



## A Computational Analysis on Steady MHD Casson Fluid Flow Across A Vertical Porous Channel Affected By Thermal Radiation Effect

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### ABSTRACT

This article investigates the MHD free convection flow of a Casson fluid that is electrically conductive, affected by thermal radiation effect in a vertical porous channel. The resulting solutions of the leading differential equations describing the velocity and temperature flows characteristics are obtained analytically using regular perturbation method. The actions of Casson fluid parameter, magnetic field effect, thermal radiation parameters on the velocity field, temperature distribution, skin friction coefficient and rate of heat transfer has been presented for relevant temperature jump and velocity slip conditions. The results obtained revealed that by taking varying values of the major controlling parameters, such as Casson parameter, thermal radiation parameter, magnetic field effect, rarefaction parameter and wall ambient temperature difference ratio parameter, the velocity and temperature profiles are substantially influenced. Additionally, when the Casson parameter is set to be 1000, the numerical computations of these findings agree very well with those of Abbas *et al.*, 2020.

**Keywords:** Free convection, Casson fluid parameter, suction/injection parameter, MHD, thermal radiation, vertical porous channel

### 1.0 INTRODUCTION

Magneto-hydrodynamics (MHD) research has evolved in prominence in recent decades owing to the concept's benefits in a range of MHD applications, namely magneto-hydrodynamic (MHD) generators, cooling baths having cooling metallic plates, electric transformers, and magneto-hydrodynamic injectors. Several nuclear power facilities already use chemical energy technology, which incorporates the use of MHD pumps to move electrically conductive fluids. In addition to these applications, an applied magnetic field can considerably increase free convection flow whenever the fluid is electrically conductive (Jha *et al.* 2015). Various research has been conducted on hydro-magnetic convective flow under a spectrum of environmental situations. In a recent work, Hamza *et al.* (2023) used homotopy perturbation method to explore the characteristics of Arrhenius-controlled heat transfer fluid on free convection

affected by an induced magnetic field. Later, Hamza *et al.* (2022) evaluated the convectively heated, MHD-free flow of a chemically reacting fluid in a vertical channel contemplated with porous media. The steady-state component was solved using the homotopy perturbation method while the unsteady state case was analyzed using a numerical technique. In a different paper, Hamza *et al.* (2021) discussed the implications of MHD on free convective slip flow of an exothermic fluid under the influence of Newtonian heating. Taid and Ahmed (2022) used the perturbation approach to resolve the combined impacts of the sores effect, heat dissipation, and chemical reaction on steady two-dimensional hydro-magnetic free convection flow across an inclined porous plate coated with porous media. Osman *et al.* (2022) outlined the action of magneto-hydrodynamics on free convection flow over an infinite inclined plate using the Laplace transformation technique. Siva *et*

al. (2021) presented a precise response to the MHD action on a heat transfer study of electroosmotic flow in a rotating microfluidic channel. When Joseph et al. (2015) inspected unsteady MHD poiseuille flow across two infinitely parallel porous plates in an inclined magnetic field, they also considered the impacts of heat and mass transport. They discovered that as the Hartman number  $Ha$  rises, the velocity slows down. Velocity rises under the effect of the thermal Grashof number  $Gr$  and the solutal Grashof number  $Gc$ . A boost in the Prandtl number,  $Pr$ , results in a decrease in temperature. As the chemical parameter  $Kc$  and the Schmidt number  $Sc$  rise, so does the concentration of a species. Geethan et al. (2016) studied the joint effects of thermal radiation, chemical reaction, and solet on MHD free convection slip flow along an inclined plate with constant temperature affected by heat source. A computational analysis of the Hall current and thermal radiation effects of MHD free convective mass and energy transfer radiating fluid flow in the solet effect and heat generation was reported by Sivaiah and Reddy (2017). Ojemer and Hamza (2022) put forth a theoretical investigation of MHD natural convection heat generating/absorbing fluid flow controlled by Arrhenius kinetic in a microchannel.

Thermal radiation plays a significant role in convective heat transfer processes which consist of high temperatures for example, gas turbines, solar power technology, and nuclear power plant. The impacts of radiation on magnetohydrodynamic (MHD) flow and its vital role in the field of mechanical and thermal engineering such as storage of thermal energy, chemical engineering, space vehicle re-entry, aeronautics, and electrical power generation, etc (Abbas et al. 2020). In light of these so many practical applications, Cess (1966) examined the effect of radiation on boundary-layer development in free convection of an absorbing-emitting gas on a vertical plate, and he presented interactions between radiation and free convection. The combined radiative and convective heat transfer in a vertical channel has been studied by Bouali and Mezrhab (2006) and Grosan and Pop (2007). Kaladhar et al. (2016) have presented mixed convection flow of couple stress fluid in a vertical channel with radiation and Solet effect employing a quasi-linearization method. Recently, Gireesha et al. (2020) have studied the

impact of thermal radiation and natural convection on nanoparticle-embedded water-based hybrid nano-fluid flow across a permeable longitudinal fin using Darcy's model. Hasan et al. (2020) have investigated the effects of thermal radiation, heat generation and induced magnetic field on hydro-magnetic free convective flow of couple stress fluid in an isoflux-isothermal vertical channel utilizing the method of undetermined coefficient.

