

**Micropolar Fluid Flow with Hall Effect and Mass Transfer in a Porous Medium**<sup>1</sup>Omokhuale, E., <sup>2</sup>Uwanta, I. J., <sup>3</sup>Ahmad, K.<sup>1</sup>Department of Mathematical Sciences, Federal University Gusau, P. M. B. 1001, Zamfara State, Nigeria.<sup>2,3</sup>Department of Mathematics, Usmanu Danfodiyo University, P. M. B. 2346, Sokoto, Nigeria.Corresponding Author's Email: [emmanuelomokhuale@yahoo.com](mailto:emmanuelomokhuale@yahoo.com)

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**Abstract**

*The unsteady flow of a micropolar fluid in a porous medium has been studied, taking the effect of Hall parameter and mass transfer into account. The resulting dimensionless partial differential equations were solved analytically using perturbation technique with a view to obtain solutions for the velocity, angular velocity, temperature and concentration distributions, as well as the Skin friction. The influence of various parameters governing the flow such as; the thermal Grashof number, Hartmann number, mass Grashof number, Schmidt number, Prandtl number, permeability Parameter, time and Hall parameter on the flow field. We observed that, the velocity increases with the increase in mass and thermal Grashof number and it decreases with increase in magnetic parameter, Hall current, permeability parameter, Schmidt number, and Prandtl number. Temperature decreases with increase in Prandtl number and concentration decreases with the increase in Schmidt number.*

**Keywords:** Micropolar fluid; Magnetohydrodynamics (MHD); Mass transfer; Porous medium; Heat transfer.

**1.0 INTRODUCTION**

The concept of micropolar fluids deals with a class of fluids that exhibit microscopic effects arising from the local structure of motions of fluid elements. These fluids contain dilute suspension of rigid macromolecules with individual motions that support stress and body moments and are influenced by spin inertia. Micropolar fluids are those, which contain micro-constituents that can undergo rotation, the presence of which can affect the hydrodynamics of the flow so that it can be distinctly non-Newtonian. It has many practical applications, for example analyzing the behavior of exotic

lubricants, the flow of polymeric fluids, liquids crystals, additive suspensions, human and animal blood, turbulent shear flow and so forth. The theory of micropolar fluids is generating a very much-increased interest and many classical flows are being re-examined to determine the effect of the fluid microstructure Sultana *et al.*(2011). The dynamics of micropolar fluids which originated from the theory of Eringen (1966) and it has remained a popular area of research. Rahman and Sultana (2008) studied radiative heat transfer flow of micropolar fluid with variable heat flux in a porous medium. Abo-Eldahab and Ghonaim (2005) investigated

radiation effect on heat transfer of polar fluid through a porous medium. Khedr *et al.* (2009) examined MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption. Kim and Lee (2002) reported the oscillatory flow of a micropolar fluid over a vertical porous plate while Sharma and Gupta (1995) studied the effects of porous medium permeability and thermal convection in micropolar fluids. El-Amin (2001) investigated MHD free convection and mass transfer flow in micropolar fluid with constant suction. Islam *et al.* (2011) examined MHD micropolar fluid flow through vertical porous medium. Patil and Kulkarni (2008) presented effects of Chemical reaction on free convection flow of a polar fluid through a porous medium in the presence of internal heat generation. Satya and Dubey (2011) investigated unsteady MHD heat and mass transfer free convection flow of a polar fluid past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion. Hassanien and Gorla (1990) analysed heat transfer to a micropolar fluid from a non-isothermal stretching sheet with suction and blowing. Kim (2001) discussed unsteady MHD micropolar flow and heat transfer over a vertical porous moving plate with variable suction. Recently, Sharma *et al.* (2013) examined effect of chemical reaction on Magneto-micropolar fluid flow from a radiative surface in the presence of variable permeability analytically. Olajuwon *et al.* (2014) studied effect of thermal radiation on Hall current o heat and mass transfer of unsteady MHD flow of a viscoelastic micropolar fluid through a porous medium analytically using perturbation method. Rawat *et al.* (2016) presented analytically MHD flow heat and mass transfer of a micropolar fluid

over a stretching sheet in a porous medium taking into cognisance the presence of variable microinertia density, heat flux and chemical reaction.

Hence, the aim of the present paper is to investigate the Micropolar fluid flow with Hall effect and mass transfer in a porous medium. The governing equations are transformed to dimensionless form using dimensionless variables and solved using perturbation method. Analytic solutions for the velocity, angular velocity, temperature and concentration were obtained as well as the Skin friction. Effect of various parameters on the velocity, temperature and concentration were studied with the aid of graphs and tables.

## 2.0 MATHEMATICAL FORMULATION OF THE RESEARCH PROBLEM

Consider the flow the two dimensional unsteady flow of a laminar incompressible, micropolar fluid past a semi- infinite vertical porous moving plate embedded in a porous medium and subjected to a transverse magnetic field in the presence of heat and mass transfer and with hall effect. It is assumed that there is no applied voltage which implies the absence of an electric field. The transversely applied magnetic field and Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy resistance terms are taken into account the constant permeability porous medium. It is also assumed here that the hole size of the porous plate is significantly larger than a characteristics microscopic length scale of porous medium.

