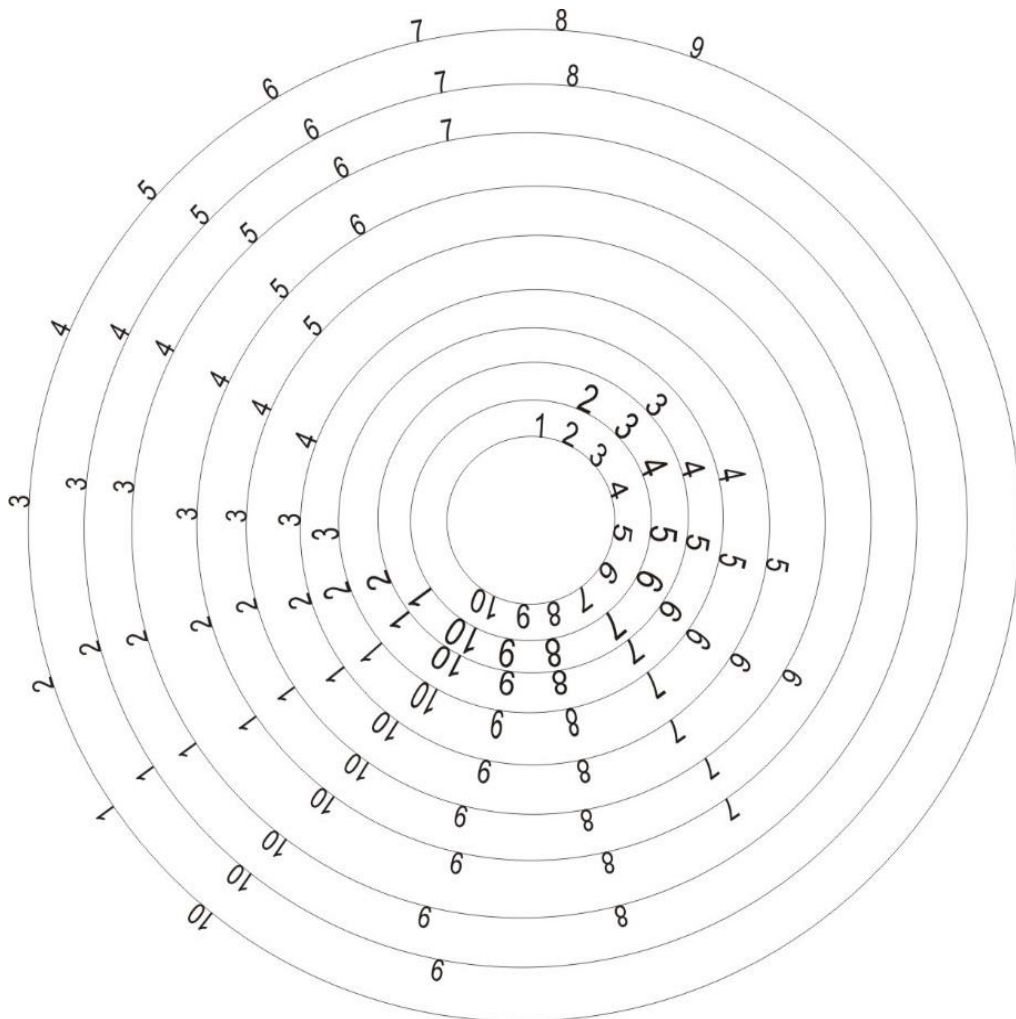


Figure 1.: Formation of Orbits for AG_{10}

2.3 GROUP ACTION WITH TWELVE (12) ORDERED SAMPLE SIZE

Let the permutation patterns of the generated function of twelve elements sample as in accordance with (3) be:

$$\begin{aligned}
 i.e \quad \xi_{12} = & \{(1); \overset{\delta_1}{\downarrow} (1234567891011) \overset{\delta_2}{\downarrow} (1357911246810) \overset{\delta_3}{\downarrow} (1471025811369) \overset{\delta_4}{\downarrow} (15292610371148) \overset{\delta_5}{\downarrow} (1611510493827) \\
 & \overset{\delta_6}{\downarrow} (1728394105116) \overset{\delta_7}{\downarrow} (1841173106295) \overset{\delta_8}{\downarrow} (1963118521074) \overset{\delta_9}{\downarrow} (1\ 10\ 864211\ 9753) \overset{\delta_{10}}{\downarrow} (1\ 11\ 10\ 98765432)\}
 \end{aligned}$$

If we can associate those structures with arbitrary subset $\Delta \subset \mathbb{R}$ of orders 12. That is $\Delta_{12} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ then we can compute the action ξ_{11} as a group action on each set of elements in Δ_{11}