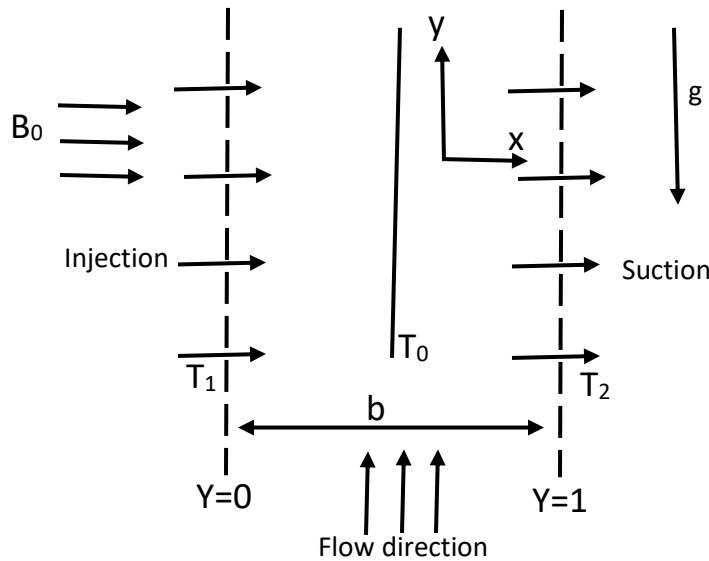
Suction/injection plays a vital function in boundary layer control, particularly in engineering, aerodynamics, space research, and medicine. These applications gained our curiosity to investigate the role of suction/injection in different types of fluids. In view of these, Jha et al. (2022) recently analyzed the impact of suction/injection on a free convective heat flow due to point/line heat source/sink in a vertical channel using Laplace transformation method. Shojaefard et al. (2005) investigated the effect of flow regulated by suction/injection on the aerodynamic lift coefficient numerically. According to the research, closing surface suction slots at the trailing edge increase lift coefficient by 10%. Saeid (2006) performed a numerical investigation on free convection flow in a vertical porous layer generated by bi-thermal sources. In the presence of a suction effect, increasing the temperature of the top element enhances heat transfer to the lower element. In another investigation, Uwanta et al. (2014) showed the influence of suction/injection on hydromagnetic natural convection flow in vertical plates. It has been noticed that suction or injection can be used to control fluid flow in the channel. According to Jha et al. (2015), increasing suction/ injection increases the wall ambient temperature difference, which reduces convection in micro-channels. Furthermore, Jha et al. (2018) deliberated on the effect of suction/injection on natural convection flow in transient magneto-hydrodynamics. During chemical reactions, suction can be used to remove reactants while injection can be applied to add reactants. Falade et al. (2017) examined the effect of suction/injection on transient oscillatory MHD flow. It is worth mentioning that injection on the hot plate causes the skin friction in the channel to increase. Moreover, Jha et al. (2019) proposed the effect of suction/injection on hydro-magnetic free convection flow. They discovered that increasing

velocity lowers injection parameters as well. Suction parameters rise as shear stress increases. Additionally, as a result of periodic heating in a vertical surface, Keshtkar et al. (2014) studied the suction/injection saturated by a nanofluid with a heat source/sink on MHD mixed convection boundary layer flow.

The ultimate goal of this research is to expand the work done by Abbas et al. (2020) by investigating the heat transfer analysis on MHD Casson fluid flow with thermal radiation in vertical porous

channel. The variations of various pertinent parameters are discussed in detail with the aid of illustrative graphs. Researches on the effects of Casson fluid is by no means exhaustive, hence the need to further explore its relevance in a growing technological innovation as we have today. These fluids have various rheological features, which makes them more relevant to chemical, biomedical sciences, chemical engineering, pharmaceutical and so on. Researches on Casson fluid can be found in Omokhuale and Jabaka (2019; 2020a and 2020b).

**2.0 FORMULATION OF THE PROBLEM**



**Figure 1: Schematic of the flow system**

By these assumptions the mass, momentum, and energy balanced equations with thermal radiation are;

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v \left( 1 + \frac{1}{\Gamma} \right) \frac{\partial^2 u}{\partial y^2} - s \frac{\partial u}{\partial y} + g\beta(T - T_0) - \frac{\sigma B_0^2 u}{\rho} - \left( 1 + \frac{1}{\Gamma} \right) = 0 \tag{2}$$

$$u \frac{\partial T}{\partial x} + s \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \tag{3}$$

Here  $s > 0$  indicates the suction velocity and  $s < 0$  indicates injection velocity.

