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Mixed Convection Flow in a Micro-channel in the Presence of Magnetic Field and Thermal Property

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ABSTRACT

The fully developed MHD mixed convection flow in a micro channel having variable thermal conductivity is considered. The velocity slip and temperature jump at both plates were taken into consideration. The effects of various flow parameters entering into the problem such as Hartmann number, variable thermal conductivity, mixed convection parameter, rarefaction parameter, fluid-wall interaction parameter and wall-ambient temperature difference ratio were discussed with the aid of line graphs. It is revealed that greater values of variable thermal conductivity tend to significantly improve the temperature and velocity gradients respectively. Furthermore, mixed convection parameter, significantly influence the flow pattern.

Keywords: MHD, Mixed convection, variable thermal conductivity, wall interaction, rarefaction, perturbation technique, thermal property

1.0 INTRODUCTION

Emerging technologies of heat transfer flow in micro – geometry in the existence of applied magnetic field has found promising applications in the development of micro-electron mechanical systems technologies (MEMS) and Nano electromechanical system technology (NEMS), some of which includes: magnetic sensors, micromixers, micro-pumps, micro-valves, micro heat exchangers amongst others. Nowadays, heat transfer enhancement and thermal efficiency are becoming important engineering topics related to conservation of the limited energy and enhancing thermodynamic performance. In recent decades, many technologies have been presented to improve the efficiency in different heat conversion devices or heat exchangers. Among the technological revolutions, micro-channels have been identified as the most essential one to transport fluids in a miniaturization system (Ganji and Malyandi, 2016, Kandlikar *et al.*, 2006).

More specifically, many authors have simulated different models in micro-channels like (Ltaifa *et al.*, 2021), where they presented mixed convection heat transfer for a laminar flow in a rectangular

inclined microchannel totally filled with a water/Al₂O₃ Nano fluid. Their results show that the higher values of the volume fraction of Al₂O₃ and the large values of inclination angles improve the heat transfer rate. Chen and Weng (2004) fundamentally highlighted the natural convection flow in a microchannel while Jha *et al.* (2014) later extended Chen and Weng's work by considering magnetic field in a vertical microchannel. Jha *et al.* (2018) investigated the impact of Hall current on hydromagnetic free convection flow in a vertical micro – channel. Jha and Malgwi (2019a) also analyzed the influence of magnetic field inclination on hydromagnetic free convection flow in an asymmetrically heated vertical micro – channel in existence of Hall current and they (2019b) later extended their work to examine the combined influence of ion – slip and induced magnetic field effects.

Naphon *et al.* (2018) experimentally studied the performance of heat transfer for jet impingement in a microchannel heat sink for the flow of laminar Nano fluids. They found that convective heat transfer went up from 18.56% to 0.015% by volume fraction. Bowers *et al.* (2018) experimentally

studied the thermal behavior of the flow of laminar Nano fluids in micro channels. They proved that convective heat transfer intensified for both Nano fluids (silica and alumina) at low particle concentrations. The improvement in heat transfer grew with the rise of Re and the hydraulic diameter of micro-channels. D'Orazio and Karimipour (2019) studied the same problem with top-bottom walls subjected to a constant heat flux. Recently, Hamza *et al.* (2021) reported free convection flow of an electrically conducting fluid in a microchannel filled with porous material having variable thermal conductivity.

Magnetohydrodynamics (MHD) is the study of electrically conducting liquids, such as salty water, electrolytes, plasma, and liquid metals. This type of fluid has a number of engineering and industrial applications such as in growth of crystals, reactor cooling, magnetic drug targeting, MHD sensors, and power generation. MHD is dependent on the intensity of magnetic induction (Jawad *et al.* 2021). Various researches have been conducted to analyze jointly or separately, several effects like MHD hybrid Nano fluid effect (Gul *et al.*, 2021), MHD Casson Nano fluid flow for mass and heat transfer (Saeed and Gul, 2021), cross diffusion flow of non-Newtonian fluids with heat source and sink (Reddy *et al.*, 2016). In this analysis, the authors have studied the combined effects of fractional and irregular for heat on the flow system and used the Fehlbeg method to determine the solution of the modeled problem. Sravanthi and Gorla (2018) discussed the influence of the source/sink and chemical response on the Maxwell MHD nanoliquid flow. In this discussion, the authors considered the flow past an exponentially convective stretched surface and employed the semi-analytical technique HAM to determine the solution problem.

Gurivireddy *et al.* (2016) highlighted the effect of thermal diffusion on MHD heat and mass transfer flow through a semi-infinite moving vertical porous plate with heat generation and chemical reaction. The heat and mass transfer of unsteady hydromagnetic free convection flow through porous medium past a vertical plate with uniform surface heat flux was analyzed by Mohamed and Aishah (2017). Ziz and Shams (2020) studied the

generation of entropy for the MHD Maxwell Nano fluid flow. In this study, the authors considered the flow system by considering slip conditions, thermal radiations, heat sources, and variable thermal conductivity.