The governing equations for the continuity, momentum, angular velocity, energy and concentration are as follows;

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial y^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + (v + v_r) \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T - T_\infty) + g\beta c(C - C_\infty) - \frac{vu^*}{K^*} - \frac{\sigma B_0 (u^* + m\omega^*)}{\rho(1 + m^2)} + 2v_r \frac{\partial \omega^*}{\partial y^*} \quad (2)$$

$$\rho j \left( \frac{\partial \omega^*}{\partial t^*} + v^* \frac{\partial \omega^*}{\partial y^*} \right) = \gamma \frac{\partial^2 \omega^*}{\partial y^{*2}} \tag{3}$$

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} \tag{4}$$

$$\frac{\partial C}{\partial t^*} + v^* \frac{\partial C}{\partial y^*} = D \frac{\partial^2 C}{\partial y^{*2}} \tag{5}$$

where  $x^*$  and  $y^*$  are the dimensional distances longitudinal and perpendicular to the plate, respectively.  $u^*$  and  $v^*$  are the components of the velocity in dimensional velocities along  $x^*$  and  $y^*$  directions, respectively.  $\rho$  is the density,  $\nu$  is the kinematic viscosity,  $\nu_r$  is the kinematic rational viscosity,  $g$  is the acceleration due to gravity,  $\beta$  and  $\beta_c$  are the coefficients of volumetric thermal expansion of the fluid,  $K^*$  is the permeability of porous medium  $\sigma$  is the

$$v = -v_0 \left( 1 + \varepsilon A e^{in^*t^*} \right) \tag{6}$$

electrical conductivity of the fluid,  $B_0$  is the magnetic induction,  $j^*$  is the micro-inertia density,  $\omega^*$  is the component of the angular velocity vector normal to the xy- plane,  $\gamma$  is the spin gradient viscosity,  $T$  is the temperature and  $\alpha$  is the effective fluid thermal diffusivity. Integrating (1) and assuming it is time dependent gives;

Where  $v_0 > 0$  and the negative sign indicate that the suction is towards the plane.

The boundary conditions of the problem are:

$$\left. \begin{aligned} y = 0 : u^* = u_p^*, T = T_\omega + \varepsilon(T_\omega - T_\infty)e^{in^*t^*}, C = C_\omega + \varepsilon(C_\omega - C_\infty)e^{in^*t^*} \\ y \rightarrow \infty : u^* \rightarrow u_\infty^* = U_\infty \left( 1 + \varepsilon e^{in^*t^*} \right), T \rightarrow T_\infty, C \rightarrow C_\infty, \omega^* \rightarrow 0 \end{aligned} \right\} \tag{7}$$

Introducing the following non-dimensional quantities

$$\left. \begin{aligned} u = \frac{u^*}{U}, v = \frac{v^*}{v_0}, y = \frac{y^* v_0}{\nu}, U_\infty = \frac{U_\infty}{U_0}, u_p = \frac{u_p^*}{u_0} \\ \omega = \frac{\nu}{u_0 v_0} \omega^*, t = \frac{t^* v_0^2}{\nu}, \theta = \frac{T - T_\infty}{T_\omega - T_\infty}, C = \frac{C - C_\infty}{C_\omega - C_\infty}, \eta = \frac{\eta^* \nu}{v_0^2} \\ K = \frac{K^* v_0^2}{\nu^2}, \gamma = \mu j^* \left( 1 + \frac{1}{2} \beta \right), j^* = \frac{v_0^2 j^*}{\nu^2}, N = \left( \frac{1}{K} + \frac{M}{(1+m^2)} \right), S = 2\beta \\ \beta = \frac{\nu}{\mu}, Pr = \frac{\nu^2}{\alpha}, M = \frac{\sigma B_0 \nu}{\rho \nu_0^2}, Gr = \frac{g \beta \nu (T_\omega - T_\infty)}{u_0 v_0}, Gc = \frac{g \beta_c \nu (C_\omega - C_\infty)}{u_0 v_0} \end{aligned} \right\} \tag{8}$$

Substituting the dimensionless variable in (7) into (2), (3), (4) and (5), and also using (6). We get (dropping the stars)

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{in^*t^*}) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + (1 + \beta) \frac{\partial^2 u}{\partial y^2} + Gr\theta + GcC + N(U_\infty - u) + S \frac{\partial \omega}{\partial y} \tag{9}$$

$$\frac{\partial \omega}{\partial t} - (1 + \varepsilon A e^{\text{int}}) \frac{\partial \omega}{\partial y} = \frac{1}{\eta} \frac{\partial^2 \omega}{\partial y^2} \tag{10}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{\text{int}}) \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{\text{int}}) \frac{\partial C}{\partial y} = \frac{1}{\text{Sc}} \frac{\partial^2 C}{\partial y^2} \tag{12}$$

With boundary conditions

$$\left. \begin{aligned} y = 0 : u = u_p, \theta = 1 + \varepsilon e^{\text{int}}, C = 1 + \varepsilon e^{\text{int}}, \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2} \\ y \rightarrow \infty : u \rightarrow U_\infty, U_\infty = 1 + \varepsilon e^{\text{int}}, \theta \rightarrow 0, C \rightarrow 0, \omega \rightarrow 0 \end{aligned} \right\} \tag{13}$$

where Gr is the thermal Grashof number, M is the Hartmann number, Gc is the mass Grashof number, Sc is the Schmidt number, Pr is the Prandtl number, K is the permeability Parameter and k is the chemical reaction parameter.