Also in equations (1) – (3)  $u$  and  $v$  shows the velocity components along  $x$  and  $y$  directions

$$qr = - \frac{4\sigma^* \partial T^4}{3k^* \partial y} \tag{4}$$

Where  $\sigma^*$  and  $k^*$  are the Stefan Boltzmann constant and Rosseland mean absorption coefficient, respectively. Consider that the

electrical conductivity and thermal conductivity of the fluid.

Using the Rosseland approximation, the radiative heat flux  $qr$  is given by

temperature difference within the flow is sufficiently small such that  $T^4$  may be expressed as a linear function of temperature in a Taylor

series about  $T_0$  and neglecting higher terms one can get

$$T^4 \cong 4T_0^3T - 3T_0^4 \tag{5}$$

Using equations (4) and (5) equation (3) becomes

$$\frac{k}{\rho c_p} \left( 1 + \frac{16\sigma^*T^3}{3kk^*} \right) \frac{\partial^2 T}{\partial y^2} - s \frac{\partial T}{\partial y} = 0 \tag{6}$$

Where  $N_d = 16\sigma^*T^3/3kk^*$  is considered as radiation parameter, so the above equation becomes

$$\frac{k}{\rho c_p} (1 + N_d) \frac{\partial^2 T}{\partial y^2} - s \frac{\partial T}{\partial y} = 0 \tag{7}$$

Introducing the dimensionless parameters

$$y = \frac{y'}{b}, u = \frac{u'}{U_0}, \theta = \frac{T - T_0}{T_1 - T_0}, \tag{8}$$

Where  $U_0 = \rho g \beta (T - T_0) b^2 / \mu$  is dimensionless velocity.

By incorporating equation (8) in equation (2) and (3) the leading equations now becomes, following Abbas et al []:

$$\left( 1 + \frac{1}{\Gamma} \right) \frac{d^2 u}{dy^2} - s \frac{du}{dy} - M^2 u + \theta = 0 \tag{9}$$

$$\frac{(1 + N_d)}{Pr} \frac{d^2 \theta}{dy^2} - s \frac{d\theta}{dy} = 0 \tag{10}$$

The relevant boundary conditions in dimensionless are as follows:

$$\left. \begin{aligned} U(Y) = (1 + \frac{1}{\Gamma}) \beta_v Kn \frac{dU}{dy}, \quad \theta(Y) = \xi + \beta_v Kn \ln \frac{d\theta}{dy}, \quad at \quad y = 0 \\ U(Y) = -(1 + \frac{1}{\Gamma}) \beta_v Kn \frac{dU}{dy}, \quad \theta(Y) = 1 - \beta_v Kn \ln \frac{d\theta}{dy}, \quad at \quad y = 1 \end{aligned} \right\} \tag{11}$$

### 3.0 ANALYTICAL SOLUTIONS

The coupled system differential equations with constant coefficients has to be computed in closed form by using regular perturbation

method. So the analytical solutions of equations (9) and (10) subject to equation (11) has been determined as follows:

$$\theta(Y) = K_1 e^{m_1 Y} + K_2 \tag{12}$$

$$U(Y) = K_3 e^{m_3 Y} + K_4 e^{-m_4 Y} + K_5 e^{m_1 Y} + K_6 \tag{13}$$

Using equations (12) and (13) respectively, the rate of heat transfer and skin friction coefficients are computed as follows:

$$Nu_0 = \frac{d\theta}{dy} = m_1 K_1 \tag{14}$$

$$Nu_1 = \frac{d\theta}{dy} = m_1 K_1 e^{m_1} \tag{15}$$

$$\tau_0 = \frac{du}{dy} = m_3 K_3 - m_4 K_4 + m_1 K_5 \tag{16}$$

$$\tau_1 = \frac{du}{dy} = m_3 K_3 e^{m_3} - m_4 K_4 e^{-m_4} + m_1 K_5 e^{m_1} \tag{17}$$

Where the constants  $K_1, K_2, K_3, K_4, K_5,$  and  $K_6$  are indicated in the appendix.

