Viscoelastic Fluid Flow with Heat and Mass Transfer in the Presence of Induced Magnetic Field

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Abstract

The unsteady flow of a viscoelastic fluid with heat and mass transfer has been studied, taking into consideration the effect of induced magnetic field. The governing equations in dimensionless form have been solved analytically using perturbation technique. Solutions were obtained for the velocity, magnetic, temperature and concentration distributions, as well as the Skin friction, Nusselt number and Sherwood number. The influence of various parameters such as the thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, viscoelasticity parameter and the frequency of oscillation on the flow field was examined. It was observed that the velocity increases with the increase in mass Grashof number, thermal Grashof number and permeability parameter, and it decreases with increase in Schmidt number and Prandtl number. The temperature becomes lower with increase in Prandtl number while the concentration falls due to the increase in Schmidt number.

Keywords: Viscoelastic; Heat transfer; Mass transfer; Magnetohydrodynamics; Induced magnetic field.

1.0 INTRODUCTION

Heat transfer is of considerable interest because it can occur in many geothermal, geophysical, technological and engineering processes such as nuclear reactors. Geothermal gases are electrically conducting and are affected by the presence of magnetic field. Viscoelastic flows arise in numerous areas of chemical industrial process, biosystems, food processing and biomedical engineering (Lima and Rey, 2006). Viscoelastic fluid flows are important in petroleum industry and in the purification of crude oils, dust particles in boundary layers. The study of viscoelastic fluid has become important in the last few years. Qualitative analyses of these studies have significant bearing on several industrial applications such as polymer sheet extrusion from a dye, drawing of plastic firms etc. When manufacturing processes at high temperature need cooling, the flows may need viscoelastic fluid to produce a good effect or reduce the temperature (Kai-Long, 2010). Seddeek (2007) studied the heat and mass transfer problems about the viscoelastic boundary layer flow over a stretching sheet with magnetic effect. Kai-long (2010) investigated heat and mass transfer for viscous flow with radiation effect past a non-linear stretching sheet. Muthucumaraswamy and Senthil(2004) presented heat and mass transfer effects on moving vertical plate in the presence of thermal radiation. We find several industrial applications of magnetic field such as polymer technology and metallurgy. Sarpakaya (1961) is the first researcher to investigate MHD flows of non-Newtonian fluids. Chaudhary and Jha (2008) considered heat and mass transfer in elastic viscous fluid past an impulsively
started infinite vertical plate with Hall Effect. Mohiddin et al. (2010) investigated the numerical study of unsteady free convective heat and mass transfer in a Walters B viscoelastic flow along a vertical cone. Computational treatment of free convection effects on perfectly conducting viscoelastic fluid was studied by El-Bary (2005). The effect of magnetic field on an oscillatory flow of a viscoelastic fluid with thermal radiation was investigated by Ambethkar and Singh (2011). Hayat et al. (2010) examined two-dimensional mixed convection boundary layer magnetohydrodynamics (MHD) stagnation-point flow through a porous medium bounded by a stretching vertical plate with thermal radiation and magnetic field. They obtained exact solution using the method of homotopy analysis. Kai-long (2007) studied the heat transfer on a stretching sheet cooled or heated by a high or low prandtl number, the buoyancy parameter, the magnetic parameter, radiation parameter, and conduction convection coefficient for second grade viscoelastic fluid. Kishore et al. (2010) studied viscoelastic buoyancy driven MHD free convective heat and mass transfer past a vertical cone with thermal radiation and viscous dissipation effects. Afify (2004) studied the effects of radiation and chemical reaction on steady free convective flow and mass transfer of an optically dense viscous, incompressible and electrically conducting fluid past a vertical isothermal cone in the presence of magnetic field. Raptis and Perdikis (2006) studied numerically the steady two dimensional flow of an incompressible viscous and electrically conducting fluid over a non-linearly semi-infinite stretching sheet in the presence of a chemical reaction and under the influence of a magnetic field.