### 3.0 METHOD OF SOLUTION

To solve (9), (10), (11) and (12) subject to the boundary conditions (13), we assume solutions of the form

$$u(y,t) = U_0(y) + U_1(y)\varepsilon e^{\text{int}} \tag{14}$$

$$\omega(y,t) = \omega_0(y) + \omega_1(y)\varepsilon e^{\text{int}} \tag{15}$$

$$\theta(y,t) = \theta_0(y) + \theta_1(y)\varepsilon e^{\text{int}} \tag{16}$$

$$C(y,t) = C_0(y) + C_1(y)\varepsilon e^{\text{int}} \tag{17}$$

where  $U_0(y), \omega_0(y), \theta_0(y), C_0(y), U_1(y), \omega_1(y), \theta_1(y)$  and  $C_1(y)$  are to be determined.

Substituting (14), (15), (16) and (17) into (9), (10), (11) and (12) respectively, Comparing harmonic and non harmonic terms, we obtain

$$(1 + \beta)U_0'' + U_0' - NU_0 = -Gr\theta_0 - GcC_0 - S\omega_0' - M \tag{18}$$

$$\omega_0'' + \eta\omega_0' = 0 \tag{19}$$

$$\theta_0'' + \text{Pr}\theta_0' = 0 \tag{20}$$

$$C_0'' + \text{Sc}C_0' = 0 \tag{21}$$

and the boundary conditions become

$$\left. \begin{aligned} y = 0 : U_0 = u_p, \omega_0 = -u_p'', \theta_0 = 1, C_0 = 1 \\ y \rightarrow \infty : U_0 = 1, \omega_0 \rightarrow 0, \theta_0 \rightarrow 0, C_0 \rightarrow 0 \end{aligned} \right\} \tag{22}$$

Also,

$$(1 + \beta)U_1'' + U_1' - (N + in)U_1 = -Gr\theta_1 - GcC_1 - S\omega_1' - M + AU_0' \tag{23}$$

$$\omega_1'' + \eta\omega_1 - in\text{Pr}\omega_1 = -A\text{Pr}\omega_0' \tag{24}$$

$$\theta_1'' + \text{Pr}\theta_1' - in\text{Pr}\theta_1 = -A\text{Pr}\theta_0' \tag{25}$$

$$C_1'' + \text{Sc}C_1 - in\text{Sc}C_1 = -A\text{Sc}C_0' \tag{26}$$

Subject to the boundary conditions

$$\left. \begin{aligned} y = 0 : U_1 = 0, \omega_0 =, \theta_1 = 1, C_1 = 1 \\ y \rightarrow \infty : U_1 = 1, \omega_1 \rightarrow 0, \theta_1 \rightarrow 0, C_1 \rightarrow 0 \end{aligned} \right\} \tag{27}$$

where the primes denotes differentiation with respect to  $y$ .

Solving (18) to (26) under the boundary conditions (22) and (27) and substituting the obtained solutions into (14) to (17). Then, the velocity distribution is given by

$$U(y,t) = \left( A_8 e^{-\left(\frac{1}{2(1+\beta)}+a\right)y} + A_9 e^{-Pr y} + A_{10} e^{-Sc y} + A_{11} e^{-\eta y} + \frac{M}{N} \right) + \varepsilon e^{\text{int}} \left( D_{21} e^{-\left(\frac{1}{2(1+\beta)}+f\right)y} + D_4 e^{-m_4 y} + D_5 e^{-Pr y} + D_6 e^{-m_3 y} + D_7 e^{-Sc y} + D_8 e^{-m_5 y} + D_9 e^{-\eta y} + D_{10} + D_{11} e^{-m_2 y} + D_{12} e^{-Pr y} + D_{13} e^{-Sc y} + D_{14} e^{-\eta y} + D_{15} \right) \tag{28}$$

The angular velocity distribution is

$$\omega(y,t) = A_6 e^{-\eta y} + \varepsilon e^{\text{int}} (B_8 e^{-m_5 y} + B_9 e^{-\eta y}) \tag{29}$$

The temperature distribution is expressed as

$$\theta(y,t) = e^{-Pr y} + \varepsilon e^{\text{int}} (B_5 e^{-m_4 y} + B_6 e^{-Pr y}) \tag{30}$$

The concentration distribution is given by

$$C(y,t) = e^{-Sc y} + \varepsilon e^{\text{int}} (B_2 e^{-m_3 y} + B_3 e^{-Sc y}) \tag{31}$$

