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Suction/Injection Effects on Pulsating Poiseuille Basic Gaseous
Fluctuating-Micro-FlowsAshafa Sani¹ and Haruna M. Jibril²¹Department of Mathematical Science, Federal University Gusau, Nigeria.²Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria.*Corresponding Author's Email: ashaphathany@gmail.com¹

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ABSTRACT

The pulsating Poiseuille fluctuating-micro-flow where the fluid flow is initiated by sinusoidal pressure gradient is investigated in this study. The flow problem is modelled and the analytical solution is obtained. The effect of suction/injection and that of other leading flow parameters such as the frequency of the fluctuating driving force, the Knudsen number and the unsteadiness are analyzed. It is found from the study that the effect of suction and that of injection are observed to be opposite; suction leads to increase while injection leads to decrease in velocity of the moving fluid.

Keywords: Suction/Injection, Poiseuille flow, sinusoidal velocity, sinusoidal pressure gradient, fluctuating micro-flows.

1.0 INTRODUCTION

Fluid flows in micro-scale devices have several applications in many fields of human endeavor, such as in industrial and medical fields. Micro pumps are used for inkjet printing, environmental testing and electronic cooling. Micro-ducts are found to have played important roles in infrared detectors, diode lasers, and in high frequency fluidic control systems. Micro devices are found useful in accelerators for automobiles airbags, keyless entry systems, blood analyzers, reactor for separating biological cells, and in dense arrays of micro-mirrors for high-definition optical displays. In medical field, small pumps are used in controlled delivery and monitoring of minute amount of medication, manufacturing of nano liters of chemicals and in development of artificial pancreas. These numerous applications of micro-fluidic systems make micro-fluidic flow a trending area of research in hydrodynamics.

In 1821, Clause Navier and George Gabriel Stokes developed equations named Navier-Stokes equations which can be used to figure out the velocity vector field that applies to a fluid, given some initial conditions. These equations arose from the application of Newton's second law of motion in combination with a fluid stress and pressure term. Later in 1823, Navier, treatise on the movement of fluids where he introduced the linear boundary conditions (this was also

proposed later by Maxwell (Maxwell, 1879)) (Navier, 1823). These were based on the assumption of no-slip boundary conditions (that all the three of the solid surfaces are equal to the respective velocity components of the surfaces) and remained the standard characterization of slip conditions used today (Lauga E., Brenner M.P. and Stone H.A., 2005). Knudsen was the first person to measure the mass flow rates in micro-channels with slip/no-slip boundary conditions (Knudsen, 1909). The slip/no-slip boundary conditions in hydrodynamics have received immense consideration, and was highly debated in the 19th century (the reader may refer to (S., Fluid mechanics in first half of this century, 1969; S., Note on the condition at the surface of contact of a fluid with a solid body, 1938) for historical reviews). However, it is recently found that control experiments with typical dimensions microns have demonstrated an apparent violation of the no-slip boundary conditions for the flow of Newtonian fluids near a solid surface (Lauga E., Brenner M.P. and Stone H.A., 2005).

Arkilic et al carried-out investigation on slip flow in micro-channels. They thoroughly investigated the effect of slip velocity on the mass flow prediction of the Navier-Stokes equations and reported that the Navier-Stokes equation fails to adequately model momentum transferred from the fluid to the channel wall, but when they

included the slip flow boundary conditions at the wall (which is derived from momentum balance), they accurately modeled the mass flow-pressure relationship (Arkilic B.A., Schmitz M.A. and Kenneth S.B., 1994). Xu et al carried out molecular dynamics' studies of velocity slip phenomena in a nano-channel and found that the velocity slip is largely determined by the fluid temperature and attractive forces of the wall (Xu C., He Y.L and Wang Y., 2005). Xie et al introduced second-order velocity slip boundary conditions into the Navier-Stokes equations (Xie C. and Fan J., 2007). Zhou J. et al investigated boundary velocity slip of pressure driven liquid flow in a micron-pipe. Their investigation reveals that a small velocity slip may occur when the pressure gradient is relatively large, and this results to errors in the micron-pipe flow estimates (Zhou J.F., Gu B.Q and Shao C.L., 2011).