### 4.0 RESULT AND DISCUSSIONS

This section is devoted to the analysis of steady fully developed free MHD Casson fluid flow influenced by thermal radiation in a vertical porous channel. The flow pattern of temperature, velocity, Nusselt number, also known as heat transfer rate and skin friction coefficient under the impact of diverse emerging parameters namely Casson fluid parameter, suction/injection parameter, thermal radiation parameter and magnetic field effect, has been discussed in detail and the computed analytical results are presented graphically as shown in Figures 2-10. The present parametric analysis has been conducted over a reasonable range of  $0.5 \leq N_d \leq 2$ ,  $1 \leq M \leq 2$ ,  $-5 \leq s \leq 5$ ,  $0.71 \leq Pr \leq 7$ . It is worth noting to state that, throughout this discussion,  $s < 0$  represent suction while  $s > 0$  implies injection. The reference values chosen for this current investigation are  $N_d = 0.2$ ,  $M = s = 1$ ,  $\Gamma = 0.5$  and  $Pr = 0.71$

Fig. 2 and 3 depicts the impact of thermal radiation on the temperature and velocity distributions. It is revealed that, increasing values of thermal radiation leads to an upward movement of the temperature gradient. However, a diminishing effect is observed in the velocity flow for growing values of thermal radiation for different ascending values of ( $\xi$ ) as illustrated in Fig. 3. It is however important to report that the effect of the radiation parameter  $N_d$  is insignificant for ( $\xi = 1$ ) on the temperature profile.

Fig. 4 and 5 display the actions of ( $\beta_v Kn$ ) on the temperature and velocity distributions for growing levels of ( $\xi$ ). It is found that the fluid velocity becomes higher near the surface of the wall. The physical argument of this behavior is that, with an enhancement in the rare fraction parameter, the velocity slip at the surface enhances which ultimately decreases the frictional effects of the surface. Furthermore, for higher values of ( $\beta_v Kn$ ), it can be seen that the temperature jump rises which results in the reduction in heat transfer from the surface to the fluid. The buoyancy impacts are

also decreased with the decrement in the heat transfer rate which deduces the flow and hence decreases the fluid velocity far away from the surface. It is worthy of note that the impact of the rare fraction ( $\beta_v Kn$ ) parameter is insignificant for ( $\xi = 1$ ) on the temperature gradient.

Fig. 6 describes the function of Casson fluid parameter on the flow movement. It is worthy of mentioning that, as the Casson parameter grows, the velocity of the fluid retards. The physical phenomenon countering this retardation is the plasticity of fluid. When parameter  $\Gamma$  reduces, momentum boundary layer thickness increases due to an increase in the plasticity of fluid. Furthermore, it is worthy to mention that when  $\Gamma$  is very large ( $\frac{1}{\Gamma}$  approaches 0), the non-Newtonian behavior of fluid fully vanishes, and fluid reduces to a purely Newtonian fluid.

Figure 7 demonstrates the influence of MHD on the velocity distribution. The pattern shows a weakening effect on the velocity as the magnetic field intensity becomes stronger, which is expectedly so as a result of the Lorentz force in the magnetic field affecting the fluid motion.

The consequence of thermal radiation on the heat transfer rate is illustrated in Figures 8a and b. It is demonstrated from these graphs that a similar trend is witnessed at the both vertical channel plates.

Figs. 9 and 10 show the deviations of skin friction for thermal radiation and Casson fluid parameters for ( $\xi = -1$  and 0). It is evident that, at  $y=0$ , increasing function of the thermal radiation parameter leads to a fall in the shear stress coefficient as displayed in Fig. 9a, whereas as shown in Fig. 9b, a reverse situation is observed at the hot plate ( $y=1$ ). Similarly, it is abundantly clear that this same trend is replicated for varying values of Casson fluid parameter on skin friction gradient.

