



Availability Determination of a Complex Repairable Series- Parallel System Using Markovian Birth-Death Process

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Received: October 2015 Revised and Accepted: November 2015

Abstract

Availability and profit of an industrial system are becoming an increasingly important issue. This paper dealt with the evaluation of availability of a redundant system. The system considered consists of four subsystems arranged in series-parallel. Performance evaluation model for the availability based on Markovian Birth-Death process using probabilistic approach was formulated. A transition diagram representing the operational behavior of the system has been constructed. Failure and repair rates of subsystems were kept constant. It was found that as the availability of a system increases, the related profit will also increase.

Keywords: *Availability; Series-Parallel; Markov; Performance Evaluation; Transition Diagram.*

1.0 INTRODUCTION

Due to the importance of series-parallel systems in various industries, determination of their availability has become increasingly an important issue (Yusuf, 2012). System availability represents the percentage of time the system is available to the users. On the other hand, failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipment/systems always available and to

emphasize on system availability at the highest order.

A large volume of literature exists on the issue of predicting performance evaluation of various systems. Kumar *et al.*(1988) discussed on the reliability analysis of the feeding system in the paper industry. Kumar *et al.* (1989) discussed on the availability analysis of the washing system in the paper industry. Kumar *et al.* (1991) discussed the behavior of urea decomposition in the fertilizer industry under the general repair policy. Kumar *et al.* (1993) studied the design and cost analysis of a refining system in a sugar industry. Srinath (1994) explained


a Markov model to determine the availability expression for a simple consisting of only one component. Gupta *et al.* (2005) studied the behavior of Cement manufacturing plant. Arora and Kumar (2005) studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg (2005) perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates. Yusuf (2012) examined the performance evaluation of a series-parallel system.


However, availability/profit of an industrial system may be enhanced using highly reliable structural design of the system or subsystem of higher availability. By improving the availability of a system or sub-system, the production and associated profit will also increase. Increase in production leads to the increase in profit. This can be achieved by maintaining availability at highest order. To achieve both production and profits, the system should remain operative for maximum possible duration.

Little literature can be found dealing with evaluation of availability of complex systems (Yusuf, 2012). This paper aimed to use availability as a means of measuring the effectiveness of the system.

The objectives of this paper are three fold. The first one is to obtain explicit expressions for availability of the system under study. The second is to capture the effect of both failure and repair rates on the availability based on assumed numerical values given to the system parameters. The third is to construct availability matrices for each subsystem for the maximum availability and optimum values of failure and repair rates.

2.2 System Notations

; This indicates the system is in full working state.

; This indicates the system is in total failure.

A, B, C, D and E represent full working states of the system.

2.0 METHODOLOGY

A typical system consists of a number of subsystems connected to each other logically either in series or in parallel in most cases. The performance of the system depends on the configuration and performance of its subsystems.

Before analyzing the failure data it is better to describe the configuration of the system and classify it into various subsystems so that the failure can be categorized. The system presented here comprised of five subsystems (i.e. A, B, C, D and E). The system model formulation was carried out based on Markovian Birth-Death process using probabilistic approach.

2.1 System Structure

Five dissimilar sub systems were employed as indicated below:

1. Subsystem A: Single unit in series whose failure cause complete failure of the entire system.
2. Subsystem B: Single unit in series whose failure cause complete failure of the entire system.
3. Subsystem C: Consist of four units in parallel on cold standby. Total failure of the system occurs only when all of the four units have failed.
4. Subsystem D: Single unit in series whose failure cause complete failure of the entire system.
5. Subsystem E: Single unit in series whose failure cause complete failure of the entire system

C1, C2, C3 and C4 denote the systems working on cold standby units. λ_i ; $i = 1, 2,3,4,5$. ; represent the failure rates of subsystems A, B, C, D, D and E respectively.

μ_i ; $i = 1,2,3,4,5$ represent the repair rate of subsystems A, B, C, D and E, respectively.

$\frac{dp_i}{dt}$, $i = 0,1,2,3,4,5,\dots,20$ represent the respective derivatives with respect to t.

$p_0(t), p_1(t), p_2(t), p_3(t)$; Stand for the probabilities of the system working with full capacity at time t.