Haddad and Al-Nimr analyzed the effect of frequency of fluctuating driven force on basic gaseous micro-flows in slip flow regime, where they considered four flow cases, namely; the transient Couette flow, the pulsating Poiseuille flow, the Stokes second problem and natural convection. In their analysis, they found the slip in velocity and jump in temperature to increase with increase in Knudsen number and/or frequency of the driving forces, though their effect is found to be negligible for sufficiently small frequency of the driving forces and/or the Knudsen number (Haddad O.M. and Al-Nimr M.A., 2005).

The first researcher to study the problem of steady flow in an incompressible viscous liquid through a porous channel with rectangular cross-section using low Reynolds number was Berman (Berman, 1953). He obtained perturbation solution by assuming normal wall velocity to be equal. Later, Sellars extended the problem studied by Berman using high Reynolds number (Sellars, 1955). Yuan worked on the same problem, but assumed different normal velocity at the wall (Terrill, 1964). The flows considered by these author (Berman, Sellars and Yuan above) are symmetrical. Terrill and Shrestha considered asymmetric flow in a channel for walls of different permeability (Terrill R.M. and Shrestha G.M., 1965). High order velocity slip boundary conditions for incompressible flow

model was developed by Beskok *et al.* (Beskok, A., Trimmer, W. and Karniadakis, G., 1996). Most recently, Hafeez and Ndikilar analyzed the problem for flow of fluid between two parallel permeable plates subjected to bottom injection and top suction. Considering the flow velocity along the two boundaries to be uniform, they found the velocity to decrease with increase in Reynolds number (Hafeez Y.H. and Ndikilar C.E., 2014).

Suction or injection of fluid through the bounding surfaces, as for example, in mass transfer, cooling can change the flow field and as a consequence affects the rate of heat transfer from the bounding surfaces. In general, suction tends to increase the skin friction and heat transfer co-efficients, whereas injection acts in the opposite manner. Injection/suction of fluid through porous heated/cooled surfaces are of general interest in practical problems involving film cooling, control of boundary layers, etc. This can lead to enhanced heating/cooling of the system and can help to delay the transition from laminar to turbulent flow (Chaudhary M.A. and Merkin J.H., 1993).

Analytical solution of the effect of MHD inclination and unsteady heat transfer in a laminar, transition and turbulent flow of a basic gaseous micro-flow past a vertically moving oscillating plate was considered by Ashafa *et al* (Ashafa S., Ahmad A.A., and Sakir A.A., 2017).

Later in the same year, Ashafa *et al* considered the effects of suction/injection on transient Couette basic gaseous fluctuating-micro-flow (Ashafa S., Abiodun O.A., and Haruna M.J., 2017). It is found in the study that both suction and injection retard the velocity of the fluid. Haruna *et al* discussed the analytical solution of the effect of suction/injection on transient natural convection micro-gas flow between two vertical parallel plates, where injection is found to accelerate while suction retards the velocity of the moving fluid (Haruna M.J., Abiodun O.A., and Ashafa S., 2016).

In this work, the effect of suction/injection on the pulsating Poiseuille flow discussed by Haddad *et al.* in (Haddad O.M. and Al-Nimr M.A., 2005) is investigated.

2.0 FORMULATION OF THE PROBLEM

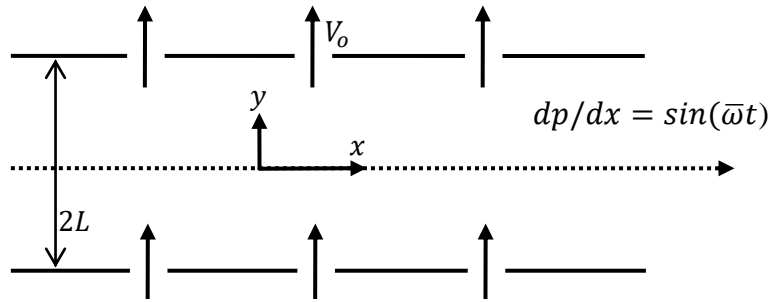


Figure 1: Schematic Diagram of the Flow Problem

Consider the pulsating Poiseuille gas flow where the driven pressure gradient moves with sinusoidal velocity (i.e. $dp/dx = \sin(\omega t)$).