The skin friction from (27) is

$$\frac{\partial U(y,t)}{\partial y} \Big|_{y=0} = \left( -\frac{1}{2(1+\beta)} + a \right) A_8 - Pr A_9 - Sc A_{10} - \eta A_{11} + \varepsilon e^{\text{int}} \left( \left( \frac{1}{2(1+\beta)} + f \right) D_{21} - m_4 D_4 - Pr D_5 - m_3 D_6 - Sc D_7 - m_5 D_8 - \eta D_9 - m_2 D_{11} - Pr D_{12} - Sc D_{13} - \eta D_{14} \right)$$

#### 4.0 RESULTS AND DISCUSSION

Micropolar fluid flow with Hall effect and mass transfer in a porous medium has been investigated and solved analytically. In order to understand the flow of the fluid, computations are performed for different parameters such as Gr, M, m, Gc, n, t, Sc, Pr,  $\beta$  and Up.

Figure 1 represents the temperature profile, Figure 2 is the concentration profile, Figures 3-6 are the angular velocity profiles and Figures 7-14 represent the velocity profiles with varying parameters respectively.

The effect of temperature for different values of (Pr = 0.71, 1, 3,7) is presented in Figure 1. The graph shows that temperature decreases with increase in Pr. In Figure 2, the effect of concentration for different values of (Sc = 0.6, 0.8, 1, 2.01) is shown. The graph shows that temperature decreases with increase in Pr. The effect of angular velocity for different values of ( $\beta = 0, 0.3, 0.6, 1$ ) is given in Figure 3. Also, the effect of angular velocity for different values of (Up = 0, 0. 2, 0.6, 1) is shown in Figure 4. Figure 5 denotes the effect of angular velocity for (n = 0.1, 0.3, 0.7, 1). The effect of angular

velocity for different values of ( $t= 0.5, 1, 1.5, 2$ ) is given in Figure 6. The graphs show that angular velocity decreases with the increase in  $\beta$ ,  $Up$ ,  $n$  and  $t$  respectively. The effect of Schmidt number for ( $Sc = 0.3, 0.6, 0.8, 2.01$ ) on velocity is presented in Figure 7. Similarly, the effect of velocity for different values of ( $Pr = 0.71, 1, 3, 7$ ) is given in Figure 8 and Figure 9 denote the effect of velocity for ( $Gr = 1, 2, 3, 5$ ). The effect of velocity for different values of ( $Gc = 1, 2, 3, 5$ ) is presented in Figure 10. The graphs show that velocity increases with the

increase in  $Gc$ ,  $Gr$  and decreases with the increase in  $Sc$  and  $Pr$ .

The effect of velocity for different values ( $M = 1, 3, 5, 10$ ) is presented in Figure 11. The effect of velocity for different values of ( $Up = 0, 0.3, 0.6, 1$ ) is shown in Figure 12, the effect of velocity for different values of ( $\beta = 0.2, 0.4, 0.8, 2$ ) is given in Figure13. Figure 14 depicts the effect of velocity for ( $m = 1, 3, 5, 10$ ). The graphs show that velocity decreases with increase in  $M$ ,  $m$  and  $\beta$  and decreases with the increase in  $Up$ .

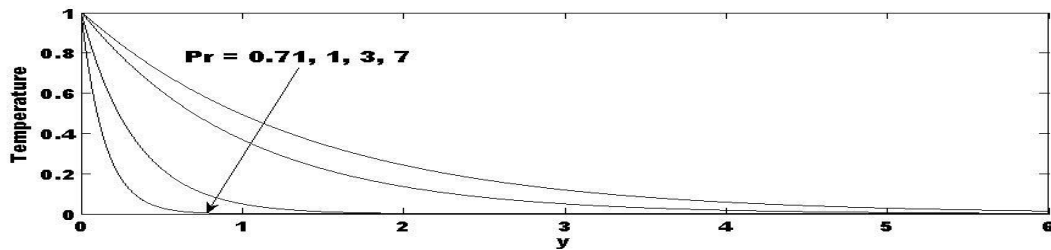


Figure 1. Temperature profiles for different values of  $Pr$ .

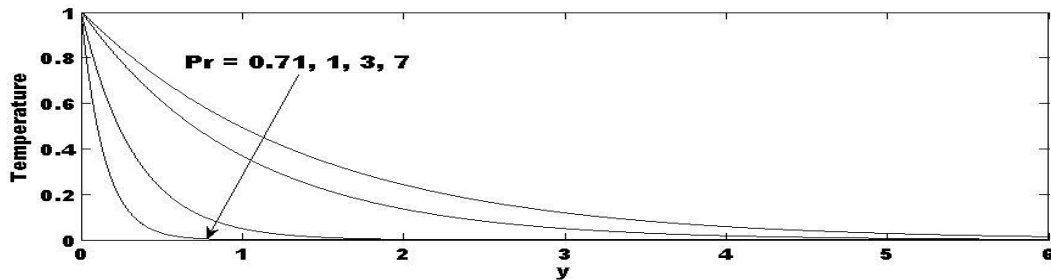


Figure 2. Concentration profiles for different values of  $Sc$ .