A_v , account for the steady state availability.

A_k , $k = 4, 5, 6, 7, 8, 9, \dots, 20$, account for probabilities of the system in failed states.

$$X_1 = \frac{\lambda_1}{\mu_1}, X_2 = \frac{\lambda_2}{\mu_2}, X_3 = \frac{\lambda_3}{\mu_3}, X_4 = \frac{\lambda_4}{\mu_4}, X_5 = \frac{\lambda_5}{\mu_5}$$

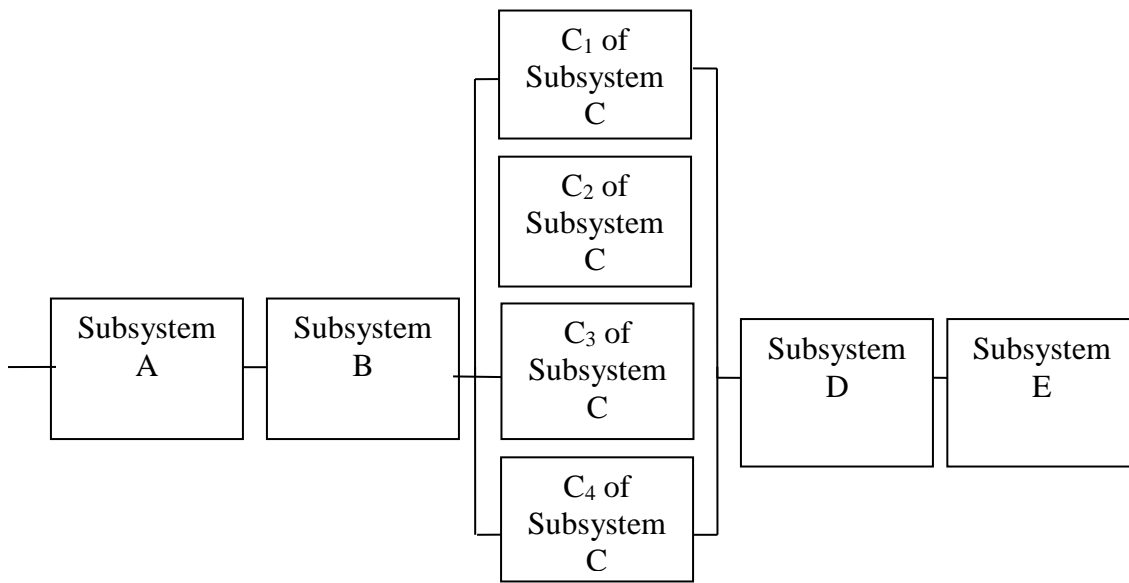


Figure1: Reliability block diagram of the system.