This flow problem has the following governing equations and boundary conditions as:

Governing equations:

$$\rho \left(\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} \sin(\omega t) + \mu \frac{\partial^2 u}{\partial y^2} \quad 2.1$$

Boundary conditions with slip velocity applied on both walls:

$$u(t, -L) = \frac{2 - \sigma_v}{\sigma_v} \lambda \frac{\partial u}{\partial y} \Big|_{y=-L} \quad 2.2$$

$$u(t, L) = - \frac{2 - \sigma_v}{\sigma_v} \lambda \frac{\partial u}{\partial y} \Big|_{y=L} \quad 2.3$$

Considering the following dimensionless variables:

$$\tau = \frac{t}{t_r}, \omega = \frac{\omega}{\omega_r}, Y = \frac{y}{L}, Kn = \frac{\lambda}{L}, S = \frac{V_0 L}{v}, U = \frac{u}{u_r} \quad 2.4$$

The governing equations and boundary conditions are transformed as follows:

Governing equations:

$$\frac{\partial U}{\partial \tau} + S \frac{\partial U}{\partial Y} = \sin(\omega \tau) + \frac{\partial^2 U}{\partial Y^2} \quad 2.5$$

Boundary conditions:

$$U(\tau, -1) = \frac{2 - \sigma_v}{\sigma_v} Kn \frac{\partial U}{\partial Y} \Big|_{Y=-1} \quad 2.6$$

$$U(\tau, 1) = \frac{2 - \sigma_v}{\sigma_v} Kn \frac{\partial U}{\partial Y} \Big|_{Y=1} \quad 2.7$$

Note that the symmetric conditions applied in (Haddad O.M. and Al-Nimr M.A., 2005) are removed, this is due to the fact that symmetry cannot be forced on flow involving suction and injection.

3.0 ANALYTICAL SOLUTION

An exact solution for these flow problems is possible by assuming the following complex solution:

$$U(\tau, Y) = \text{Im} \{ \exp(i\omega\tau) V(Y) \} \quad 3.1$$

where 'Im' represents the imaginary part of the complex solution and $i = \sqrt{-1}$. Differentiating Eq. (3.1) and substituting into the governing equations and boundary conditions, we transformed them into ordinary differential equations having the following solutions:

$$V(Y) = \frac{1}{i\omega} \left[1 - \frac{\cosh(Y\delta_3)}{\cosh(\delta_3) + a_3 \sinh(\delta_3)} \right] \quad 3.2$$

Substituting Eq. (3.7) into Eq. (3.1), we obtained:

$$U(\tau, Y) = \text{Im} \left(\frac{\exp(i\omega\tau)}{i\omega} \left[1 - \frac{\cosh(Y\delta_3)}{\cosh(\delta_3) + a_3 \sinh(\delta_3)} \right] \right) \quad 3.3$$

From Eq. (3.8), the skin friction on the walls are obtained as follows:

$$\tau_1 = \frac{\partial U}{\partial Y} \Big|_{Y=1} = \text{Im} \left(\frac{\exp(i\omega\tau)}{i\omega} \left[\frac{-\delta_3 \sinh(\delta_3)}{\cosh(\delta_3) + a_3 \sinh(\delta_3)} \right] \right) \quad 3.4$$

$$\tau_{-1} = \frac{\partial U}{\partial Y} \Big|_{Y=-1} = \text{Im} \left(\frac{\exp(i\omega\tau)}{i\omega} \left[\frac{\delta_3 \sinh(\delta_3)}{\cosh(\delta_3) + a_3 \sinh(\delta_3)} \right] \right) \quad 3.5$$

4.0 DISCUSSION AND RESULTS

In order to get physical insight of the problem, we employ MATLAB programming to generate and analyze results. For the sake of accuracy, our results were compared with (Haddad O.M. and Al-Nimr M.A., 2005) and they are found to be in good agreement. When values of suction/injection are assumed to be zero, our results reverse to those in (Haddad O.M. and Al-Nimr M.A., 2005). This validates the accuracy of our results.