Numerous intermediate situations between forced convection and free convection coexist with comparable orders of magnitude, called mixed convection. Among the examples of this diversity, we can cite: slow flows in pipes (as in water radiators) or along a wall, isotherm jets, and atmospheric flows and currents (more generally a lot of highly isotherm flows). Heat transfer by mixed convection in micro channels has been treated for several decades (Avci and Aydin 2007, Avramenko *et al.*, 2017).

Jha *et al.* (2020) performed a theoretical analysis on fully developed hydromagnetic flow of heat generating/absorbing fluid in a vertical micro channel in the presence of Hall and ion – slip effects. Results from the analysis reveal that in the existence of heat generation/absorption, the primary velocity and the temperature distribution could be enhanced by growing the ion – slip and Hall current parameter, whereas its influence on velocity is contrast along the secondary direction. In this context, Khanafer *et al.* (2019) simulated mixed convection heat transfer in a differentially heated cavity of the cylinders speed. They found that the average Nusselt number increased for high values of the rotation speed of the cylinder, the Reynolds number (Re) and the Richardson number (Ri).

This simulation also showed that the heat transfer rate was comparable to the values of the cylinder speed for high Richardson number. Tavakoli *et al.* (2019) numerically investigated mixed convection heat transfer inside rectangular microchannels with two-phases. The obtained results showed that an improvement in Reynolds number, the slip coefficient number, Grashof number (Gr) and the volume fraction of nanoparticles would enhance the heat transfer rate. Masoud *et al.* (2017) presented an experimental study dealing with nanofluid thermal conductivity composed of MWCNT-CuO/water.

Xu and Sun (2019) conducted a model of a hybrid nanofluid in a vertical microchannel under mixed convection. They stated that the nanoparticle volumetric fractions strongly affected the dynamic and thermal behaviors. Additionally, they demonstrated that the increase in Ri and the nanoparticles volume fractions would enhance the average heat transfer rate. Manay and Mandev (2019) investigated the mixed convection of nanofluids in circular microchannels. They found that the suspension of the volume fraction of silicon dioxide nanoparticles enhanced the heat transfer rate. Therefore, it would decrease regardless Re . Recently, Bilal *et al.* (2022) studied mixed convective magnetohydrodynamics (MHD) Newtonian nanofluid flow through a porous vertical channel with radiation effects. An induced magnetic field is retained for the conducting and non-conducting walls of the channel. Ojemeru and Hamza investigated a chemically reacting fluid in a vertical micro-channel limited to an applied transverse magnetic field that has fully developed steady natural convection flow under the impact of internal heat generation or absorption effects. Analysis of exponential heat source and induced magnetic field on the second-class convection of

Casson fluid in a microchannel was presented by Hasibi *et al.* (2022).

2.0 FORMULATION OF THE PROBLEM

The impact of variable thermal conductivity on steady fully developed mixed convection flow in a vertical fully developed mixed convection flow in a vertical micro-channel is presented. The x -axis is parallel to the gravitational acceleration g but in the opposite direction while the y -axis is orthogonal to the vertical plate. The schematic diagram of the present flow model is illustrated in figure 1. A uniform magnetic field of strength B_0 is assumed to be applied in the direction perpendicular to the flow direction. It is assumed that the magnetic Reynolds number is very small, which correspond to negligible induced magnetic field compared to the externally applied one. The fluid is also assumed to be incompressible and electrically conducting. The plates are heated asymmetrically with one plate maintained at a temperature T_1 while the other plate at temperature T_2 where $T_1 > T_2$. Following Jha *et al.* (2014) and under the basic assumption of the Boussinesq's approximation, the velocity and temperature equations for the present problem in dimensionless form are as follows:

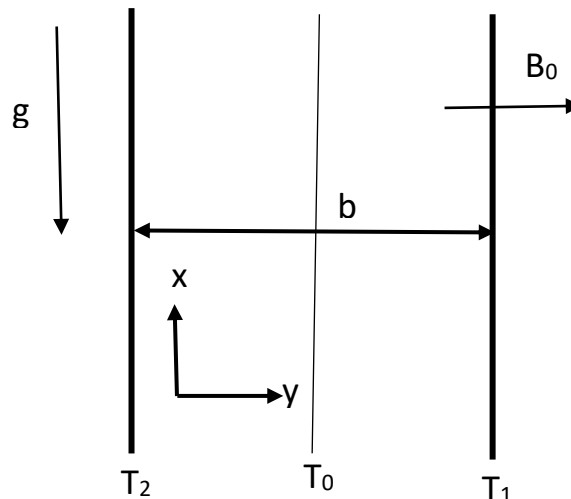


Figure 1: schematic diagram of the flow model

$$v \frac{d^2 U}{dy^2} + g \beta (T - T_0) - \frac{\sigma B_0^2 b^2}{\nu \rho} u = 0 \quad (1)$$