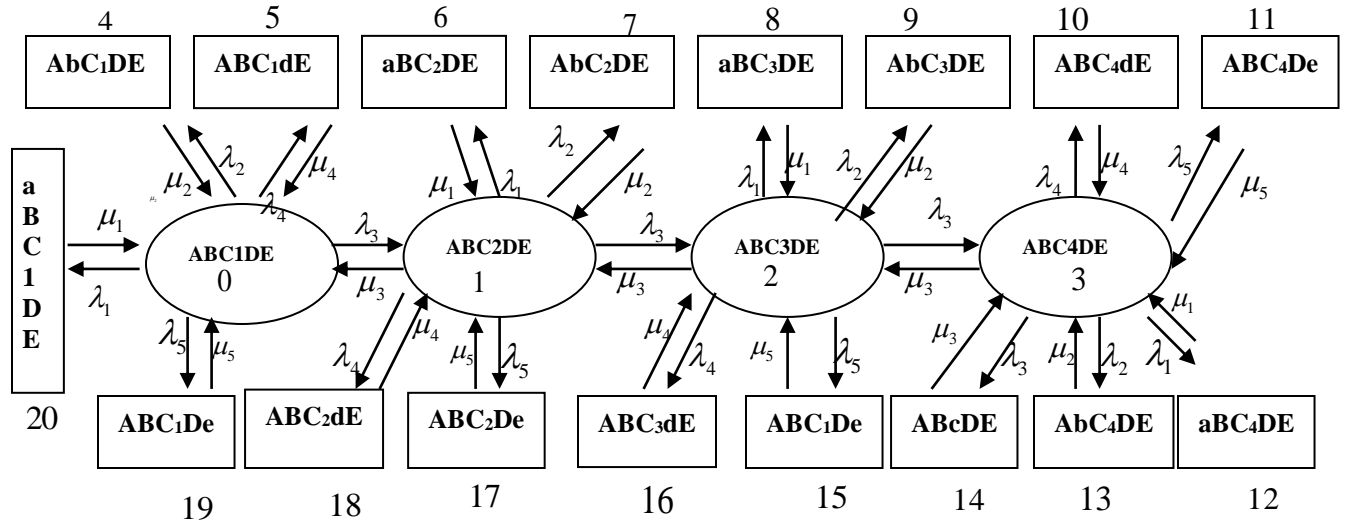


Figure 2: Transition diagram of the system.

2.3 Simulation Modeling

The following differential equations associated with the transition diagram (i.e. fig.2) were formed:

$$\frac{dp_0(t)}{dt} = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)p_0(t) + \mu_1 p_{20}(t) + \mu_2 p_4(t) + \mu_3 p_1(t) + \mu_4 p_5(t) + \mu_5 p_{19}(t) \quad (1)$$

$$\frac{dp_1(t)}{dt} = -(\mu_3 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)p_1(t) + \lambda_3 p_0(t) + \mu_1 p_6(t) + \mu_2 p_7(t) + \mu_3 p_2(t) + \mu_4 p_{18}(t) + \mu_5 p_{17}(t) \quad (2)$$

$$\frac{dp_2(t)}{dt} = -(\mu_3 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)p_2(t) + \lambda_3 p_1(t) + \mu_1 p_8(t) + \mu_2 p_9(t) + \mu_3 p_3(t) + \mu_5 p_{15}(t) + \mu_4 p_{16}(t) \quad (3)$$

$$\frac{dp_3(t)}{dt} = -(\mu_3 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)p_3(t) + \lambda_3 p_2(t) + \mu_1 p_{12}(t) + \mu_2 p_{13}(t) + \mu_3 p_{14}(t) + \mu_4 p_{10}(t) + \mu_5 p_{11}(t) \quad (4)$$