Fig. 2 (a) shows the effect of injection on the velocity curve as the channel advances from the bottom wall ($y = -1$), to the top of the channel ($y = 1$). It is found that injection retards

the velocity of the moving fluid; this is because velocity of the injected fluid counters the velocity of the moving fluid. In short, increasing the injection velocity decreases the velocity of the moving fluid.

Fig. 2 (b) highlights the effect of suction on the fluid velocity curve. Generally, it is deduced from the figure that suction increases the velocity of the fluid; increase in suction to higher values will make the velocity of fluid at the center higher than that on the plate. As also seen from the figure, using center as the reference point, velocity is observed to have increased with increase in channel length and reaches maximum (at about $y = \pm 0.5$ to ± 0.8) for $S = 0, -1, -2$ and -3 , then decreased to the wall. This effect is more enhanced at the center than on the walls.

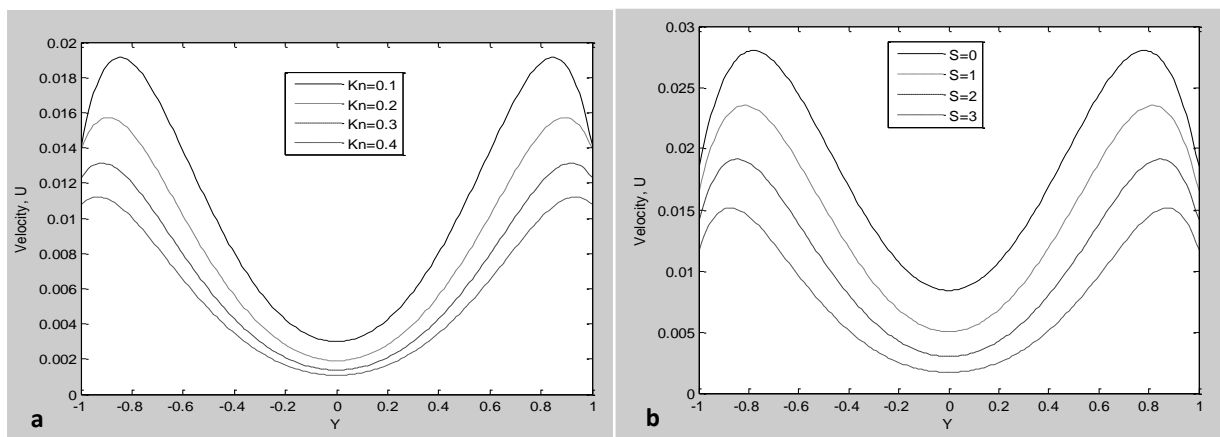


Figure 2: Velocity Profiles for $Kn = 10^{-1}$, $\omega = 10$ and $\omega\tau = \pi/2$

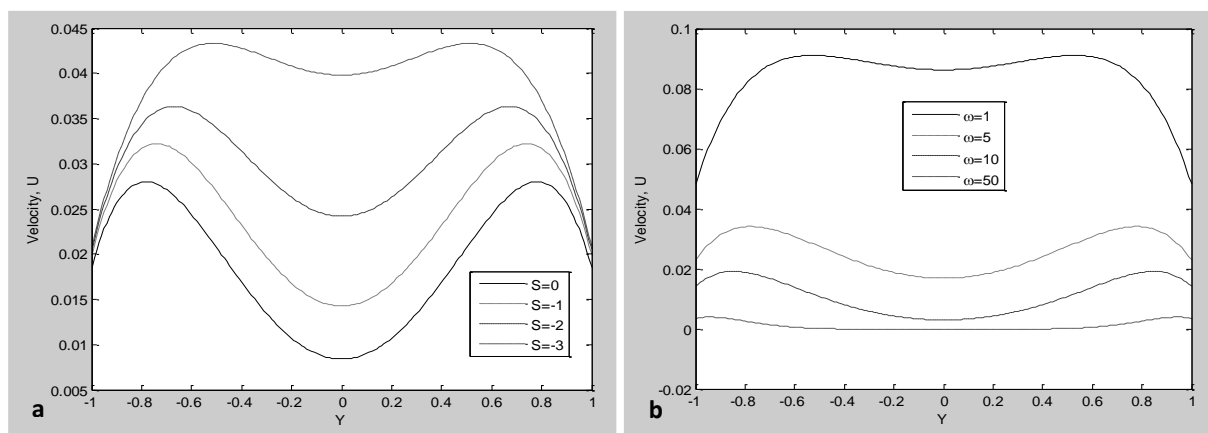


Figure 3: Velocity Profiles for $Kn = 10^{-1}$, $\omega\tau = \pi/2$ and (a) $S = 2$ (b) $S = -2$ (c) $S = 2$ (d) $S = -2$