$$\frac{1}{\rho C_p} \frac{d}{dy} \left[k_f \frac{dT}{dy} \right] = 0 \tag{2}$$

$$\left. \begin{aligned} U_{(y=0)} &= \frac{2-fv}{fv} \lambda \frac{du}{dy}, T_{(y=0)} = T_2 + \frac{2-ft}{ft} \frac{2\gamma}{\gamma+1} \frac{\lambda}{Pr} \frac{dT}{dy}, \text{ at } y = 0 \\ U_{(y=b)} &= \frac{-2-fv}{fv} \lambda \frac{du}{dy}, T_{(y=b)} = T_1 - \frac{2-ft}{ft} \frac{2\gamma}{\gamma+1} \frac{\lambda}{Pr} \frac{dT}{dy}, \text{ at } y = b \end{aligned} \right\} \tag{3}$$

The dimensional quantities used in the flow problem is given below as follows:

$$\left. \begin{aligned} \beta_v &= \frac{2-fv}{fv}, \beta_t = \frac{2-ft}{ft} \frac{2\gamma_s}{\gamma_s+1} \frac{1}{Pr}, Y = \frac{y}{b}, \theta = \frac{T-T_0}{T_1-T_0}, U = \frac{u}{U_0}, P = \frac{du}{dx}, \\ \varepsilon &= \varepsilon^* (T_w - T_0), M^2 = \frac{\sigma B_0^2 b^2}{\rho \nu}, \lambda = \frac{QC_0^* AEH^2}{RT_0^2} e^{\left(\frac{-E}{RT_0}\right)}, Gr = \frac{g \beta (T_1 - T_0) x^3}{\nu^2}, Re = \frac{u_0 x}{\nu}, \\ Kn &= \frac{\lambda}{b}, In = \frac{\beta_t}{\beta_v}, \xi = \frac{T_2 - T_0}{T_1 - T_0}, Pr = \frac{\nu}{\alpha}, \varepsilon = \frac{RT_0}{E}, U_0 = \frac{\rho g \beta (T_1 - T_0) b^2}{\mu} \end{aligned} \right\} \tag{4}$$

Substituting (4) into equations (1) to (3) we get, the following dimensionless equations

$$\frac{d^2 U}{dY^2} - M^2 U = -\frac{Gr}{Re} \theta + P \tag{5}$$

$$(1 + \varepsilon \theta) \frac{d^2 \theta}{dY^2} + \varepsilon \left(\frac{d\theta}{dY} \right)^2 = 0 \tag{6}$$

The corresponding boundary conditions for the momentum and energy equations are as follows:

$$\left. \begin{aligned} U(Y) &= \beta_v Kn \frac{dU}{dY}, \theta(Y) = \xi + \beta_v Kn In \frac{d\theta}{dY}, \text{ at } Y = 0 \\ U(Y) &= -\beta_v Kn \frac{dU}{dY}, \theta(Y) = 1 - \beta_v Kn In \frac{d\theta}{dY}, \text{ at } Y = 1 \end{aligned} \right\} \tag{7}$$

The dimensionless quantities used in the equations are defined in the nomenclature.

3.0 METHOD OF SOLUTION

We assume the solution of U and θ to be:

$$\left. \begin{aligned} U(Y) &= u_0 + \varepsilon u_1 + o(\varepsilon) \\ \theta(Y) &= \theta_0 + \varepsilon \theta_1 + o(\varepsilon) \end{aligned} \right\} \tag{8}$$

Substituting equations (8) into equations (5) and (6), subject to equation (7), we have the solutions for temperature and velocity as follows:

$$U(Y) = A_1 e^{MY} + A_2 e^{-MY} + \alpha_1 Y + \alpha_2 + \varepsilon [A_3 e^{MY} + A_4 e^{-MY} + \alpha_3 + \alpha_4 Y^2 + \alpha_5 Y] \tag{9}$$

$$\theta(Y) = C_1 Y + C_2 + \varepsilon \left[-\frac{C_1^2 Y^2}{2} + C_3 Y + C_4 \right] \tag{10}$$

The steady state skin friction and Nusselt number at both plates are obtained as follows:

$$\left(\tau_0 \right) = \frac{dU}{dY} \Big|_{Y=0} = A_1 M_1 - A_2 M_2 + \alpha_1 + \varepsilon [A_5 M_1 - A_2 M_2 + \alpha_4] \tag{11}$$

$$(\tau_1) = \frac{dU}{dY} \Big|_{Y=1} = A_1 M_1 e^{M_1} - A_2 M_2 e^{-M_2} + \alpha_1 + \varepsilon \left[A_5 M_1 e^{M_1} - A_2 M_2 e^{-M_2} + 2\alpha_5 \right] \quad (12)$$

$$Nu_0 = \frac{d\theta}{dY} \Big|_{Y=0} = C_1 + \varepsilon \left[\frac{C_1^2}{2} \right] \quad (13)$$

$$Nu_1 = \frac{d\theta}{dY} \Big|_{Y=1} = C_1 - \varepsilon \left[\frac{C_1^2}{2} \right] \quad (14)$$