Salehet et al. (2010) examined heat and mass transfer in MHD viscous elastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Geetanjali et al. (2011) investigated gravity flow of dusty viscoelastic fluid through an inclined channel with effect of magnetic field. Sayed-Ahmed et al. (2011) discussed the free convection and mass transfer flow of Rivlin – Erickson fluid in the presence of uniform magnetic field.

Motivated by all these studies and applications, we investigated the flow of a viscoelastic fluid with heat and mass transfer in the presence of induced magnetic field analytically by perturbation method.

**2.0 MATHEMATICAL FORMULATION**

Consider the flow of incompressible memory fluid in a fixed plane with heat and mass transfer, under the influence of an induced magnetic field and constant suction. The x-axis is taken along the plane in the upward direction and a straight line perpendicular to that of the y-axis. All fluid properties are assumed constant. Since the fluid is conducting, the magnetic Reynolds number is much less than unity and hence the induced magnetic field is not neglected.

The governing equations for the momentum, induced magnetic, energy and concentration is as follows;

$$\frac{\partial u}{\partial t} - v_0 \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial u}{\partial y} = g \beta(T - T_\infty) + v \frac{\partial^2 u}{\partial y^2} - k_0 \left( \frac{\partial^2 u}{\partial t^2} - v_0 \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial^2 u}{\partial y^2} \right) + \frac{\sigma \mu e^2}{\rho} + s^2 \frac{\partial H}{\partial y} + g \beta_y (C - C_\infty) \quad (1)$$

$$\frac{\partial H}{\partial t} - v_0 \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial H}{\partial y} = \beta_y \frac{\partial^2 H}{\partial y^2} \quad (2)$$

$$\frac{\partial T}{\partial t} - v_0 \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial T}{\partial y} = \left( \frac{K_T}{\rho C_p} \right) \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$\frac{\partial C}{\partial t} - v_0 \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

where u and v are the components of velocity in the x and y direction respectively, g is the acceleration due to gravity, $\beta$ and $\beta_y$ are the coefficient of volume expansion, $k_0$ is the kinematic viscoelasticity, $\rho$ is the density, $\mu$ is the viscosity, $\nu$ is the kinematic viscosity, $K_T$ is the thermal conductivity, $C_p$ is the specific heat in the fluid at constant pressure, $\sigma$ is the electrical conductivity of the fluid, $\mu_e$ is the magnetic permeability, D is the molecular diffusivity, $T_\infty$ is the temperature of the plane and $T_\infty$ is the temperature of the fluid far away from plane. $C_\infty$ is the concentration of the plane and $C_\infty$ is the concentration of the fluid far away from the plane.

And $v_0 > 0$, the negative sign indicate that the suction is towards the plane.
The boundary conditions of the problem are:
\[ y = 0: U = 0, H = H_\infty + \varepsilon (H_\infty - H_\infty) e^{i\omega t}, T = T_\infty + \varepsilon (T_\infty - T_\infty) e^{i\omega t}, C = C_\infty + \varepsilon (C_\infty - C_\infty) e^{i\omega t} \]
\[ y \to \infty : U = 0, H = \theta = 0, C \to 0 \]  
(5)

Introducing the following non-dimensional quantities
\[ \tilde{y} = \frac{y v_0}{v}; \quad \tilde{t} = \frac{t v_0^2}{\nu}; \quad \tilde{u} = \frac{u}{v_0}; \quad \tilde{\theta} = \frac{T - T_\infty}{T_\infty - T_\infty}; \quad \tilde{C} = \frac{C - C_\infty}{C_\infty - C_\infty} \]
\[ Gr = \frac{\nu g \beta (T_\infty - T_\infty)}{v_0^3}; \quad Pr = \frac{\mu C_p}{\nu}; \quad K = \frac{k_0 v_0^2}{\nu^2}; \quad S^2 = \frac{\sigma \mu e^2 v}{\rho v_0^2} \]
\[ Rm = \frac{\beta v_0^2}{v^2}; \quad Gc = \frac{\nu g \beta' (C_\infty - C_\infty)}{v_0^3}; \quad Sc = \frac{v}{D}; \quad H = \frac{H - H_\infty}{H_\infty - H_\infty} \]
(6)