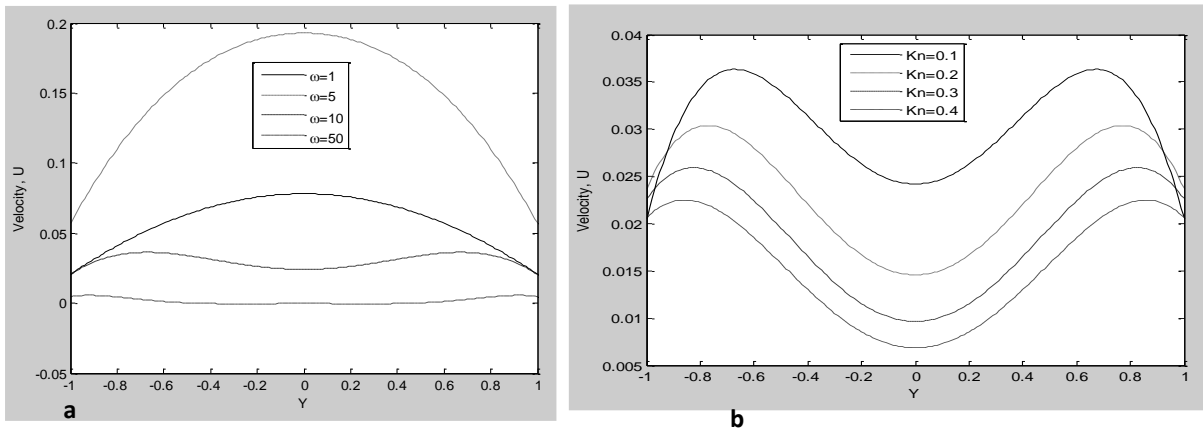


Figure 4: Velocity Profiles for $\omega = 10$, $\omega\tau = \pi/2$ and (a) $S = 2$ (b) $S = -2$ (c) $S = 2$ (d) $S = -2$

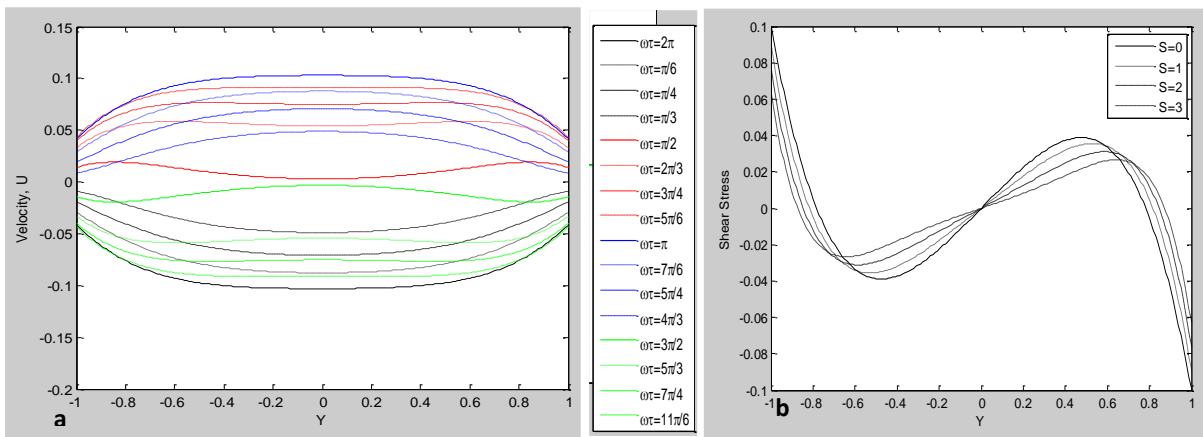


Figure 5: Velocity Profiles for $Kn = 10^{-1}$, $\omega = 10$ and (a) $S = 2$ (b) $S = -2$ (c) $S = 2$ (d) $S = -2$