4.0 RESULTS AND DISCUSSION

In the present study, the regular perturbation technique has been employed to obtain the analytical expressions of momentum and energy equations, skin friction and Nusselt number. To see the physical impact of the Hartmann number, (M), Wall Interaction parameter (In), mixed convection parameter (Gre), pressure parameter (P), and Variable thermal conductivity (ε). We have plotted the line graph for both the velocity and temperature. The present parametric study has been performed over reasonable ranges of $0 \leq BvKn \leq 0.1$ and $0 \leq In \leq 10$.

Fig. 2: shows the velocity variation for different values of Hartmann number (M), from this figure, it is found out that the effect of the Hartmann number is to decrease the fluid velocity. This is the classical Hartmann effect. Furthermore, the effect of Hartmann number on the micro channel slip velocity becomes significant with the increase of the wall-ambient temperature difference ratio (ξ).

Fig. 3 displays the influence of (In) wall interaction parameter on temperature profile for different value of ξ . It seen that for $\xi = -1$ and $\xi = 0$ the temperature profile increase with an increase in (In) up to a point 0.5 on Y where the trend changes to decrease the temperature. But a different scenario for $\xi = 1$, where the temperature profile remains the same for any increase in (In). Fig. 4 demonstrate the effect on mixed convection parameter (Gre) on velocity profile for values of $\xi = -1, 0, 1$ and it is shown from the figure that for $\xi = -1$, the velocity fluid decrease with an increase in Gre up to a certain point on $y = 0.6$ where the trend changes to increase the velocity profile. But the velocity fluid increase for every increase in Gre for both $\xi = 0$ and 1.

Fig. 5 shows the velocity variation for different values of fluid-wall interaction parameter (In). It clearly reveals that velocity slip increases on the microchannel surfaces with increase of fluid wall interaction parameter while the impact of fluid-wall interaction parameter on the microchannel slip velocity become significant with the decrease of the wall-ambient temperature difference ratio (ξ). Fig. 6 illustrate the impact of variable thermal conductivity (ε) on temperature, as for every increase in ε also increase the temperature profile for $\xi = -1$ and $\xi = 0$ but the temperature remain constant for every increase in $\xi = 1$. Fig. 7 shows the velocity variation for different values of rarefaction parameter ($BvKn$). It is observed that as $BvKn$ increases, the velocity slip at the wall increases which reduces the retarding effect of the wall. This yields an observable increase in the gas velocity near the wall. Also, as $BvKn$ increases, the temperature jump increases and reduces the amount of heat transfer from the wall to the fluid.

Fig. 8 and 9: displays the variation of skin-friction with respect to fluid-wall interaction parameter (In) and the departure from the continuum regime ($BvKn$) for different values of wall-ambient temperature difference ratio (In) at the microchannel plates $Y = 0$ and $Y = 1$, respectively. It evident from these figures that the skin-friction increases with the increase of the values of fluid-wall interaction parameter. Fig. 10 and 11 presents the variation of rate of heat transfer at the microchannel plates $Y = 0$ and $Y = 1$ respectively, for different values of fluid-wall interaction parameter In . It is seen from this figures that the rate of heat transfer decreases with increase of fluid-wall interaction parameter (In).