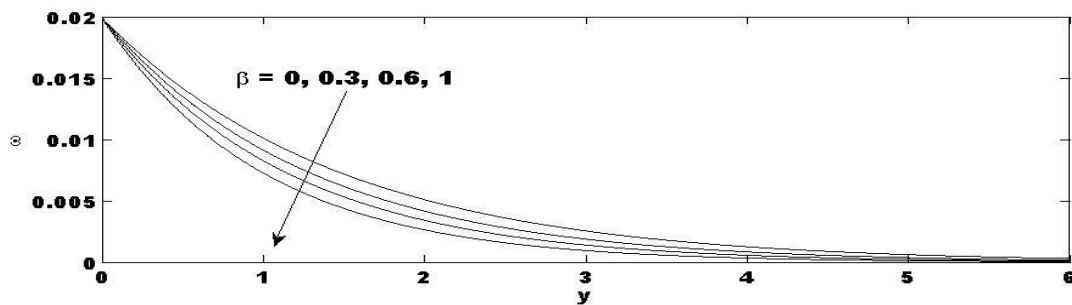


Figure 3. Angular velocity profiles plotted for different values of  $\beta$ .

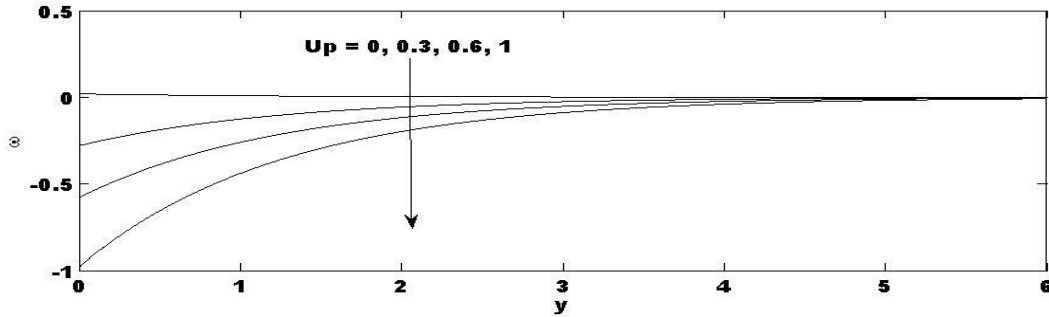


Figure 4. Angular velocity profiles for different values of  $U_p$ .

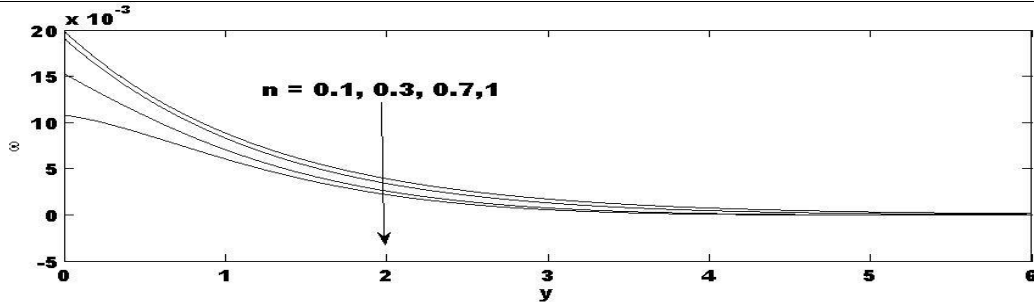


Figure 5. Angular velocity profiles for different values of  $n$ .

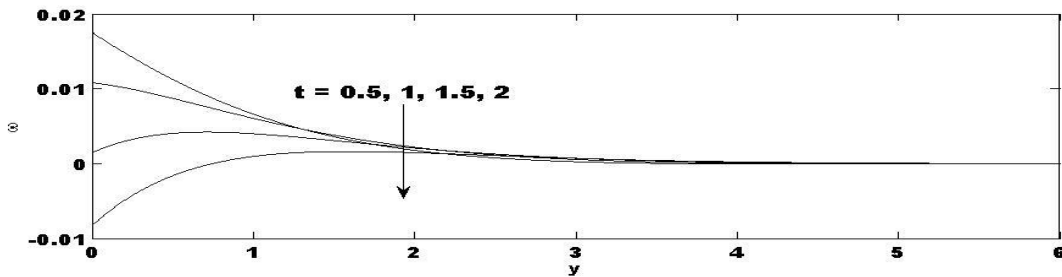


Figure 6. Angular velocity profiles for different values of  $t$ .