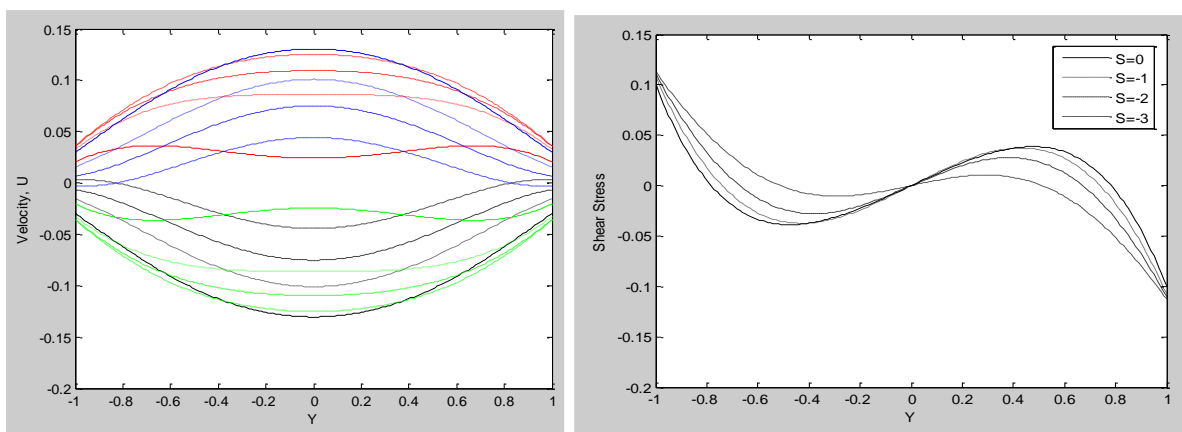


Figure 6: Skin Friction Profiles for $Kn = 10^{-1}$, $\omega = 10$ and $\omega\tau = \pi/2$

Fig. 3 (a) conspicuously shows the effect of frequency of the fluctuating driving force on the flow for constant injection velocity. It is

observed that increase in values of the driving force leads to decrease in the velocity of the fluid.

Fig. 3 (b) illustrates the effect of frequency of the fluctuating driving force on the flow velocity curve for constant suction velocity. It is found that increase in frequency of the fluctuating driving force from 1 to 5 increases the fluid velocity, while for values greater than 5, the velocity of the fluid decreases and continue to decrease with increasing frequency, though for higher frequencies, this effect is almost insignificant.

Fig. 4 (a) displays the effect of Knudsen number on the fluid velocity curve for injection velocity $S = 2$. It is found that velocity of the moving fluid decreases with increase in Knudsen number. For all the values of Kn used, the velocity is higher on the plates when compared to the center. From the center, the velocity is found to have increased with increase in channel length, reaches maximum at about $y = \pm 0.85$ to ± 0.95 then decreases towards the plates.

Fig. 4 (b) demonstrated the effect of Knudsen number on the flow for suction velocity $S = -2$. It is found at the center that increase in Knudsen number leads to decrease in fluid velocity. Same is observed on the plates - except for $Kn = 0.1$ which crossover $Kn = 0.2$ at $y = \pm 0.9$, $Kn = 0.3$ at $y = \pm 0.95$ and $Kn = 0.4$ at $y = \pm 1$. The velocity is also found to be minimum at the center, and increases with length to reach maximum at about $y = \pm 0.7$ to ± 0.85 then decreases to the walls.

The time at which the velocity attains maximal/minimal value(s) is important. For the sake of analysis, special values of $\omega\tau$ (i. e $2\pi, \pi/6, \pi/4, \pi/3, \pi/2, 2\pi/3, 3\pi/4, 5\pi/6, \pi, 7\pi/6, 5\pi/4, 4\pi/3, 3\pi/2, 5\pi/3, 7\pi/4$ and $11\pi/6$) are considered in this paper. The velocity curve is found to be maximal/minimal

with difference of π between the respective values of $\omega\tau$, and also oscillates between the maximal and minimal velocities as the values of $\omega\tau$ increased periodically. It is also found (from Fig. 6 (a-b)) that values whose difference is π were generally symmetrical about the velocity axis.