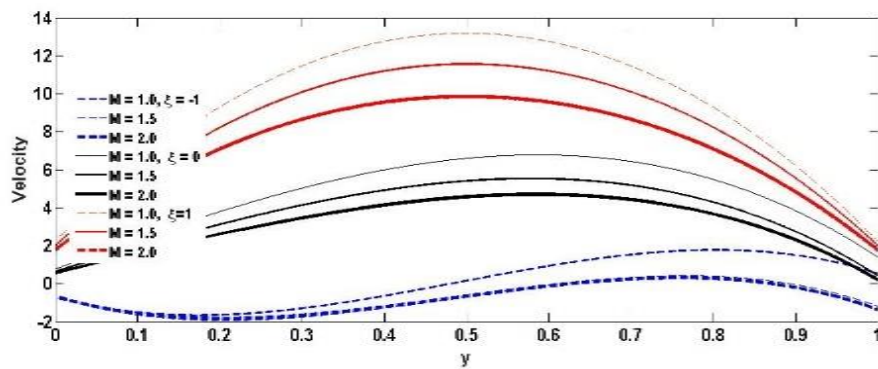


Figure 2: Velocity profile as a function M for $BvKn = 0.05$, $In = 1.667$

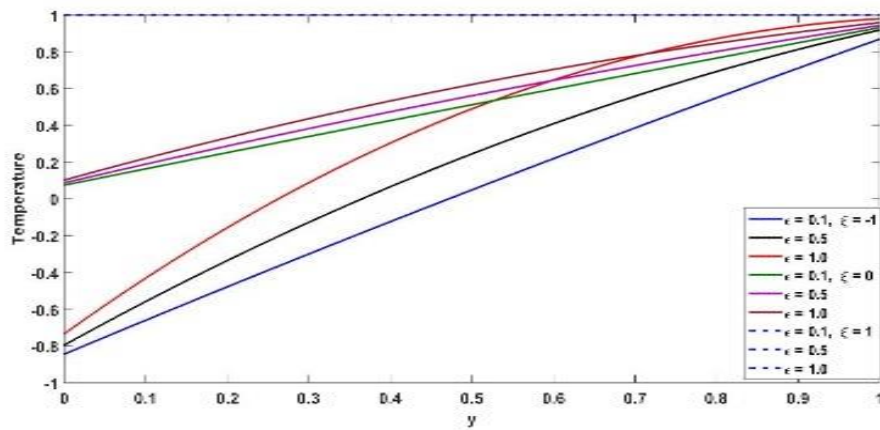


Figure 3: Effects variable thermal conductivity on Temperature Profile when other parameters are fixed.

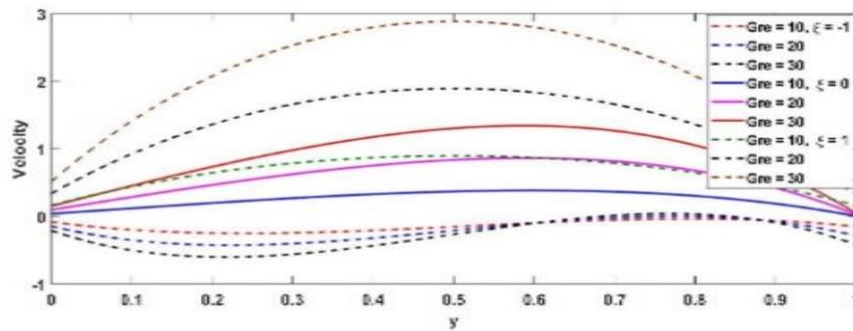


Figure 4: Velocity profile as a function Gre for $Bvkn = 0.05$, $In = 1.667$.