Substituting the dimensionless variables in (6) into (1) to (5), we get (dropping the bars)
\[ \frac{\partial^3 u}{4 \partial t^3} - \left(1 + \varepsilon e^{i\omega t}\right) \frac{\partial^2 u}{\partial y^2} = Gr \theta + \frac{\partial^2 u}{\partial y^2} + K \left[ \frac{\partial^3 u}{4 \partial t^3} - v_0 \left(1 + \varepsilon e^{i\omega t}\right) \frac{\partial^3 u}{\partial y^3} \right] + S^2 \frac{\partial^2 H}{\partial y^2} - GcC \]
(7)
\[ \frac{\partial^3 H}{4 \partial t^3} - \left(1 + \varepsilon e^{i\omega t}\right) \frac{\partial^2 H}{\partial y^2} = \frac{1}{Rm} \frac{\partial^2 H}{\partial y^2} \]
(8)
\[ \frac{\partial^3 T}{4 \partial t^3} - \left(1 + \varepsilon e^{i\omega t}\right) \frac{\partial^2 T}{\partial y^2} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} \]
(9)
\[ \frac{\partial^3 C}{4 \partial t^3} - \left(1 + \varepsilon e^{i\omega t}\right) \frac{\partial^2 C}{\partial y^2} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \]
(10)
with boundary conditions
\[ y = 0: U = 0, H = 1 + \varepsilon e^{i\omega t}, \theta = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \]
\[ y \to \infty : U = 0 = H = \theta = 0, C \to 0 \]  
(11)

where Gr is the thermal Grashof number, Gc is the mass Grashof number, Sc is the Schmidt number, Pr is the Prandtl number, Rm is the Reynolds number, K is the viscoelastic parameter, and S is the permeability.

### 3.0 METHOD OF SOLUTIONS

To solve (7) to (10) subject to the boundary conditions (11), we take solutions of the form
\[ U(y,t) = U_0(y) + U_1(y) e^{i\omega t} \]
(12)
\[ H(y,t) = H_0(y) + H_1(y) e^{i\omega t} \]
(13)
\[ \theta(y,t) = \theta_0(y) + \theta_1(y) e^{i\omega t} \]
(14)
\[ C(y,t) = C_0(y) + C_1(y) e^{i\omega t} \]
(15)
where \( U_0(y), H_0(y), \theta_0(y), C_0(y), U_1(y), H_1(y), \theta_1(y), \) and \( C_1(y) \) are to be determined.

Substituting (12) to (17) into (7)-(11), respectively. Comparing harmonic and non-harmonic terms, we obtained
\[ KU''_0 + U''_0 + U'_0 = -S^2 H''_0 - Gr \theta_0 - GcC_0 \]
(16)
\[ H''_0 + Rm H'_0 = 0 \]
(17)
\[ \theta''_0 + Pr \theta'_0 = 0 \]
(18)
\[ C''_0 + Sc C'_0 = 0 \]
(19)
and boundary conditions give
y = 0 : \( U_0 = 0, H_0 = \theta_0 = C_0 = 1 \) \[20\]

\( y \to \infty : U_0 \to 0, H_1 = \theta_1 = C_1 \to 0 \)

Also,

\[
KU''_i + U'_i - \frac{i\omega}{4} KU''_i + U'_i - \frac{i\omega U'_i}{4} = -KU''_0 - S^2 H''_0 - U'_0 - Gc\theta_i - GcC_i
\] \[21\]

\[
H'_1 + RmH'_1 - \frac{i\omega Rm}{4} H'_1 = -RmH'_0
\] \[22\]

\[
\theta'_i + Pr\theta'_i - \frac{i\omega Pr}{4} \theta_i = -Pr\theta'_0
\] \[23\]

\[
C'_i + ScC'_i - \frac{i\omega Sc}{4} C_i = -ScC'_0
\] \[24\]

with boundary conditions

\[
y = 0 : U_i = 0, H_i = \theta_i = C_i = 1
\]

\[
y \to \infty : U_i \to 0, H_i = \theta_i = C_i \to 0
\] \[25\]

where the primes denotes differentiation with respect to \( y \).