Fig. 5 (a) indicates the change in velocity of the flow with respect to time, for constant injection velocity. This effect is observed more enhanced at the center than on the walls. The velocity of the flow is found to be maximum on the channel walls when $\omega\tau = 5\pi/6$ and minimum when $\omega\tau = 11\pi/6$, while at the center of the channel, the velocity is found to be maximum and minimum when $\omega\tau = \pi$ and 2π respectively.

Fig. 5 (b) figures out the effect of $\omega\tau$ on the velocity distribution of the flow for constant suction velocity. Maximum velocity at the center and on the walls are achieved respectively when $\omega\tau = \pi$ and $5\pi/6$, while the minimum velocity are respectively found at $\omega\tau = 2\pi$ and $11\pi/6$.

Fig. 6 (a) illustrates the effect of injection on shear stress curve of the flow problem at $y=-1$. On the wall, increasing injection velocity lead to a decrease in velocity of the fluid. The velocity decreased downward up to about $y = -0.6$, where it reverses and approaches zero at the center of the channel.

Fig. 6 (b) showed the effect of suction velocity on shear stress as the length of the channel increases from the bottom wall to the upper one. Increasing suction velocity increases the shear stress at $y = -1$ and decreases it at $y = 1$. The shear stress is found zero at the center of the channel.

Table 1: Numerical Values of Skin Friction and Velocity ($\omega = 10, Kn = 10^{-1}$ and $\omega\tau = \pi/2$)

S	Y	τ_y	Velocity	Y	τ_y	Velocit y	S	Y	τ_y	Velocit y	Y	τ_y	Velocit y
0	1	-0.0999	0.0186	0	0	0.0084	0	1	-0.0999	0.0186	0	0	0.0084
1	1	-0.0889	0.0165	0	0	0.0050	-1	1	-0.1074	0.0199	0	0	0.0143
2	1	-0.0761	0.0141	0	0	0.0030	-2	1	-0.1104	0.0205	0	0	0.0242
3	1	-0.0631	0.0117	0	0	0.0017	-3	1	-0.1125	0.0209	0	0	0.0398

5. 0 CONCLUSION

The following conclusion was drawn from the study:

- i. Effect of suction and that of injection are opposite, suction leads to increase while injection leads to decrease in velocity of the moving fluid.
- ii. For injection, higher value of ω lead to decrease in velocity of the fluid, while for suction, velocity increases when $\omega = 1$ to $\omega = 5$, and then decreases as ω becomes significant.
- iii. For Injection, increasing Kn leads to general decrease in velocity, while for suction, this happens only at the center of the channel, but on the wall,

the velocity increases when $Kn = 0.2$, and from then decreases with increasing Kn .

- iv. For both suction/injection the maximum/minimum velocities are yield at the center when $\omega\tau = \pi$ and 2π respectively, while at $\omega\tau = 5\pi/6$ and $11\pi/6$ on the wall. For suction, the velocity is more pronounced at the center, while the reverse of this is found on the wall.
- v. Injection velocity increase lead to a decrease in shear stress at $y=-1$, while increasing suction velocity does the reverse.

Nomenclature

Kn	Knudsen number ($= \lambda / L$)	V_o	Suction/injection
L	Reference length ($= L$)	X	Dimensionless axial coordinate ($= x / L$)
P	Pressure	x	Axial coordinate
S	Dimensionless suction/injection parameter	Y	Dimensionless transverse coordinate ($= y / L$)
t	Time	y	Transverse coordinate
t_r	Reference time ($= L^2 / \nu$)	Greek symbols	
U	Dimensionless axial velocity ($= u / u_o$) for case I and ($= u / u_r$) for case II	λ	Mean-free-path length
u	Axial velocity (in x -direction)	ρ	Density
u_o	Velocity of the moving wall	μ	Dynamic viscosity
u_r	Reference velocity ($= -L^2 (\partial p / \partial x) / \mu$)	ν	Kinematic viscosity
V	Complex solution function for velocity	σ_v	Tangential momentum accommodation coefficient ($= 0.7$)
		ω	Frequency

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