$\Delta_{i \in z}^{\xi_i}, i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$ that is (when ξ_{12} acts on sets Δ_{12}) and obtained as follows:

FOR

$$\begin{aligned} \Delta_1^{\delta_1} &= 1^1 = 1 \\ \Delta_2^{\delta_1} &= 2^{(1)} = 2 \\ \Delta_3^{\delta_1} &= 3^{(1)} = 3 \quad (1234567891011) \\ \Delta_4^{\delta_1} &= 4^{(1)} = 4 \\ \Delta_5^{\delta_1} &= 5^{(1)} = 5 \\ \Delta_6^{\delta_1} &= 6^{(1)} = 6 \\ \Delta_7^{\delta_1} &= 7^{(1)} = 7 \\ \Delta_8^{\delta_1} &= 8^{(1)} = 8 \\ \Delta_9^{\delta_1} &= 9^{(1)} = 9 \\ \Delta_{10}^{\delta_1} &= 10^{(1)} = 10 \\ \Delta_{11}^{\delta_1} &= 11^{(1)} = 11 \\ \Delta_{12}^{\delta_1} &= 12^{(1)} = 12 \end{aligned}$$

FOR

$$\begin{aligned} \Delta_1^{\delta_3} &= (1)^{(1471025811369)} = 2 \\ \Delta_2^{\delta_3} &= (2)^{(1471025811369)} = 3 \\ \Delta_3^{\delta_3} &= (3)^{(1471025811369)} = 4 \quad (4567891011123) \\ \Delta_4^{\delta_3} &= (4)^{(1471025811369)} = 5 \\ \Delta_5^{\delta_3} &= (5)^{(1471025811369)} = 6 \\ \Delta_6^{\delta_3} &= (6)^{(1471025811369)} = 7 \\ \Delta_7^{\delta_3} &= (7)^{(1471025811369)} = 8 \\ \Delta_8^{\delta_3} &= (8)^{(1471025811369)} = 9 \\ \Delta_9^{\delta_3} &= (9)^{(1471025811369)} = 10 \\ \Delta_{10}^{\delta_3} &= (10)^{(1471025811369)} = 11 \\ \Delta_{11}^{\delta_3} &= (11)^{(1471025811369)} = 12 \\ \Delta_{12}^{\delta_3} &= (12)^{(1357911246810)} = 1 \end{aligned}$$

FOR

$$\begin{aligned} \Delta_1^{\delta_2} &= (1)^{(1357911246810)} = 2 \\ \Delta_2^{\delta_2} &= (2)^{(1357911246810)} = 3 \\ \Delta_3^{\delta_2} &= (3)^{(1357911246810)} = 4 \quad (3456789101112) \\ \Delta_4^{\delta_2} &= (4)^{(1357911246810)} = 5 \\ \Delta_5^{\delta_2} &= (5)^{(1357911246810)} = 6 \\ \Delta_6^{\delta_2} &= (6)^{(1357911246810)} = 7 \\ \Delta_7^{\delta_2} &= (7)^{(1357911246810)} = 8 \\ \Delta_8^{\delta_2} &= (8)^{(1357911246810)} = 9 \\ \Delta_9^{\delta_2} &= (9)^{(1357911246810)} = 10 \\ \Delta_{10}^{\delta_2} &= (10)^{(1357911246810)} = 11 \\ \Delta_{11}^{\delta_2} &= (11)^{(1357911246810)} = 12 \\ \Delta_{12}^{\delta_2} &= (12)^{(1357911246810)} = 1 \end{aligned}$$

AG

$$\begin{aligned} \Delta_1^{\delta_4} &= (1)^{(1592610371148)} = 3 \\ \Delta_2^{\delta_4} &= (2)^{(1592610371148)} = 4 \\ \Delta_3^{\delta_4} &= (3)^{(1592610371148)} = 5 \quad (56789101112311) \\ \Delta_4^{\delta_4} &= (4)^{(1592610371148)} = 6 \\ \Delta_5^{\delta_4} &= (5)^{(1592610371148)} = 7 \\ \Delta_6^{\delta_4} &= (6)^{(1592610371148)} = 8 \\ \Delta_7^{\delta_4} &= (7)^{(1592610371148)} = 9 \\ \Delta_8^{\delta_4} &= (8)^{(1592610371148)} = 10 \\ \Delta_9^{\delta_4} &= (9)^{(1592610371148)} = 11 \\ \Delta_{10}^{\delta_4} &= (10)^{(1592610371148)} = 12 \\ \Delta_{11}^{\delta_4} &= (11)^{(1592610371148)} = 1 \\ \Delta_{12}^{\delta_4} &= (12)^{(1357911246810)} = 2 \end{aligned}$$