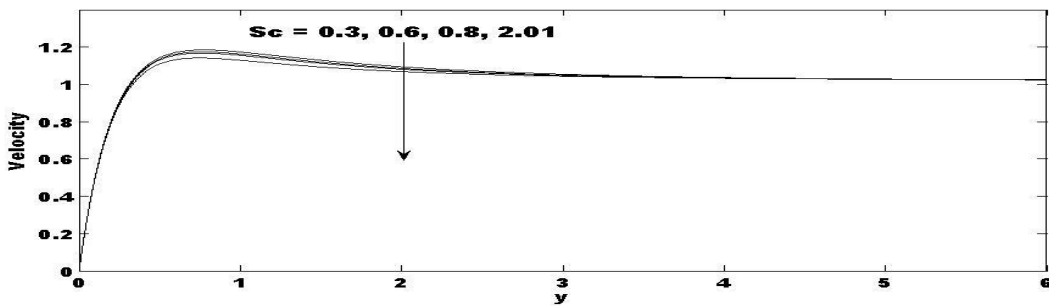


Figure 7. Velocity profiles for different values of  $Sc$ .

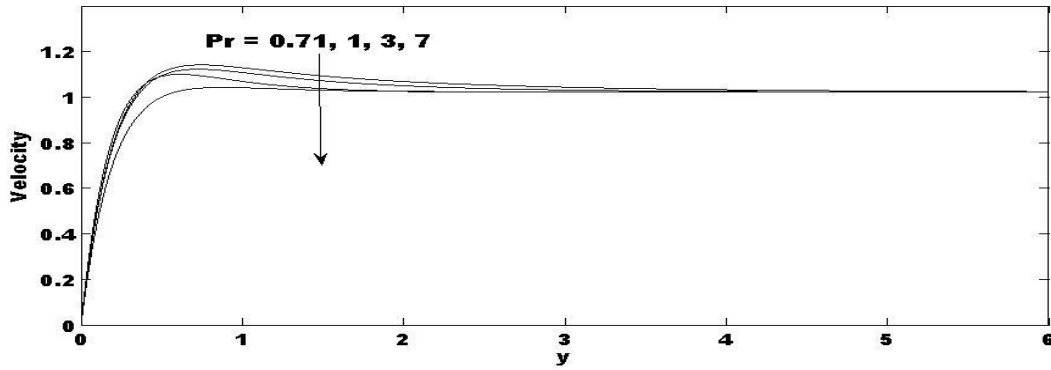


Figure 8. Velocity profiles for different values of Pr.

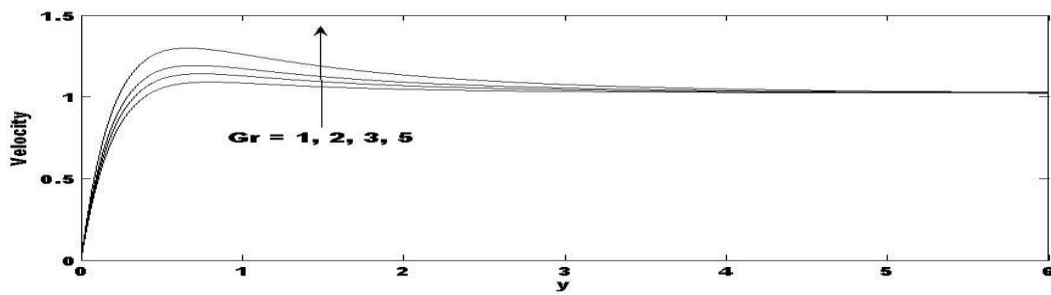


Figure 9. Velocity profiles for different values of Gr.

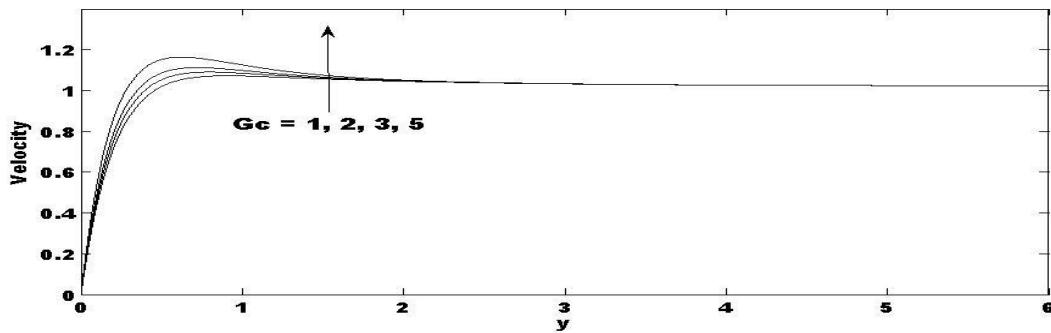


Figure 10. Velocity profiles for different values of Gc.

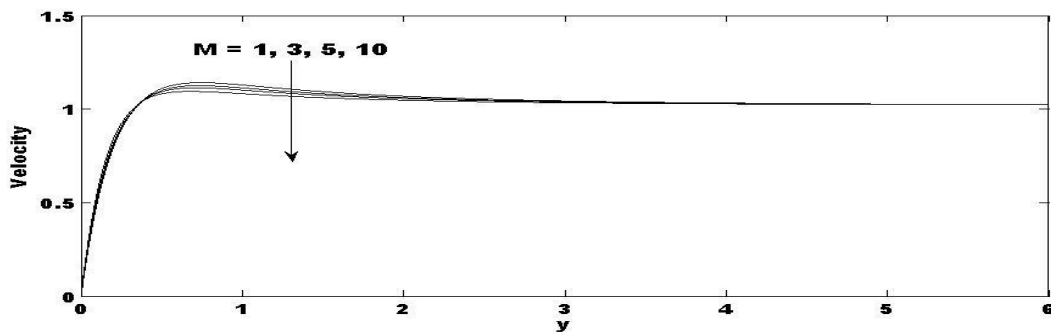


Figure 11. Velocity profiles for different values of M.