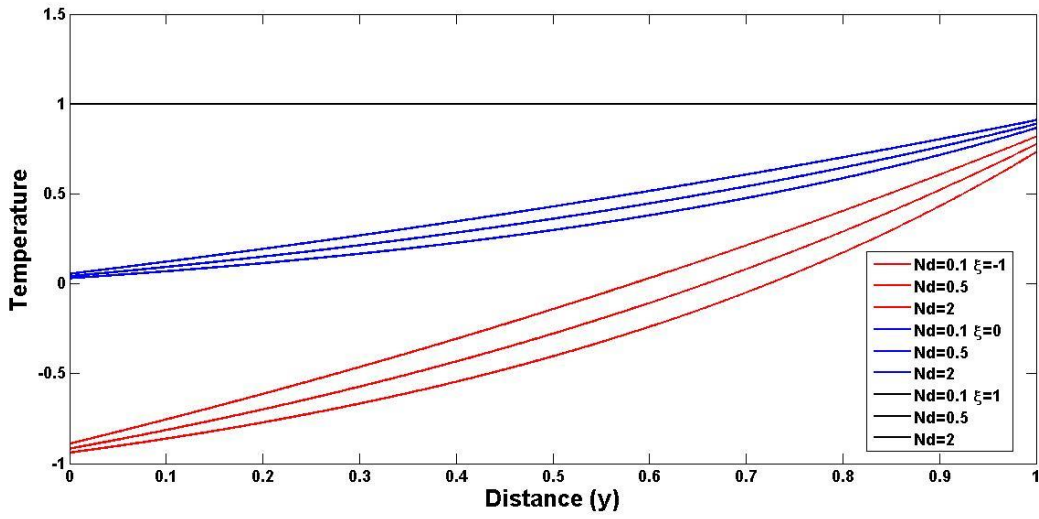


Figure 2: Action of  $N_d$  on temperature profile

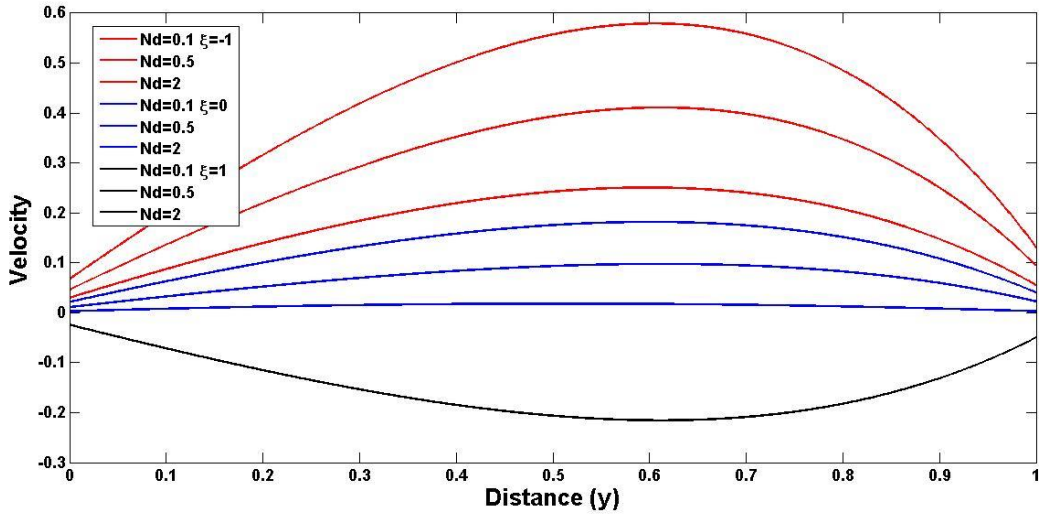


Figure 3: Action of  $N_d$  on velocity profile

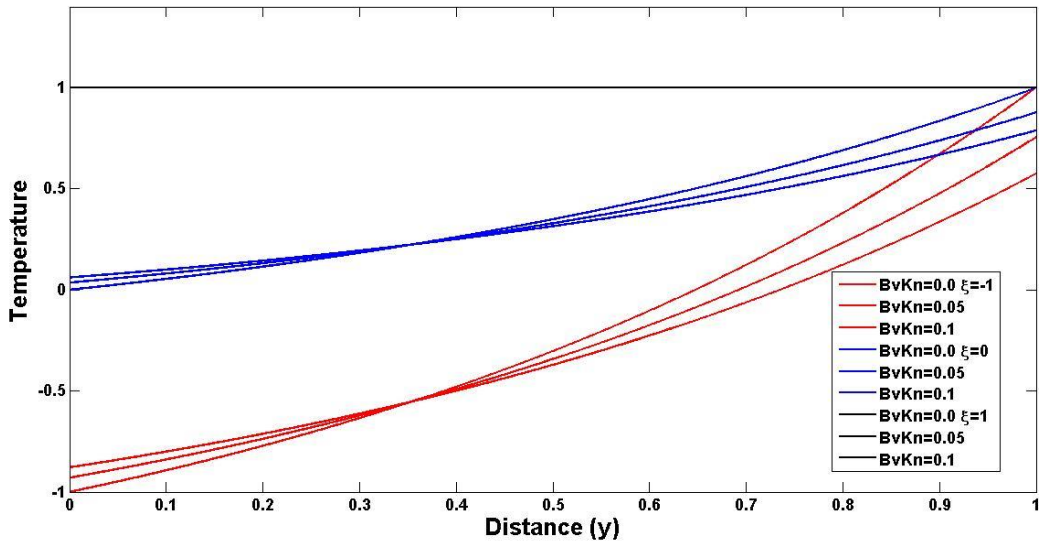


Figure 4: Action of BvKn on temperature profile

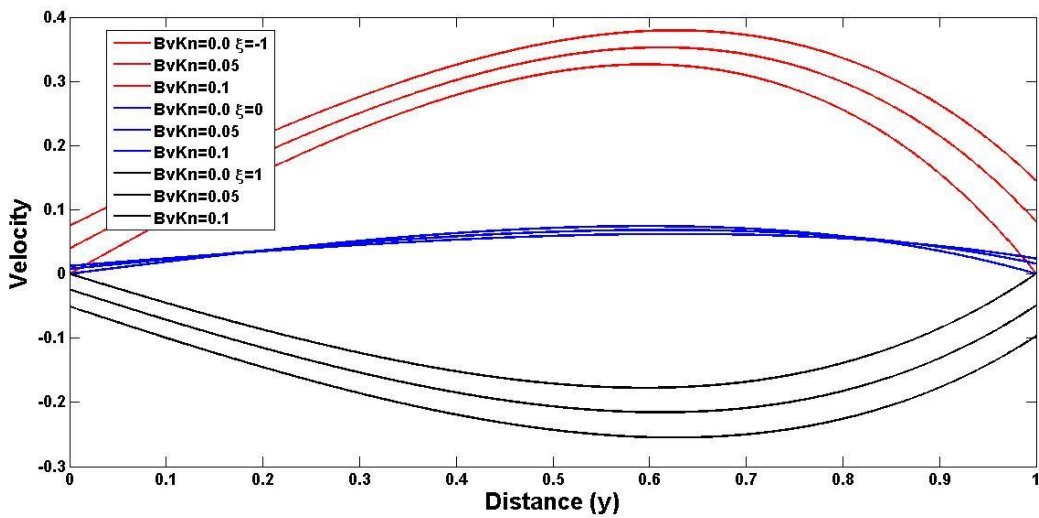


Figure 5: Action of BvKn on velocity profile

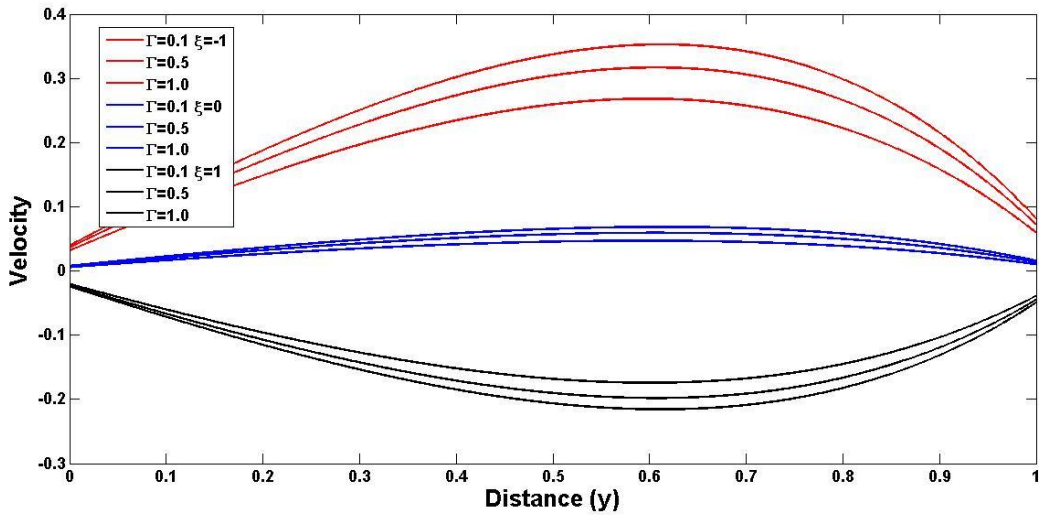


Figure 6: Action of  $\Gamma$  on velocity profile

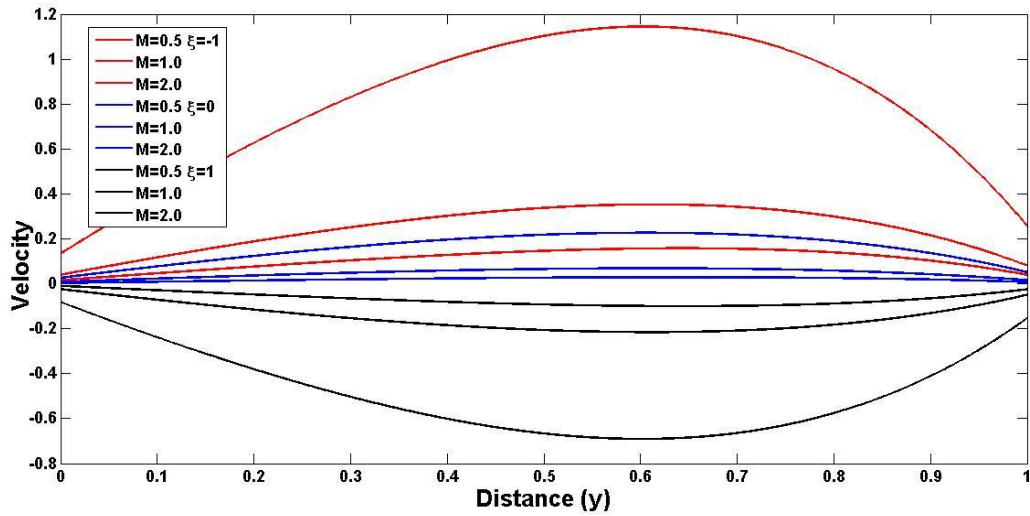


Figure 7: Action of  $M$  on velocity profile

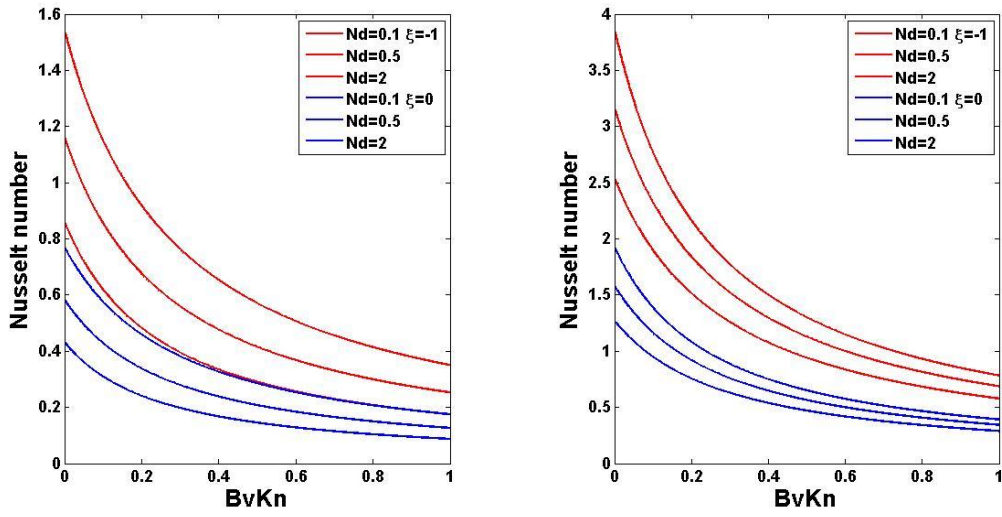


Figure 8: Actions of  $Nd$  on Nusselt number at (a)  $y=0$  and at (b)  $y=1$

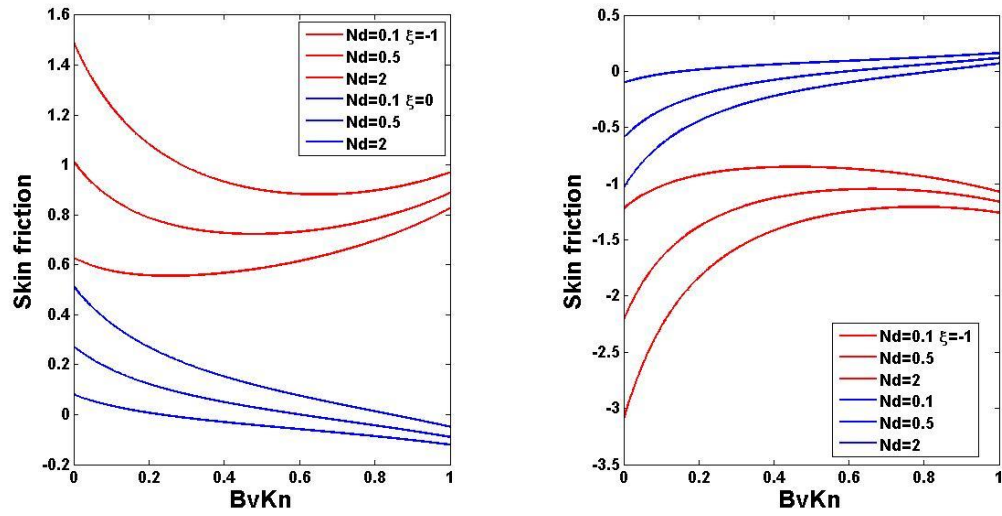


Figure 9: Actions of  $Nd$  on shear stress at (a)  $y=0$  and at (b)  $y=1$

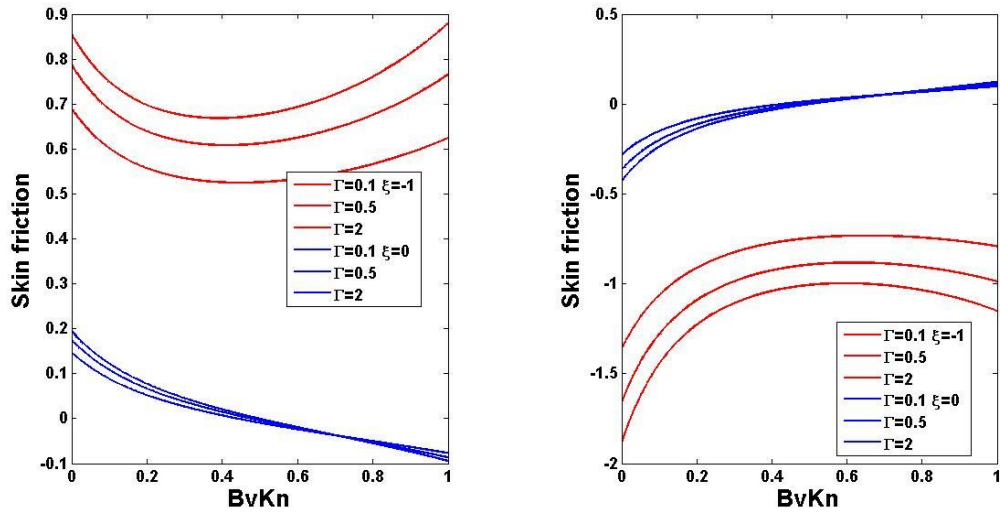


Figure 10: Actions of  $\Gamma$  on shear stress at (a)  $y=0$  and at (b)  $y=1$