<p>AG 5</p> $\Delta_1^{\delta_5} = (1)^{(1611\ 510\ 493827)} = 4$ $\Delta_2^{\delta_5} = (2)^{(1611\ 510\ 493827)} = 5$ $\Delta_3^{\delta_5} = (3)^{(1611\ 510\ 493827)} = 6 \quad (6789\ 10\ 111\ 2345)$ $\Delta_4^{\delta_5} = (4)^{(1611\ 510\ 493827)} = 7$ $\Delta_5^{\delta_5} = (5)^{(1611\ 510\ 493827)} = 8$ $\Delta_6^{\delta_5} = (6)^{(1611\ 510\ 493827)} = 9$ $\Delta_7^{\delta_5} = (7)^{(1611\ 510\ 493827)} = 10$ $\Delta_8^{\delta_5} = (8)^{(1611\ 510\ 493827)} = 11$ $\Delta_9^{\delta_5} = (9)^{(1611\ 510\ 493827)} = 12$ $\Delta_{10}^{\delta_5} = (10)^{(1611\ 510\ 493827)} = 1$ $\Delta_{11}^{\delta_5} = (11)^{(1611\ 510\ 493827)} = 2$ $\Delta_{12}^{\delta_5} = (12)^{(1357911\ 246810)} = 3$	<p>AG 8</p> $\Delta_1^{\delta_8} = (1)^{(1728394105116)} = 8$ $\Delta_2^{\delta_8} = (2)^{(1728394105116)} = 9$ $\Delta_3^{\delta_8} = (3)^{(1728394105116)} = 10 \quad (78910111\ 1\ 23456)$ $\Delta_4^{\delta_8} = (4)^{(1728394105116)} = 11$ $\Delta_5^{\delta_8} = (5)^{(1728394105116)} = 12$ $\Delta_6^{\delta_8} = (6)^{(1728394105116)} = 1$ $\Delta_7^{\delta_8} = (7)^{(1728394105116)} = 2$ $\Delta_8^{\delta_8} = (8)^{(1728394105116)} = 3$ $\Delta_9^{\delta_8} = (9)^{(1728394105116)} = 4$ $\Delta_{10}^{\delta_8} = (10)^{(1728394105116)} = 5$ $\Delta_{11}^{\delta_8} = (11)^{(1728394105116)} = 6$ $\Delta_{12}^{\delta_8} = (12)^{(1357911\ 246810)} = 7$
<p>AG 6</p> $\Delta_1^{\delta_6} = (1)^{(1728394105116)} = 5$ $\Delta_2^{\delta_6} = (2)^{(1728394105116)} = 6$ $\Delta_3^{\delta_6} = (3)^{(1728394105116)} = 7 \quad (78910111\ 1\ 2345678)$ $\Delta_4^{\delta_6} = (4)^{(1728394105116)} = 8$ $\Delta_5^{\delta_6} = (5)^{(1728394105116)} = 9$ $\Delta_6^{\delta_6} = (6)^{(1728394105116)} = 10$ $\Delta_7^{\delta_6} = (7)^{(1728394105116)} = 11$ $\Delta_8^{\delta_6} = (8)^{(1728394105116)} = 12$ $\Delta_9^{\delta_6} = (9)^{(1728394105116)} = 1$ $\Delta_{10}^{\delta_6} = (10)^{(1728394105116)} = 2$ $\Delta_{11}^{\delta_6} = (11)^{(1728394105116)} = 3$ $\Delta_{12}^{\delta_6} = (12)^{(1357911\ 246810)} = 4$	<p>AG 9</p> $\Delta_1^{\delta_9} = (1)^{(110\ 8642\ 119753)} = 9$ $\Delta_2^{\delta_9} = (2)^{(110\ 8642\ 119753)} = 10$ $\Delta_3^{\delta_9} = (3)^{(110\ 8642\ 119753)} = 11 \quad (10\ 111\ 1\ 23456789)$ $\Delta_4^{\delta_9} = (4)^{(110\ 8642\ 119753)} = 12$ $\Delta_5^{\delta_9} = (5)^{(110\ 8642\ 119753)} = 1$ $\Delta_6^{\delta_9} = (6)^{(110\ 8642\ 119753)} = 2$ $\Delta_7^{\delta_9} = (7)^{(110\ 8642\ 119753)} = 3$ $\Delta_8^{\delta_9} = (8)^{(110\ 8642\ 119753)} = 4$ $\Delta_9^{\delta_9} = (9)^{(110\ 8642\ 119753)} = 5$ $\Delta_{10}^{\delta_9} = (10)^{(110\ 8642\ 119753)} = 6$ $\Delta_{11}^{\delta_9} = (11)^{(110\ 8642\ 119753)} = 7$ $\Delta_{12}^{\delta_9} = (12)^{(1357911\ 246810)} = 8$
<p>AG 7</p> $\Delta_1^{\delta_7} = (1)^{(1728394105116)} = 6$ $\Delta_2^{\delta_7} = (2)^{(1728394105116)} = 7$ $\Delta_3^{\delta_7} = (3)^{(1728394105116)} = 8 \quad (78910111\ 1\ 23456)$ $\Delta_4^{\delta_7} = (4)^{(1728394105116)} = 9$ $\Delta_5^{\delta_7} = (5)^{(1728394105116)} = 10$ $\Delta_6^{\delta_7} = (6)^{(1728394105116)} = 11$ $\Delta_7^{\delta_7} = (7)^{(1728394105116)} = 12$ $\Delta_8^{\delta_7} = (8)^{(1728394105116)} = 1$ $\Delta_9^{\delta_7} = (9)^{(1728394105116)} = 2$ $\Delta_{10}^{\delta_7} = (10)^{(1728394105116)} = 3$ $\Delta_{11}^{\delta_7} = (11)^{(1728394105116)} = 4$ $\Delta_{12}^{\delta_7} = (12)^{(1357911\ 246810)} = 5$	
<p>AG 8</p> $\Delta_1^{\delta_8} = (1)^{(1728394105116)} = 7$	