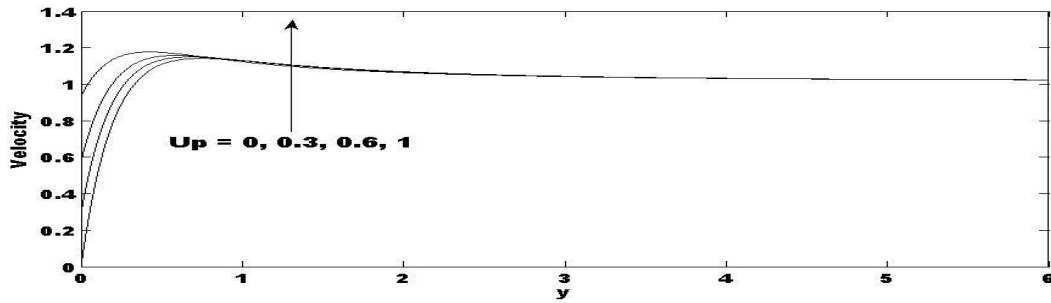


Figure 12. Velocity profiles for different values of  $U_p$ .

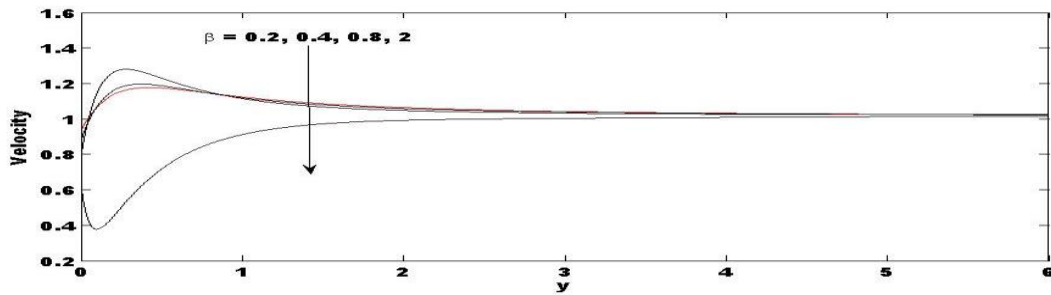


Figure 13. Velocity profiles for different values of  $\beta$ .

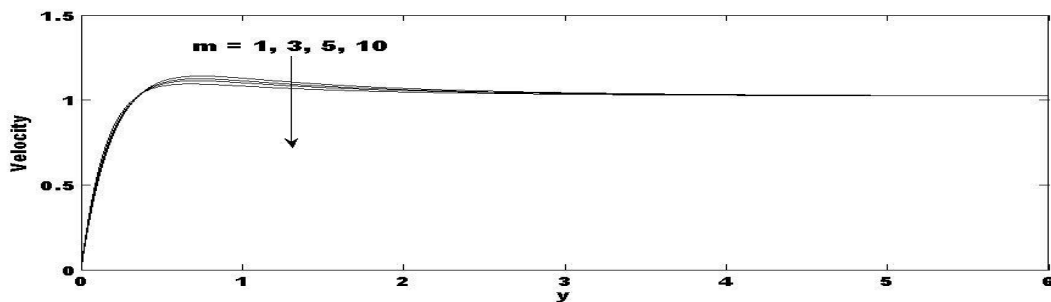


Figure 14. Velocity profiles for different values of  $m$ .

Table 1 represents the Skin friction, It shows that Skin friction increases with increase in  $G_c$  and  $G_r$  and decreases with the increase in  $Sc$ ,  $\omega$ ,  $Pr$ ,  $m$ ,  $K$ ,  $\varepsilon$  and  $M$ .

Table 1: Skin friction  $\tau$

$\varepsilon$	$N$	$Pr$	$Sc$	$K$	$G_c$	$G_r$	$M$	$m$	$\beta$	$\tau$
0.2	1	0.71	0.3	0.1	2	3	4	1	0.3	0.4213
<b>0.4</b>	1	0.71	0.3	0.1	2	3	4	1	0.3	0.4011
0.2	<b>3</b>	0.71	0.3	0.1	2	3	4	1	0.3	0.3924
0.2	1	<b>1</b>	0.3	0.1	2	3	4	1	0.3	0.4156
0.2	1	0.71	<b>0.6</b>	0.1	2	3	4	1	0.3	0.3973
0.2	1	0.71	0.3	<b>0.3</b>	2	3	4	1	0.3	0.4172
0.2	1	0.71	0.3	0.1	<b>4</b>	3	4	1	0.3	0.4036
0.2	1	0.71	0.3	0.1	2	<b>5</b>	4	1	0.3	0.6217
0.2	1	0.71	0.3	0.1	2	3	<b>8</b>	1	0.3	0.5251