$$\frac{dp_4(t)}{dt} = -\mu_1 p_2(t) + \lambda_2 p_0(t) \quad (5)$$

$$\frac{dp_5(t)}{dt} = -\mu_4 p_5(t) + \lambda_4 p_0(t) \quad (6)$$

$$\frac{dp_6(t)}{dt} = -\mu_1 p_6(t) + \lambda_1 p_1(t) \quad (7)$$

$$\frac{dp_7(t)}{dt} = -\mu_2 p_7(t) + \lambda_2 p_1(t) \quad (8)$$

$$\frac{dp_8(t)}{dt} = -\mu_1 p_8(t) + \lambda_1 p_2(t) \quad (9)$$

$$\frac{dp_9(t)}{dt} = -\mu_2 p_9(t) + \lambda_2 p_2(t) \quad (10)$$

$$\frac{dp_{10}(t)}{dt} = -\mu_4 p_{10}(t) + \lambda_4 p_3(t) \quad (11)$$

$$\frac{dp_{11}(t)}{dt} = -\mu_5 p_{11}(t) + \lambda_5 p_3(t) \quad (12)$$

$$\frac{dp_{12}(t)}{dt} = -\mu_1 p_{12}(t) + \lambda_1 p_3(t) \quad (13)$$

$$\frac{dp_{13}(t)}{dt} = -\mu_2 p_{13}(t) + \lambda_2 p_3(t) \quad (14)$$

$$\frac{dp_{14}(t)}{dt} = -\mu_3 p_{14}(t) + \lambda_3 p_3(t) \quad (15)$$

$$\frac{dp_{15}(t)}{dt} = -\mu_5 p_{15}(t) + \lambda_5 p_2(t) \quad (16)$$

$$\frac{dp_{16}(t)}{dt} = -\mu_4 p_{16}(t) + \lambda_4 p_2(t) \quad (17)$$

$$\frac{dp_{17}(t)}{dt} = -\mu_5 p_{17}(t) + \lambda_5 p_1(t) \quad (18)$$

$$\frac{dp_{18}(t)}{dt} = -\mu_4 p_{18}(t) + \lambda_4 p_1(t) \quad (19)$$

$$\frac{dp_{19}(t)}{dt} = -\mu_5 p_{19}(t) + \lambda_5 p_0(t) \quad (20)$$

$$\frac{dp_{20}(t)}{dt} = -\mu_1 p_{20}(t) + \lambda_1 p_0(t) \quad (21)$$

Setting $\frac{dp_0(t)}{dt} = 0$ as $t \rightarrow \infty$ in equations 1 – 21 and solve recursively, the probability of full working states p_0 is determined using condition normalizing indicated below:

$$\sum_{i=0}^{20} p_i = 1 \quad (41)$$

Thus,

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + \dots + p_{20} = 1 \quad (42)$$

$$\therefore P_0 = \frac{1}{1 + (X_1 + X_2 + X_3 + X_4 + X_5)(1 + X_3 + X_3^2 + X_3^3)} \quad (43)$$

Where;

$$X_1 = \frac{\lambda_1}{\mu_1}, X_2 = \frac{\lambda_2}{\mu_2}, X_3 = \frac{\lambda_3}{\mu_3}, X_4 = \frac{\lambda_4}{\mu_4}, X_5 = \frac{\lambda_5}{\mu_5}$$

2.4 Steady-state availability of the system

Availability = Summation of all working states

$$A_v = \sum_{i=0}^3 P_i \tag{44}$$

$$\Rightarrow A_v = P_0 + P_1 + P_2 + P_3 \tag{45}$$

$$= P_0 + X_3 P_0 + X_3^2 P_0 + X_3^3 P_0$$

$$= P_0 (1 + X_3 + X_3^2 + X_3^3)$$

$$\therefore A_v = \frac{1 + X_3 + X_3^2 + X_3^3}{1 + (X_1 + X_2 + X_3 + X_4 + X_5)(1 + X_3 + X_3^2 + X_3^3)} \tag{46}$$

3.0 RESULTS and DISCUSSION

Explicit expression for the availability model was developed and used for the evaluation of performance of different subsystems of the series-parallel system in this study. Using the model, Tables 1-5 were constructed to show the relationship between failure and repair rates on the system availability. Tables 1-5 and Figures 3 to 12 revealed the effect of failure and repair rates of subsystems A, B, C, D and E on the availability of the system. It is observed that for some known constant values of failure / repair rates of the subsystems, as failure rate of subsystem increases, the subsystem availability decreases. Similarly as repair rate of the subsystem increases, the subsystem availability increases

The Availability decreases as the failure rate increases. Similarly as availability increases, so also the repair rates. Table 6 shows the optimum values of Failure/Repair rates of Subsystems A, B, C, D and E on the availability of the system

Table1. Availability matrix for subsystem A.

λ_1 / μ_1	0.1000	0.1850	0.2700	0.3550	0.4400
0.0500	0.4506	0.5027	0.5251	0.5376	0.5456
0.0680	0.4168	0.4792	0.5074	0.5234	0.5337
0.0860	0.3877	0.4579	0.4908	0.5098	0.5223
0.1040	0.3624	0.4383	0.4752	0.4970	0.5114
0.1220	0.3402	0.4204	0.4606	0.4848	0.5009

$\mu_2 = 0.3990$
 $\lambda_2 = 0.1090$
 $\mu_3 = 0.2520$
 $\lambda_3 = 0.1050$
 $\mu_4 = 0.4250$
 $\lambda_4 = 0.0990$
 $\mu_5 = 0.3900$
 $\lambda_5 = 0.0760$