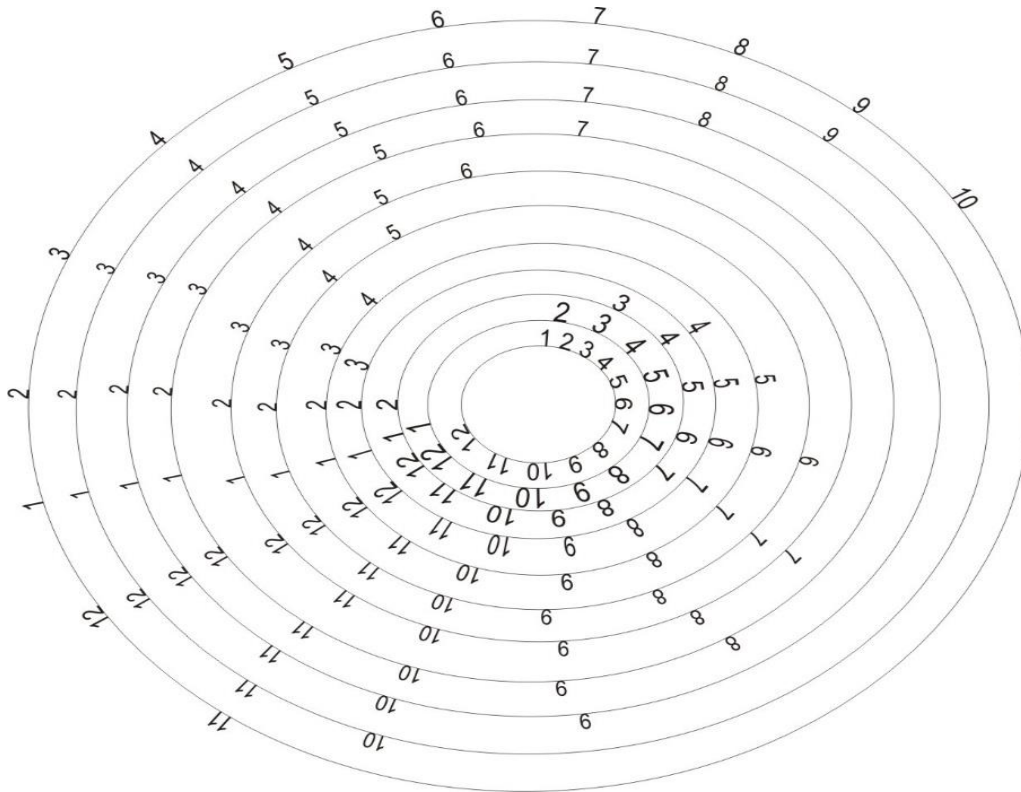


Figure2: Formation of Orbits for AG₁₂

2.4GROUP ACTION WITH FOURTEEN (14) ORDERED SAMPLE SIZE

Let the permutation patterns of the generated function of fourteen elements sample as in accordance with (3) be:

$$i.e \ \xi_{14} = \{(1); \overset{\delta_1}{\downarrow} (1234567891011) \overset{\delta_2}{\downarrow} (1357911246810) \overset{\delta_3}{\downarrow} (1471025811369) \overset{\delta_4}{\downarrow} (15292610371148) \overset{\delta_5}{\downarrow} (1611510493827) \\ \overset{\delta_6}{\downarrow} (1728394105116) \overset{\delta_7}{\downarrow} (1841173106295) \overset{\delta_8}{\downarrow} (1963118521074) \overset{\delta_9}{\downarrow} (1108642119753) \overset{\delta_{10}}{\downarrow} (1111098765432)\}$$