Solving (16) to (24) subject to the boundary conditions (20) and (25). And substituting the obtained solutions into (12) to (15) respectively. Then, the velocity field can be expressed as

\[
U(y,t) = \left[A_2 e^{-y} + A_3 e^{-Rmy} + A_4 e^{-Pr} + A_5 e^{-Sc} + K(B_2 e^{-y} + B_3 y e^{-y} + B_4 e^{-Rmy} + B_5 e^{-Pr} + B_6 e^{-Sc})ight]
\]

\[
+ e^{i\omega y} \left[D_{14} e^{-m^y} + D_{15} e^{-m^y} + D_{16} e^{-Rm^y} + D_{17} e^{-Pr} + D_{18} e^{-Sc} + D_{19} e^{-s} + D_{20} e^{-m^y} + D_{21} e^{-m^y}
\]

\[
+ K(D_{31} e^{-m^y} + E_1 y e^{-m^y} + E_2 e^{-m^y} + E_3 e^{-Rm^y} + E_4 e^{-Pr} + E_5 e^{-Sc} + E_6 e^{-s} + E_7 e^{-m^y} + E_8 e^{-m^y}
\]

\[
+ E_9 e^{-y} + E_{10} y e^{-y})\right]
\] \[26\]

the induced magnetic field is

\[
H(y,t) = e^{-Rmy} + e^{i\omega y} (D_8 e^{-m^y} + D_9 e^{-Rm^y})
\] \[27\]

And, the temperature field is given as

\[
\theta(y,t) = e^{-Pr} + e^{i\omega y} (D_6 e^{-m^y} + D_7 e^{-Pr})
\] \[28\]

Similarly, the concentration distribution is

\[
C(y,t) = e^{-Sc} + e^{i\omega y} (D_6 e^{-m^y} + D_7 e^{-Sc})
\] \[29\]

The Skin friction, Nusselt number and Sherwood number is obtained by differentiating (26) to (29) and evaluated at \( y = 0 \), respectively.

\[
- \frac{\partial U(y,t)}{\partial y} \bigg|_{y=0} = [A_8 + RmA_9 + PrA_10 + ScA_1 + K(B_2 + B_3 + RmB_4 + PrB_5 + ScB_6)]
\]

\[
+ e^{i\omega y} \left[m_4 D_{14} + m_5 D_{15} + RmD_{16} + PrD_{17} + ScD_{18} + D_{19} + m_2 D_{20} + m_1 D_{21} + K(m_4 D_{31} + E_1
\]

\[
+ m_3 E_2 + RmE_3 + PrE_4 + ScE_5 + E_6 + m_2 E_7 + m_1 E_8 + E_9 + E_{10})\]
\] \[30\]

\[
- \frac{\partial \theta(y,t)}{\partial y} \bigg|_{y=0} = Pr + e^{i\omega y} (m_2 D_5 + PrD_6)
\] \[31\]

\[
- \frac{\partial C(y,t)}{\partial y} \bigg|_{y=0} = Sc + e^{i\omega y} (m_1 D_2 + ScD_3)
\] \[32\]

4.0 RESULTS AND DISCUSSION

Viscoelastic fluid flow with heat and mass transfer in the presence of induced magnetic field has been formulated and solved analytically. In order to understand the flow of the fluid, computations are
performed for different parameters such as Gr, Gc, Sc, Pr, ω and ε.

4.1 Velocity Profiles
Figures 1-7 represent the velocity profiles, figure 8 depicts the temperature profiles and Figure 9 shows the concentration profiles with varying parameters respectively.

The effect of velocity for different values of (Sc = 0.2, 0.3, 0.6, 0.8, 1, 2.01) is presented in Figure 1, the graph shows that velocity decreases with increase in Sc.

The effect of velocity for different values of (ω = 1, 2, 3, 4, 5) is given in Figure 2. From the graph it is seen that, the velocity decreases with the increase in ω.