0.2	1	0.71	0.3	0.1	2	3	4	1	0.3	0.4125
0.2	1	0.71	0.3	0.1	2	3	4	<b>3</b>	0.3	0.3981
0.2	1	0.71	0.3	0.1	2	3	4	1	0.3	0.4201

## 5.0 CONCLUSION

We have examined and solved the governing equations for the micropolar fluid flow with Hall effect and mass transfer in a porous medium analytically using perturbation technique to obtain approximate solutions for the velocity, angular velocity, temperature and concentration. The effect of physical parameters namely; thermal Grashof number, Hartmann number, mass Grashof number, Schmidt number, Prandtl number, permeability Parameter, time and Hall parameter on the flow field was examined. From the study, it was observed that the velocity is higher due to increase of  $G_c$ ,  $G_r$  and  $U_p$ , and it decreases higher values of  $M$ ,  $m$ ,  $K$ ,  $Sc$ ,  $Pr$  and  $\beta$ . The temperature is lower when  $Pr$  is increased and the concentration falls when  $Sc$  becomes significant.

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### APPENDIX

$$a = \sqrt{\frac{1}{(1+\beta)^2} + \frac{4N}{(1+\beta)}}, A_{10} = \frac{-Gc}{(1+\beta)\left(Sc^2 - \frac{Sc}{(1+\beta)} - \frac{N}{(1+\beta)}\right)}, b = \sqrt{\frac{Pr^2 + 4inPr}{4}}$$

$$A_9 = \frac{-Gr}{(1+\beta)\left(Pr^2 - \frac{Pr}{(1+\beta)} - \frac{N}{(1+\beta)}\right)}, A_{11} = \frac{S\eta}{(1+\beta)\left(\eta^2 - \frac{\eta}{(1+\beta)} - \frac{N}{(1+\beta)}\right)}, A_{12} = \frac{M}{N}$$

$$A_8 = u_p - A_9 - A_{10} - A_{11} - 1, d = \sqrt{\frac{Sc^2 + 4inSc}{4}}, B_3 = -\frac{ASc}{in}, B_2 = 1 - B_3, m_3 = \frac{Sc}{2} + d, B_6 = -\frac{APr}{in},$$

$$B_5 = 1 - B_6, m_4 = \frac{Pr}{2} + b, e = \sqrt{\frac{\eta^2 + 4in\eta}{4}}, B_8 = -\frac{A\eta}{in}, B_5 = 1 - B_6, m_5 = \frac{\eta}{2} + e,$$

$$f = \sqrt{\frac{1}{(1+\beta)^2} + \frac{(4N + in)}{(1+\beta)}}, D_4 = \frac{-GrB_6}{(1+\beta)\left(m_4^2 - \frac{m_4}{(1+\beta)} - \frac{(N + in)}{(1+\beta)}\right)}, D_{15} = \frac{AM}{(N + in)}$$

$$D_5 = \frac{-GrB_6}{(1+\beta)\left(Pr^2 - \frac{Pr}{(1+\beta)} - \frac{(N + in)}{(1+\beta)}\right)}, D_6 = \frac{-GcB_3}{(1+\beta)\left(m_3^2 - \frac{m_3}{(1+\beta)} - \frac{(N + in)}{(1+\beta)}\right)},$$

$$\begin{aligned}
D_7 &= \frac{-GcB_5}{(1+\beta)\left(Sc^2 - \frac{Sc}{(1+\beta)} - \frac{(N+in)}{(1+\beta)}\right)}, D_8 = \frac{-SB_8}{(1+\beta)\left(m_5^2 - \frac{m_5}{(1+\beta)} - \frac{(N+in)}{(1+\beta)}\right)}, \\
D_9 &= \frac{-SB_9}{(1+\beta)\left(\eta^2 - \frac{\eta}{(1+\beta)} - \frac{(N+in)}{(1+\beta)}\right)}, D_{10} = \frac{M}{(N+in)}, D_{11} = \frac{-AA_8}{(1+\beta)\left(m_2^2 - \frac{m_2}{(1+\beta)} - \frac{(N+in)}{(1+\beta)}\right)}, \\
D_{12} &= \frac{-AA_9}{(1+\beta)\left(Pr^2 - \frac{Pr}{(1+\beta)} - \frac{(N+in)}{(1+\beta)}\right)}, D_{13} = \frac{-AA_{10}}{(1+\beta)\left(Sc^2 - \frac{Sc}{(1+\beta)} - \frac{(N+in)}{(1+\beta)}\right)}, \\
D_{14} &= \frac{-AA_{11}}{(1+\beta)\left(\eta^2 - \frac{\eta}{(1+\beta)} - \frac{(N+in)}{(1+\beta)}\right)}, \\
D_{21} &= -D_4 - D_5 - D_6 - D_7 - D_8 - D_9 - D_{11} - D_{12} - D_{13} - D_{14} - 1
\end{aligned}$$


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