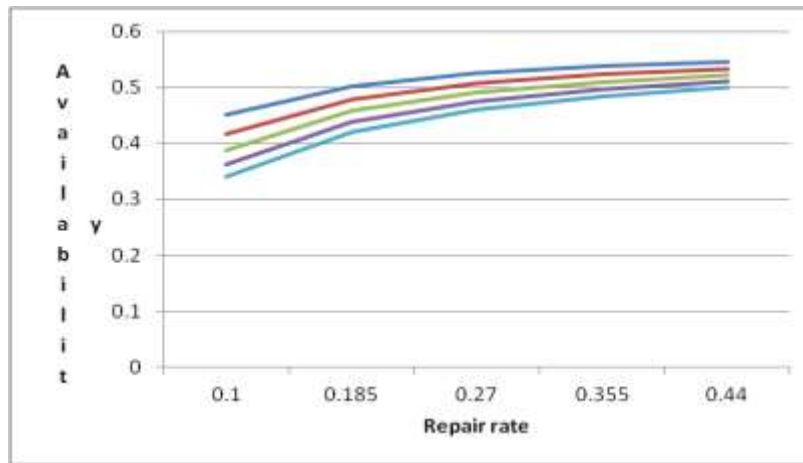


Figure 3. Effect of μ_1 on system availability.

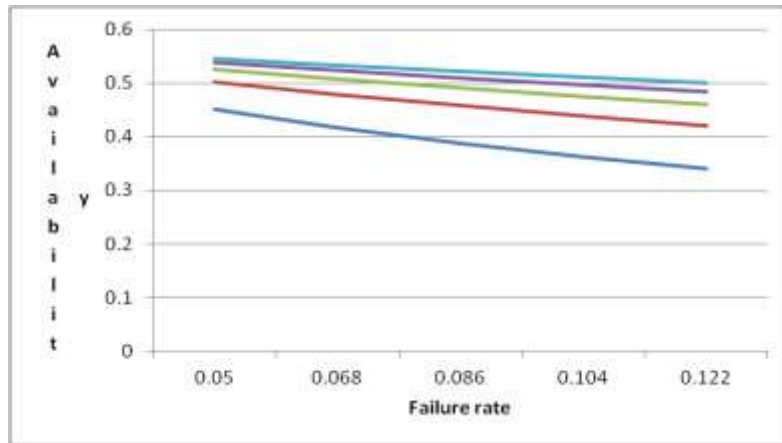


Figure 4. Effect of λ_1 on system availability.

Table 2. Availability matrix for subsystem B.

λ_2 / μ_2	0.1000	0.2000	0.3000	0.4000	0.5000
0.0750	0.4126	0.4882	0.5199	0.5374	0.5484
0.1000	0.3741	0.4601	0.4983	0.5199	0.5338
0.1250	0.3421	0.4351	0.4785	0.5036	0.5199
0.1500	0.3151	0.4126	0.4601	0.4882	0.5067
0.1750	0.2921	0.3824	0.4431	0.4737	0.4942

$\mu_1 = 0.4390$
 $\lambda_1 = 0.0510$
 $\mu_3 = 0.0990$
 $\lambda_3 = 0.1050$
 $\mu_4 = 0.4250$
 $\lambda_4 = 0.0670$
 $\mu_5 = 0.2500$
 $\lambda_5 = 0.0990$

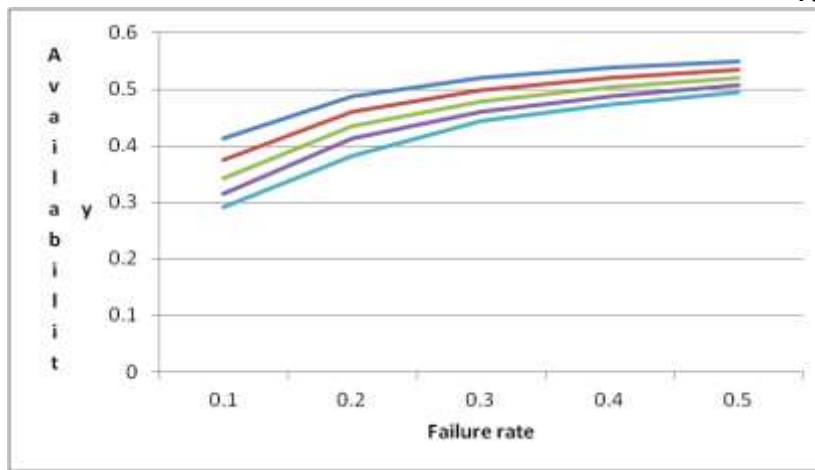


Figure 5. Effect of μ_2 on system Availability.