If we can associate those structures with arbitrary subset $\Delta \subset \mathbb{R}$ of orders 12. That is $\Delta_{14} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ then we can compute the action ξ_{14} as a group action on each set of elements in Δ_{14}

$\Delta_{i \in \mathbb{Z}}, i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14$
 that is (when ξ_{14} acts on sets Δ_{14})
 and obtained as follows:

FOR $\Delta_1^{\delta_1} = 1^1 = 1$
 $\Delta_2^{\delta_1} = 2^{(1)} = 2$
 $\Delta_3^{\delta_1} = 3^{(1)} = 3$ (1234567891011)
 $\Delta_4^{\delta_1} = 4^{(1)} = 4$
 $\Delta_5^{\delta_1} = 5^{(1)} = 5$
 $\Delta_6^{\delta_1} = 6^{(1)} = 6$
 $\Delta_7^{\delta_1} = 7^{(1)} = 7$
 $\Delta_8^{\delta_1} = 8^{(1)} = 8$
 $\Delta_9^{\delta_1} = 9^{(1)} = 9$
 $\Delta_{10}^{\delta_1} = 10^{(1)} = 10$
 $\Delta_{11}^{\delta_1} = 11^{(1)} = 11$
 $\Delta_{12}^{\delta_1} = 12^{(1)} = 12$
 $\Delta_{13}^{\delta_1} = 13^{(1)} = 13$
 $\Delta_{14}^{\delta_1} = 14^{(1)} = 14$

FOR $\Delta_1^{\delta_3} = (1)^{(1471025811369)} = 3$
 $\Delta_2^{\delta_3} = (2)^{(1471025811369)} = 4$
 $\Delta_3^{\delta_3} = (3)^{(1471025811369)} = 5$ (45678910111 23)
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 $\Delta_8^{\delta_3} = (8)^{(1471025811369)} = 10$
 $\Delta_9^{\delta_3} = (9)^{(1471025811369)} = 11$
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FOR $\Delta_1^{\delta_2} = (1)^{(1357911 246810)} = 2$
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AG $\Delta_1^{\delta_4} = (1)^{(15926 10 371148)} = 4$
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<p>AG 5</p> $\Delta_1^{\delta_5} = (1)^{(1611\ 510\ 493827)} = 5$ $\Delta_2^{\delta_5} = (2)^{(1611\ 510\ 493827)} = 6$ $\Delta_3^{\delta_5} = (3)^{(1611\ 510\ 493827)} = 7 \quad (6789\ 10\ 111\ 2345)$ $\Delta_4^{\delta_5} = (4)^{(1611\ 510\ 493827)} = 8$ $\Delta_5^{\delta_5} = (5)^{(1611\ 510\ 493827)} = 9$ $\Delta_6^{\delta_5} = (6)^{(1611\ 510\ 493827)} = 10$ $\Delta_7^{\delta_5} = (7)^{(1611\ 510\ 493827)} = 11$ $\Delta_8^{\delta_5} = (8)^{(1611\ 510\ 493827)} = 12$ $\Delta_9^{\delta_5} = (9)^{(1611\ 510\ 493827)} = 13$ $\Delta_{10}^{\delta_5} = (10)^{(1611\ 510\ 493827)} = 14$ $\Delta_{11}^{\delta_5} = (11)^{(1611\ 510\ 493827)} = 1$ $\Delta_{12}^{\delta_2} = (12)^{(1357911\ 246810)} = 2$ $\Delta_{13}^{\delta_2} = (13)^{(1357911\ 246810)} = 3$ $\Delta_{14}^{\delta_2} = (14)^{(1357911\ 