**5.0 VALIDATION**

In order to validate this current investigation, the numerical computation of the comparison between the work of Abbas et al. (2020) with the present work has been constructed. An excellent agreement was established as demonstrated in Table 1 for velocity distribution at both plates

Table 1: Numerical computations of the comparison between the work of Abbas et al. (2020) with the present work for velocity profile for different  $\xi = -1, 0, 1$  at  $\beta_v Kn = 0.05$ ,  $In = 1.667$ ,  $Pr = 1.5$ ,  $N_d = 0.2$ ,  $M = s = 1$ , setting  $\Gamma = 10000$ .

| Y   | Abbas et al. (2020)<br>U(Y) |           |           | Present work<br>U(Y) |           |           |
|-----|-----------------------------|-----------|-----------|----------------------|-----------|-----------|
|     | $\xi = -1$                  | $\xi = 0$ | $\xi = 1$ | $\xi = -1$           | $\xi = 0$ | $\xi = 1$ |
| 0.1 | 0.6012                      | 0.2741    | -0.0530   | 0.6012               | 0.2741    | -0.0530   |
| 0.2 | 0.9158                      | 0.4154    | -0.0850   | 0.9158               | 0.4154    | -0.0850   |
| 0.3 | 1.1560                      | 0.5216    | -0.1127   | 1.1560               | 0.5216    | -0.1127   |
| 0.4 | 1.3194                      | 0.5922    | -0.1350   | 1.3194               | 0.5922    | -0.1350   |
| 0.5 | 1.4016                      | 0.6257    | -0.1502   | 1.4016               | 0.6257    | -0.1502   |

**6.0 CONCLUSION**

A fully developed steady state MHD free convection flow of viscous reactive Casson fluid with suction and injection effects under the influence of magnetic field has been performed. The analytical solution has been determined for temperature, velocity, heat transfer rate and skin friction gradients using regular perturbation scheme and various graphs has been plotted to illustrate the functions of pertinent parameters. The findings of this research are summarized below:

- i The functions of thermal radiation and rarefaction effects is to drastically speed up the temperature and velocity profiles respectively.
- ii Fluid velocity is a diminishing function of Casson parameter and magnetic field respectively