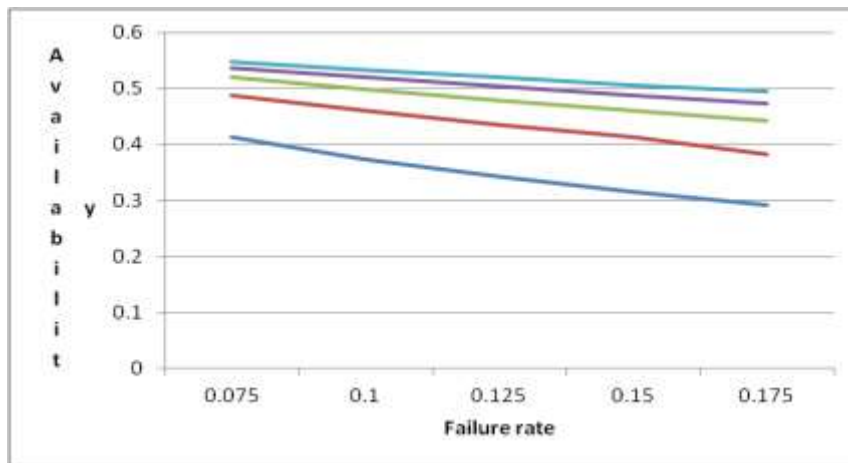


Figure 6. Effect of λ_2 on system Availability.

Table 3. Availability matrix for subsystem C.

λ_3 / μ_3	0.1250	0.2000	0,2750	0.3500	0.4250
0.0400	0.4523	0.4535	0.4537	0.4537	0.4537
0.0575	0.4486	0.4527	0.4534	0.4536	0.4537
0.0750	0.4418	0.4512	0.4529	0.4534	0.4536
0.9250	0.4319	0.4485	0.4520	0.4530	0.4534
0.1100	0.4195	0.4446	0.4505	0.4524	0.4533

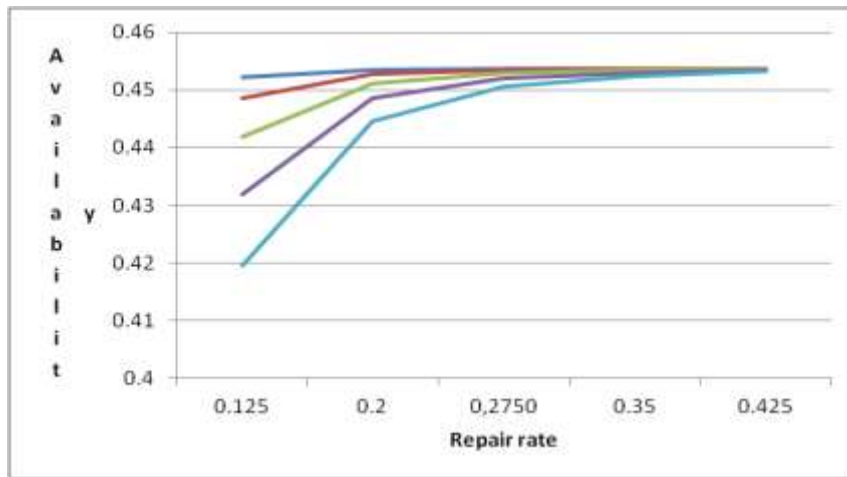


Figure 7. Effect of μ_3 on system availability.