246810)} = 4$	<p>AG 8</p> $\Delta_1^{\delta_8} = (1)^{(1728394105116)} = 9$ $\Delta_2^{\delta_8} = (2)^{(1728394105116)} = 10$ $\Delta_3^{\delta_8} = (3)^{(1728394105116)} = 11 \quad (7891011\ 1\ 23456)$ $\Delta_4^{\delta_8} = (4)^{(1728394105116)} = 12$ $\Delta_5^{\delta_8} = (5)^{(1728394105116)} = 13$ $\Delta_6^{\delta_8} = (6)^{(1728394105116)} = 14$ $\Delta_7^{\delta_8} = (7)^{(1728394105116)} = 1$ $\Delta_8^{\delta_8} = (8)^{(1728394105116)} = 2$ $\Delta_9^{\delta_6} = (9)^{(1728394105116)} = 3$ $\Delta_{10}^{\delta_6} = (10)^{(1728394105116)} = 4$ $\Delta_{11}^{\delta_6} = (11)^{(1728394105116)} = 5$ $\Delta_{12}^{\delta_2} = (12)^{(1357911\ 246810)} = 6$ $\Delta_{13}^{\delta_2} = (13)^{(1357911\ 246810)} = 7$ $\Delta_{14}^{\delta_2} = (14)^{(1357911\ 246810)} = 8$
<p>AG 6</p> $\Delta_1^{\delta_6} = (1)^{(1728394105116)} = 6$ $\Delta_2^{\delta_6} = (2)^{(1728394105116)} = 7$ $\Delta_3^{\delta_6} = (3)^{(1728394105116)} = 8 \quad (7891011\ 1\ 2345678)$ $\Delta_4^{\delta_6} = (4)^{(1728394105116)} = 9$ $\Delta_5^{\delta_6} = (5)^{(1728394105116)} = 10$ $\Delta_6^{\delta_6} = (6)^{(1728394105116)} = 11$ $\Delta_7^{\delta_6} = (7)^{(1728394105116)} = 12$ $\Delta_8^{\delta_6} = (8)^{(1728394105116)} = 13$ $\Delta_9^{\delta_6} = (9)^{(1728394105116)} = 14$ $\Delta_{10}^{\delta_6} = (10)^{(1728394105116)} = 1$ $\Delta_{11}^{\delta_6} = (11)^{(1728394105116)} = 2$ $\Delta_{12}^{\delta_2} = (12)^{(1357911\ 246810)} = 3$ $\Delta_{13}^{\delta_2} = (13)^{(1357911\ 246810)} = 4$ $\Delta_{14}^{\delta_2} = (14)^{(1357911\ 246810)} = 5$	<p>AG 9</p> $\Delta_1^{\delta_9} = (1)^{(110\ 8642\ 119753)} = 10$ $\Delta_2^{\delta_9} = (2)^{(110\ 8642\ 119753)} = 11$ $\Delta_3^{\delta_9} = (3)^{(110\ 8642\ 119753)} = 12 \quad (10\ 11\ 1\ 23456789)$ $\Delta_4^{\delta_9} = (4)^{(110\ 8642\ 119753)} = 13$ $\Delta_5^{\delta_9} = (5)^{(110\ 8642\ 119753)} = 14$ $\Delta_6^{\delta_9} = (6)^{(110\ 8642\ 119753)} = 1$ $\Delta_7^{\delta_9} = (7)^{(110\ 8642\ 119753)} = 2$ $\Delta_8^{\delta_9} = (8)^{(110\ 8642\ 119753)} = 3$ $\Delta_9^{\delta_9} = (9)^{(110\ 8642\ 119753)} = 4$ $\Delta_{10}^{\delta_9} = (10)^{(110\ 8642\ 119753)} = 5$ $\Delta_{11}^{\delta_9} = (11)^{(110\ 8642\ 119753)} = 6$ $\Delta_{12}^{\delta_2} = (12)^{(1357911\ 246810)} = 7$ $\Delta_{13}^{\delta_2} = (13)^{(1357911\ 246810)} = 8$ $\Delta_{14}^{\delta_2} = (14)^{(1357911\ 246810)} = 9$