- iii. Both the thermal radiation and Casson fluid parameters are clearly seen to weaken the shear stress at the wall ( $y=0$ ), whereas a counter attribute is noticed at  $y=1$ .
- iv The skin friction is profoundly decreasing for higher values of at  $y=0$ , whereas an opposite situation is observed at  $y=1$

v. The rate of heat transfer is minimized when thermal radiation parameter escalates

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**Appendix**

$$c1 = (1 + \frac{1}{\Gamma}), \quad c2 = \frac{s}{c1}, \quad c3 = \frac{M^2}{c1}, \quad a2 = 1 - m1BvKnLn, \quad a3 = m1BvKnLne^{m1}$$

$$a4 = a2 - e^{m1} - a3, \quad a5 = a2 - \xi a3 - \xi e^{m1}, \quad a6 = c1BvKnk6m1 - k6 - k7,$$

$$a7 = 1 - m3BvKnc1, \quad a8 = 1 + m4BvKnc1, \quad a9 = \frac{a6}{a7}, \quad a10 = \frac{a8}{a7},$$

$$a11 = -k6c1BvKnm1e^{m1} - k6e^{m1} - k7, \quad a12 = k6c1BvKnm3e^{m3} - e^{m3}, \quad a13 = e^{-m4} - c1BvKnm4e^{-m4},$$

$$a14 = a13 - a10a12, \quad a15 = a11 - a9a12,$$