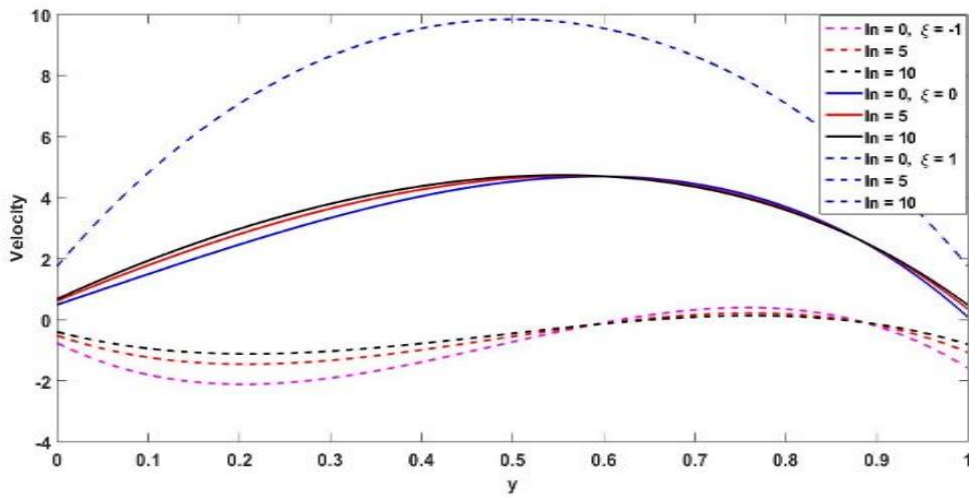


Figure 5: Velocity profile as a function \ln for $Bvkn = 0.05$. $M = 2.0$

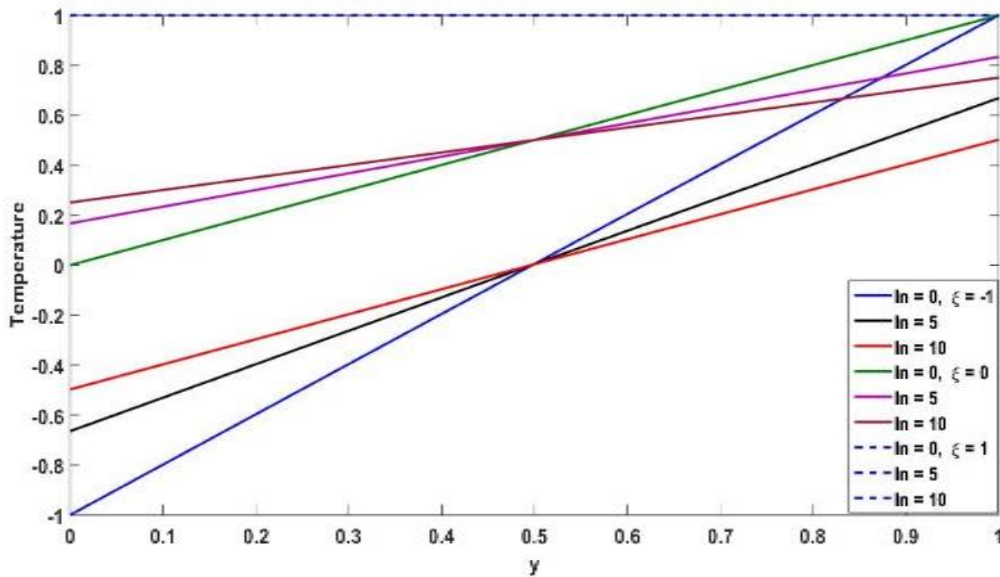


Figure 6: Effects fluid wall interaction parameter on Temperature Profile when other parameters are fixed.

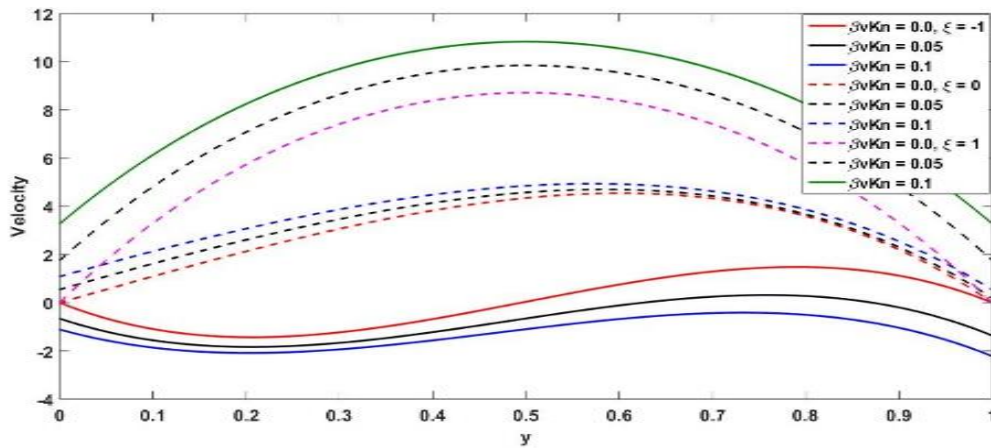


Figure 7: Velocity profile as a function $BvKn$ for $ln = 1.667, M = 2.0$.

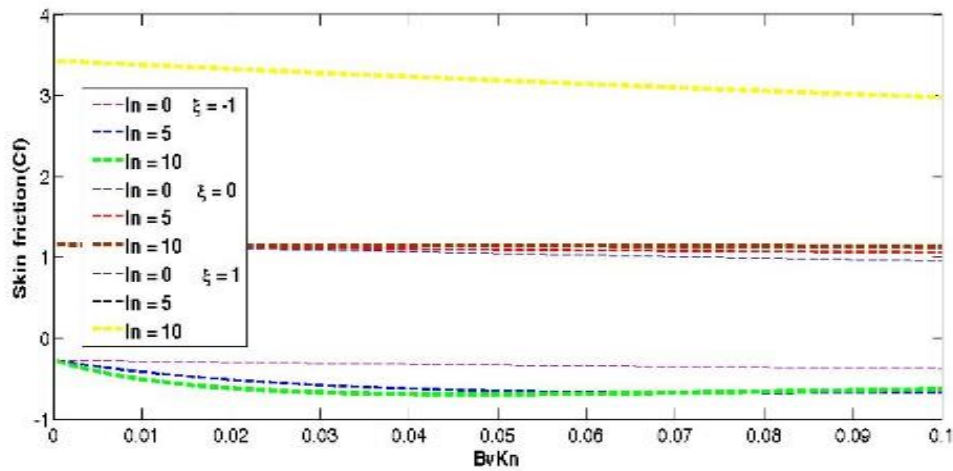


Figure 8: Variation of skin friction (cf_0) for different values of ln .