<p>AG 9</p> $\Delta_1^{\delta_8} = (1)^{(1728394105116)} = 11$ $\Delta_2^{\delta_8} = (2)^{(1728394105116)} = 12$ $\Delta_3^{\delta_8} = (3)^{(1728394105116)} = 13 \quad (7891011 \ 1 \ 23456)$ $\Delta_4^{\delta_8} = (4)^{(1728394105116)} = 14$ $\Delta_5^{\delta_8} = (5)^{(1728394105116)} = 1$ $\Delta_6^{\delta_8} = (6)^{(1728394105116)} = 2$ $\Delta_7^{\delta_8} = (7)^{(1728394105116)} = 3$ $\Delta_8^{\delta_8} = (8)^{(1728394105116)} = 4$ $\Delta_9^{\delta_8} = (9)^{(1728394105116)} = 5$ $\Delta_{10}^{\delta_8} = (10)^{(1728394105116)} = 6$ $\Delta_{11}^{\delta_8} = (11)^{(1728394105116)} = 7$ $\Delta_{12}^{\delta_2} = (12)^{(1357911 \ 246810)} = 8$ $\Delta_{13}^{\delta_2} = (13)^{(1357911 \ 246810)} = 7$ $\Delta_{14}^{\delta_2} = (14)^{(1357911 \ 246810)} = 9$	<p>AG 11</p> $\Delta_1^{\delta_8} = (1)^{(1728394105116)} = 13$ $\Delta_2^{\delta_8} = (2)^{(1728394105116)} = 14$ $\Delta_3^{\delta_8} = (3)^{(1728394105116)} = 1 \quad (7891011 \ 1 \ 23456)$ $\Delta_4^{\delta_8} = (4)^{(1728394105116)} = 2$ $\Delta_5^{\delta_8} = (5)^{(1728394105116)} = 3$ $\Delta_6^{\delta_8} = (6)^{(1728394105116)} = 4$ $\Delta_7^{\delta_8} = (7)^{(1728394105116)} = 5$ $\Delta_8^{\delta_8} = (8)^{(1728394105116)} = 6$ $\Delta_9^{\delta_8} = (9)^{(1728394105116)} = 7$ $\Delta_{10}^{\delta_8} = (10)^{(1728394105116)} = 8$ $\Delta_{11}^{\delta_8} = (11)^{(1728394105116)} = 9$ $\Delta_{12}^{\delta_2} = (12)^{(1357911 \ 246810)} = 10$ $\Delta_{13}^{\delta_2} = (13)^{(1357911 \ 246810)} = 11$ $\Delta_{14}^{\delta_2} = (14)^{(1357911 \ 246810)} = 12$
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<p>AG 10</p> $\Delta_1^{\delta_9} = (1)^{(1108642119753)} = 12$ $\Delta_2^{\delta_9} = (2)^{(1108642119753)} = 13$ $\Delta_3^{\delta_9} = (3)^{(1108642119753)} = 14 \quad (10 \ 111 \ 23456789)$ $\Delta_4^{\delta_9} = (4)^{(1108642119753)} = 1$ $\Delta_5^{\delta_9} = (5)^{(1108642119753)} = 2$ $\Delta_6^{\delta_9} = (6)^{(1108642119753)} = 3$ $\Delta_7^{\delta_9} = (7)^{(1108642119753)} = 4$ $\Delta_8^{\delta_9} = (8)^{(1108642119753)} = 5$ $\Delta_9^{\delta_9} = (9)^{(1108642119753)} = 6$ $\Delta_{10}^{\delta_9} = (10)^{(1108642119753)} = 7$ $\Delta_{11}^{\delta_9} = (11)^{(1108642119753)} = 8$ $\Delta_{12}^{\delta_2} = (12)^{(1357911 \ 246810)} = 9$ $\Delta_{13}^{\delta_2} = (13)^{(1357911 \ 246810)} = 10$ $\Delta_{14}^{\delta_2} = (14)^{(1357911 \ 246810)} = 11$	<p>AG 12</p> $\Delta_1^{\delta_9} = (1)^{(1108642119753)} = 14$ $\Delta_2^{\delta_9} = (2)^{(1108642119753)} = 1$ $\Delta_3^{\delta_9} = (3)^{(1108642119753)} = 2 \quad (10 \ 111 \ 23456789)$ $\Delta_4^{\delta_9} = (4)^{(1108642119753)} = 3$ $\Delta_5^{\delta_9} = (5)^{(1108642119753)} = 4$ $\Delta_6^{\delta_9} = (6)^{(1108642119753)} = 5$ $\Delta_7^{\delta_9} = (7)^{(1108642119753)} = 6$ $\Delta_8^{\delta_9} = (8)^{(1108642119753)} = 7$ $\Delta_9^{\delta_9} = (9)^{(1108642119753)} = 8$ $\Delta_{10}^{\delta_9} = (10)^{(1108642119753)} = 9$ $\Delta_{11}^{\delta_9} = (11)^{(1108642119753)} = 10$ $\Delta_{12}^{\delta_2} = (12)^{(1357911 \ 246810)} = 11$ $\Delta_{13}^{\delta_2} = (13)^{(1357911 \ 246810)} = 12$ $\Delta_{14}^{\delta_2} = (14)^{(1357911 \ 246810)} = 13$
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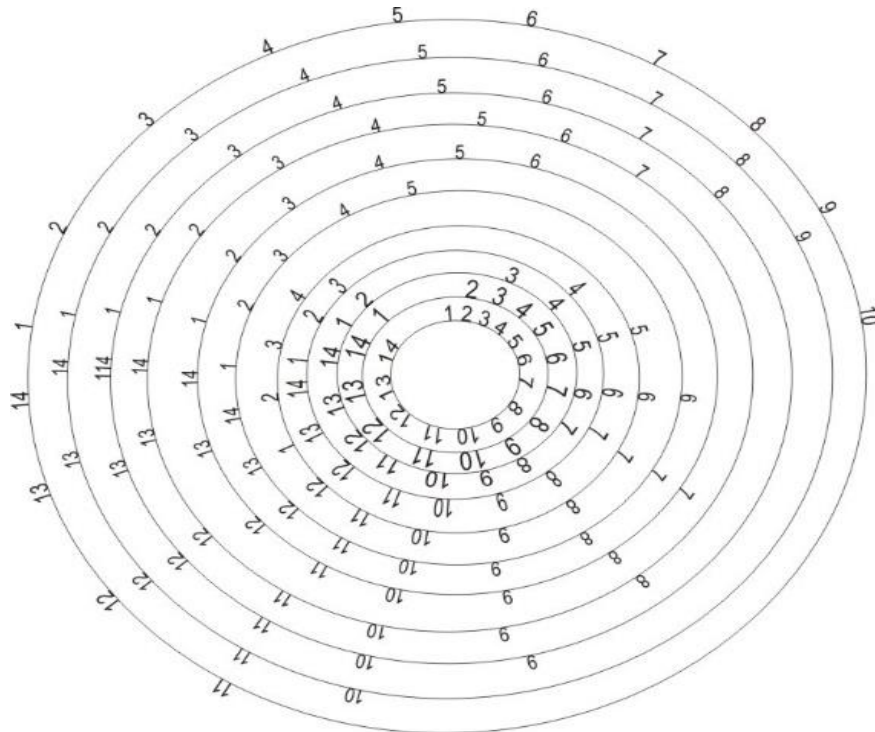