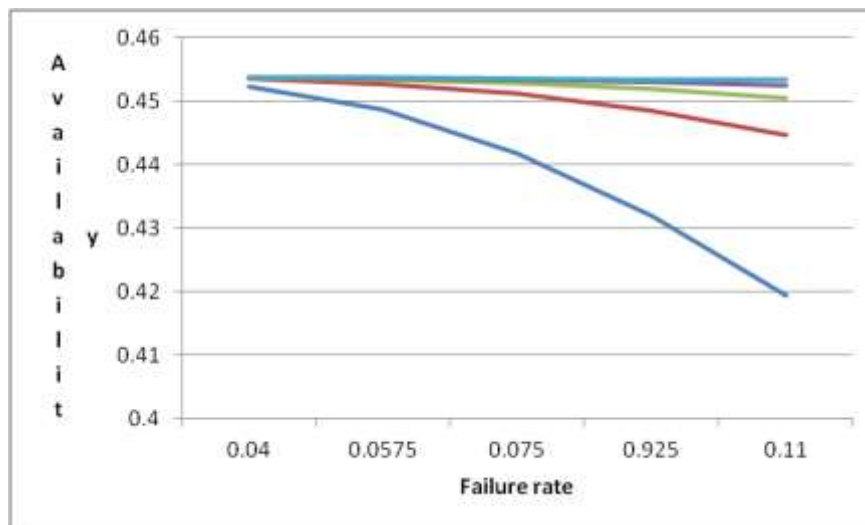


Figure 8. Effect of λ_3 on system availability.

Table 4. Availability matrix for subsystem D.

λ_4 / μ_4	0.1550	0.2300	0.3050	0.3800	0.4550
0.0500	0.5746	0.6116	0.6322	0.6454	0.6546
0.0650	0.5443	0.5881	0.6132	0.6294	0.6408
0.0800	0.5171	0.5664	0.5952	0.6141	0.6275
0.0950	0.4924	0.5462	0.5783	0.5996	0.6148
0.1100	0.4700	0.5274	0.5623	0.5857	0.6026

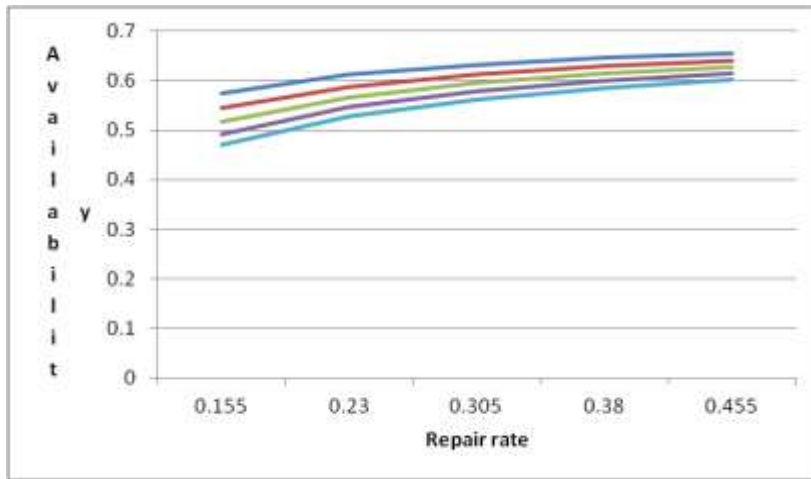


Figure 9. Effect of μ_4 on system availability

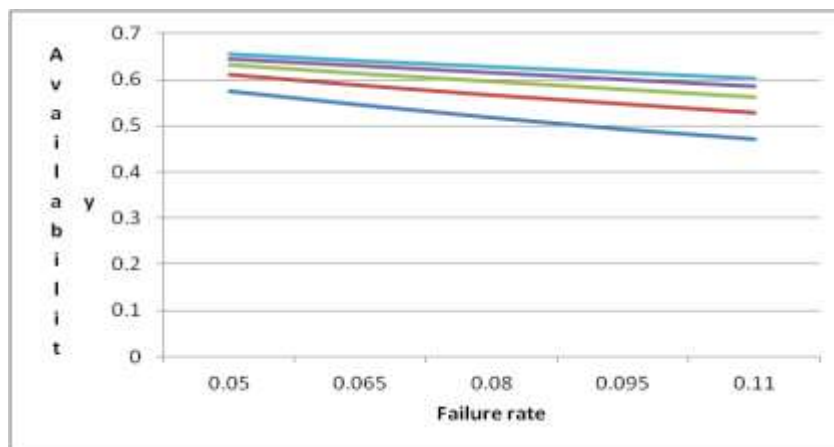


Figure 10. Effect of λ_4 on system availability.

Table 5. Availability matrix for subsystem E.