Figure 3 denotes the effect of velocity for different values of (Gr = 1, 2, 3, 4, 5), it is clear that velocity increases with the increase in Gr.

The effect of velocity for different values of (Gc = 1, 2, 3, 4, 5) is shown in Figure 4, it is found that velocity increases with increase in Gc.

Figure 5 depicts the effect of velocity for (Pr = 0.71, 0.94, 3, 7, 10), the graph shows that velocity decreases with the increase in Pr.

The effect of velocity for different values of (K = 0.01, 0.05, 0.08, 0.1, 0.3) is presented in Figure 6, it is seen that velocity increases with the increase in K.

The effect of velocity for different values of (ε = 0.2, 0.02, 0.002, 0.0002, 0.00002) is shown in Figure 7, it is observed that velocity decreases with the decrease in ε.
Figure 3. Velocity profiles for different values of Gr.

Figure 4. Velocity profiles for different values of Gc.

Figure 5. Velocity profiles for different values of Pr.

Figure 6. Velocity profiles for different values of K.
4.2 Temperature Profiles
In figure 8, the effect of temperature for different values of (Pr = 0.71, 0.94, 1, 3, 7, 10) is given. The graph shows that the temperature increases with increasing Pr.

4.3 Concentration Profiles
Figure 9 depicts the effect of concentration for (Sc = 0.22, 0.3, 0.6, 0.8, 1, 2.01), it is seen that the concentration increases with the increase in Sc.

<table>
<thead>
<tr>
<th>Table 1: Skin friction $\tau$</th>
</tr>
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<td>Tables 1 to 3 represent the Skin friction, Nusselt number and Sherwood number respectively. Table 1 depicts that the Skin friction increases with increase in Gc, Gr, K and $\varepsilon$ and decreases with increase in Sc, $\omega$ and Pr. Table 2 represents that the rate of heat transfer decreases with increase in Pr, and increases with increase in $\omega$ and $\varepsilon$. Table 3 shows that the Sherwood number increases with increase in $\omega$ and $\varepsilon$ and decreases with the increase in Sc.</td>
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### Table 2: Nusselt number

<table>
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<tr>
<th>ε</th>
<th>ω</th>
<th>Gc</th>
<th>Sc</th>
<th>K</th>
<th>Gr</th>
<th>S</th>
<th>Rm</th>
<th>Pr</th>
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<td>0.05</td>
<td>0.71</td>
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<td>1</td>
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<td>0.1</td>
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<td>0.05</td>
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<td>0.05</td>
<td>0.71</td>
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<td>0.3</td>
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<td>0.1</td>
<td>0.05</td>
<td>0.71</td>
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### Table 3: Sherwood number

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<th>Sc</th>
<th>Sh</th>
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### 5.0 SUMMARY AND CONCLUSION

We have examined viscoelastic fluid flow with heat and mass transfer in the presence of induced magnetic field analytically using perturbation technique; the governing equations in dimensionless form were solved analytically using perturbation method to obtain the velocity, temperature and concentration expressions. Computations was carried out in order to investigate the effect of physical parameters namely; thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, viscoelasticity parameter and the frequency of oscillation on the flow field. It was observed that, the velocity increases with the rise in Gc, Gr, ε and K, and it decreases with increase in Sc, S, ω and Pr. The temperature becomes lower as Pr is increased and the concentration falls with higher values of Sc. The Skin friction increases with increase in Gc, Gr, K and ε and decreases with increase in Sc, S, ω and Pr. The rate of heat transfer decreases with increase in Pr, and increases with increase in ω and ε. The Sherwood number increases with increase in ω and ε and decreases with the increase in Sc.

### 6.0 RECOMMENDATION

This work examined viscoelastic fluid flow with heat and mass transfer taking into consideration the presence of induced magnetic field analytically using perturbation method. It is however recommended that other methods could be used to examine and extend the study and also the solution provided in this study for various material effects would serve as an insight for understanding more problems; viscoelastic fluid flow in a vertical channel with constant heat flux, effects of radiation and variable thermal conductivity on viscoelastic fluid flow in a micro-channel.

### 7.0 REFERENCES