Figure3: Formation of Orbits for N_{14}

3.0 RESULT AND DISCUSSIONS

3.1 ALGEBRAIC THEORETIC PROPERTIES OF GROUP ACTION

The followings provided some group theoretic properties

Proposition 3.1: Let G be a group acting on the set X of AUNU numbers, then the group forms a regular action

Proof: Let G be a group that acts on a set X , and N is normal subgroup of G , that is

$$N = \{g \in G, g^{\wedge}x = x \forall x \in X\}$$

Define a new quotient Group G/N by

$$G/N = \{gN : g \in G\}$$

If G/N acts on X and it consist of all distinct left cosets of N i.eg $N = Ng$ since N is normal in G from the definition G/N acts on X by $(gN) \wedge x = g^{\wedge}x$ if $g_1g_2 \in N$ and $N \in G$, then

$$(g_1N) \wedge x = (g_2N) \wedge x \implies g_1 \wedge x = g_2 \wedge x \implies g_1N = g_2N$$

$$g_2^{-1}g_1N = g_2^{-1}g_2N \text{ and}$$

$$g_2^{-1}g_1N = N \text{ so } g_2^{-1}g_1 \in N$$

Note that $N = \{g \in G : g^{\wedge}x = x\}$

Then, for any $x \in X$ as $g_2^{-1}g_1 \in N$, then

$$\begin{aligned} (g_2 \wedge g_1) \wedge x &= x \\ g_2 \wedge (g_2 \wedge g_1) \wedge x &= \\ g_2 \wedge x &= (g_2 \wedge g_2 \wedge g_1) \wedge x \\ &= g_2 \wedge x \implies g_1 \wedge x = g_2 \wedge x \end{aligned}$$

Now if $(gN) \wedge x = g^{\wedge}x$ for all $x \in X$, then $g^{\wedge}x = x$ for all $x \in X$ so $g \in N$

That is $gN = N$ is the identity element of G/N and hence G act faithfully on X That is if $g \in G$ and $g \wedge x = x$ for all $x \in X$ then $g = e$ (where e is the identity element).

Proposition 3.2: Let G be a group acting on the set X of AUNU numbers. The group action forms a free actions

Proof: Let $x \in X$ then $g \in G$ $x \rightarrow g \wedge x = g x = x = ex$, then $g = e$.

Assume that for each $x \in X$, G_x is trivial and let $y \in X$ such that $g_1 \wedge y = g_2 \wedge y \forall g_1, g_2 \in G$ and $g_2^{-1} g_1 \in G_y = \{e\}$ (identity element of G). Hence the operation is free. \square

Proposition 3.3: Let G be a group acting on the set X of AUNU numbers. The group action forms a transitive action

Proof: Assume for all $x \in g \wedge (x) = x \forall g \in G$, and Let $y, z \in X$.

Since $y \in X$ there exist Z and transitivity follows:

Assume transitivity, and let $x \in X$ then for all $y \in X$ there exist $g \in G$ with $g \wedge x = y$ and hence $y \in g \wedge x = g x$ \square

Theorem 3.1: Let G be a group acting on the set X of AUNU numbers: The group actions are Strong and faithful

Proof: Considering the propositions of (1), (2) and (3) and by the definition of the group action then the results follows. \square

4.0 CONCLUSION:

This Paper has explored some group algebraic theories of permutation groups which led to obtain some Propositions and theorems relevant to the study of algebraic structures. The paper also provided some geometric structures of orbits from

the constructions which are relevant to the study of simple finite group structures particularly the Sylow p -subgroups and homeomorphisms of groups. The research further examined the structure of the orbits formed which led to obtain some group theoretic consequences of permutation groups.

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