λ_5 / μ_5	0.1000	0.1950	0.2900	0.3850	0.4800
0.0550	0.4098	0.4606	0.4808	0.4919	0.4988
0.0660	0.3921	0.4487	0.4722	0.4851	0.4932
0.0770	0.3759	0.4376	0.4639	0.4784	0.4877
0.0880	0.3610	0.4271	0.4559	0.4720	0.4823
0.0990	0.3472	0.4170	0.4481	0.4657	0.4770

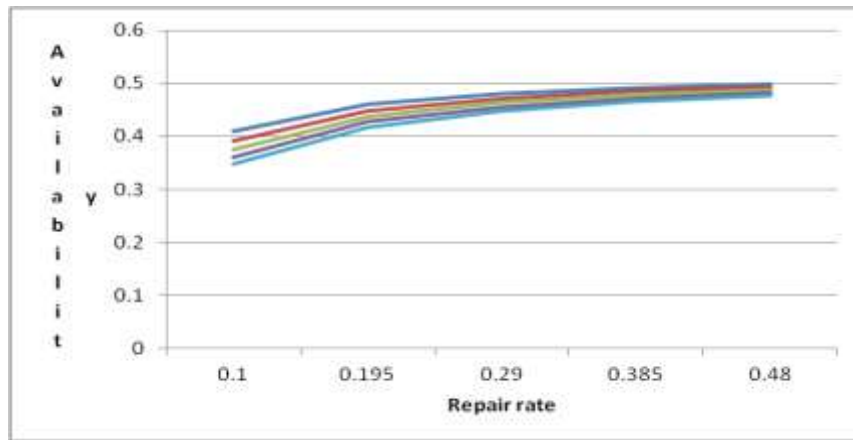


Figure 11. Effect of μ_5 on system availability.

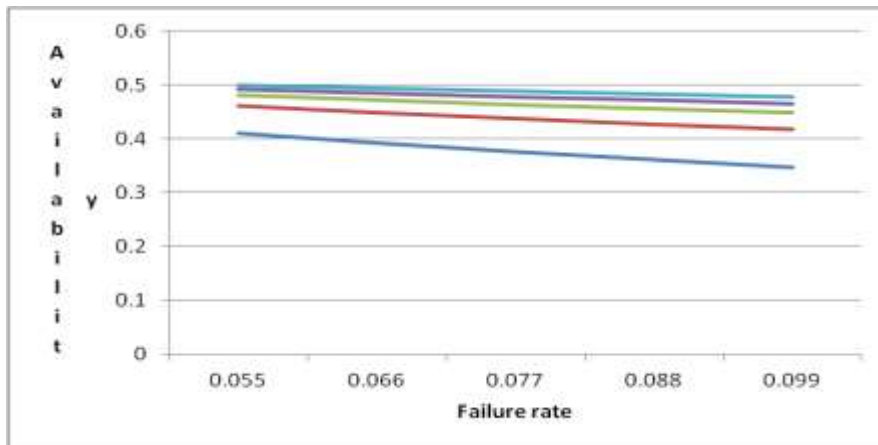


Figure 12. Effect of λ_5 on system availability.

Table 6. Optimum values of Failure/Repair rates of Subsystems of Series-Parallel system.

S/N	Subsystem	Failure rate λ_i	Repair rate μ_i	Maximum Availability (%)
1	A	0.0500	0.4400	54
2	B	0.0750	0.5000	54
3	C	0.0400	0.4250	45
4	D	0.0500	0.4550	65
5	E	0.0550	0.4800	43

4.0 CONCLUSIONS AND RECOMMENDATIONS

The model will assist maintenance engineers and managers for proper maintenance utilization. The results of this study will be beneficial to the plant management for the availability analysis of series-parallel system and hence to decide about the maintenance priorities of various subsystems of any similar system.

From Table 6, it was observed that the most critical subsystem as far as maintenance is concerned and required

immediate attention is subsystem C, as the effect of failure rates on system availability is lower than that of subsystems A,B, D and E. Therefore, on the basis of repair rates, the maintenance priority should be given as per the following order:

Subsystem C, Subsystem E, Subsystem A, Subsystem B and subsystem D respectively.

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