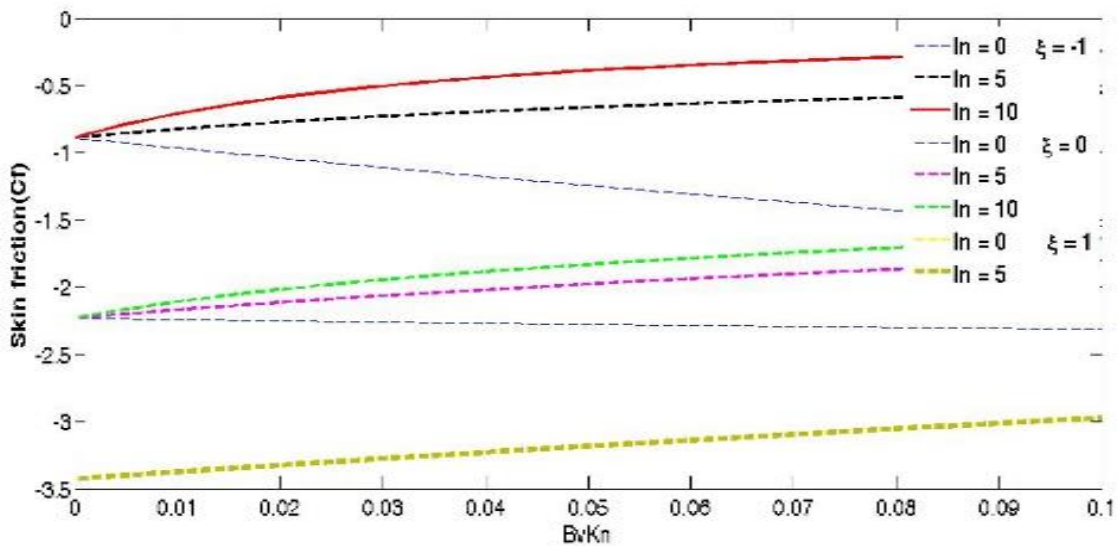


Figure 9: Variation of skin friction (cf_1) for different values of ln .

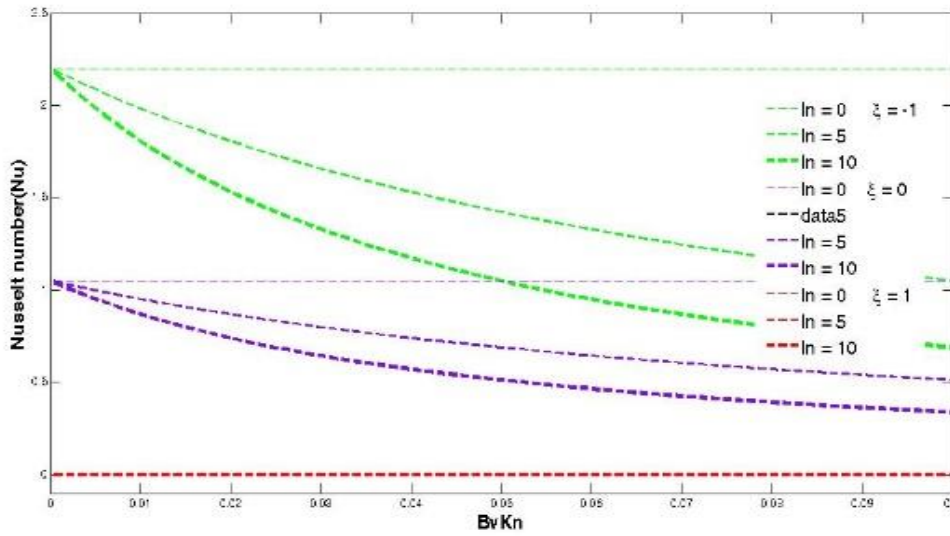


Figure 10: Variation of Nusselt number (Nu_0) for different values of ln .

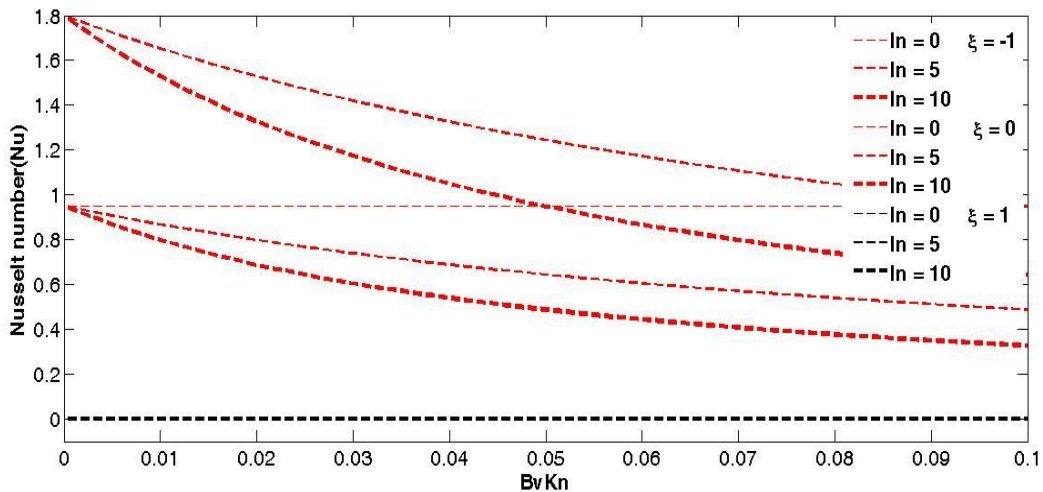


Figure 11: Variation of Nusselt number (Nu_1) for different values of ln .

5.0 VALIDATION

The numerical computation of the comparison between the works of Jha *et al.* (2014) with the present work revealed an excellent agreement as shown in table1 for velocity

Table 1: Shows numerical computations of comparison between the work of Jha *et al.* (2014) with the present work for velocity profile for different $\xi = 0$ and 1 at $\beta_v Kn = 0.05, ln = 1.667, M = 2$ as λ approaches 0

Y	Jha <i>et al.</i> (2014) U(Y)		Present Results U(Y)	
	$\xi = 0$	$\xi = 1$	$\xi = 0$	$\xi = 1$
0.1	0.0156	0.0244	0.0156	0.0244
0.2	0.0153	0.0239	0.0153	0.0239
0.3	0.0150	0.0235	0.0150	0.0235

0.4

0.0148

0.0230

0.0148

0.0230

6.0 CONCLUSION

A theoretical study on mixed convection flow in micro-channel has been presented in this work. Closed-form expressions for velocity, temperature, skin-friction and rate of heat transfer which is expressed as a Nusselt number are obtained by solving the present mathematical model. The effect of rarefaction parameter, fluid-wall interaction parameter, and Hartmann number are discussed with the aid of line graphs. The outcomes revealed that increase in the effects of rarefaction and fluid wall interaction leads to increase of the volume flow rate while it decreases with an increasing function of Hartmann number (M). It is also interesting to note that the role of Hartmann number on volume flow rate is insignificant for the case of symmetric heating ($\xi = 1$). Also, growing numbers of variable thermal conductivity tends to enhance the temperature and velocity respectively. Furthermore, at higher mixed convection parameter values, the slip velocity increases on the hot wall and decreases on the cold wall

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Nomenclature

b= channel width

g=gravitational acceleration

B_0 =constant magnetic flux density

C_p, C_v = specific heats at constant pressure and constant volume respectively

f_p, f_v = thermal and tangential momentum accommodation

coefficients, respectively

In = fluid - wall interaction parameter

Kn = Knudsen number $\frac{\lambda}{b}$

ε = dimensionless variable thermal conductivity

M = Hartmann number

$\frac{Gr}{Re}$ = mixed convection parameter (Gre)

P = pressure term

Pr = Prandtl number

Nu = dimensionless heat transfer rate

T = temperature of the fluid

T_0 = reference temperature

u = velocity component in x direction

U = dimensionless velocity

Greek letters

β = thermal expansion coefficient

β_t, β_v = dimensionless variables

μ = dynamic viscosity

α = thermal diffusivity

τ = skin friction coefficient

γ_s = ratios of specific heats (C_p, C_v)

ξ = wall-ambient temperature difference ratio

σ = electrical conductivity of the fluid

ρ = density

ν = fluid kinematic viscosity

Appendix

$$C_1 = \frac{1 - \xi}{1 + 2\beta_v kn \ln}, \quad C_2 = \xi + \beta_v kn \ln C_1,$$

$$C_3 = \frac{C_1^2}{2}, \quad C_4 = \frac{C_1^2}{2} \beta_v kn \ln,$$

$$A_1 = \frac{a_2 - A_2 a_2}{a_3}, \quad A_2 = \frac{a_3 a_4 - a_1 a_6}{a_3},$$

$$A_3 = -A_4 a_7 + a_8, \quad A_4 = \frac{a_{11}}{a_{12}},$$

$$\alpha_1 = \frac{Gre C_1}{M^2}, \quad \alpha_2 = \frac{Gre C_2 - P}{M^2},$$

$$\alpha_3 = \frac{M^2 Gre C_1^2 \beta_v kn \ln + 2 Gre C_1^2}{2 M^2}, \quad \alpha_4 = \frac{Gre C_1^2}{2 M^2},$$

$$\alpha_5 = -\frac{Gre C_1^2}{2 M^2}, \quad a_1 = \beta_v kn \alpha_1 - \alpha_1,$$

$$a_2 = 1 + M^2 \beta_v kn, \quad a_3 = 1 - M^2 \beta_v kn,$$

$$a_4 = -\alpha_2 - \alpha_1 (1 + \beta_v kn)$$

$$a_5 = a_1 e^{-m_1}, \quad a_6 = a_2 e^{-m_2}, \quad a_7 = \frac{1 + M^2 \beta_v kn}{1 - M^2 \beta_v kn},$$

$$a_8 = \frac{-\alpha_3 + M^2 \beta_v kn}{1 - M^2 \beta_v kn}, \quad a_9 = \alpha_3 + \alpha_4 + \alpha_5,$$

$$a_{10} = \alpha_4 + 2\alpha_5, \quad a_{11} = -a_8 e^{m_1} - a_9 - a_8 m_1 e^{m_1} \beta_v kn - a_{10} \beta_v kn$$