



Application of Linear Programming Technique for Optimal Production in Profit Maximization

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ABSTRACT

In this paper, we are concerned with applying linear programming technique to determine optimum product mix of Gusau Sweet Factory Limited to maximize profit. Data on the quantity of major raw materials used in the production of the three different sweets: Toffee, Lollipop and Candy, cost and selling prices and the profit of each product were collected from the extract of the financial record of the company. The problem was formulated as a linear programming problem (LPP) and solved using the revised simplex algorithm. The results show that Toffee and lollipop should be produced daily for the company to realize an optimum profit.

Keywords: Optimization, Linear programming model, Decision variables, Objective function.

1.0 INTRODUCTION

Decision making is a common phenomenon in the business world; managers are typically faced with uncertainty about the consequences of their decisions in an environment with several decision alternatives. In such situation, a good analysis will include risk assessments where the risk associated with some decisions is the direct result of uncertainty surrounding the outcome from that decision. In an attempt to address these problems, the first step in the decision analysis process is problem formulation, which Linear programming technique (LP) can be deployed to solve such decision problem. Linear programming is a subset of mathematical programming that is concerned with efficient allocation of limited resources to known activities with the objective of meeting the desired goal of either maximization of profit or minimization of cost (Taha, 1997). Linear programming is also used in operations management to find optimum solution to several types of problems like allocation of resources, transportation problems and responsibility problems where it works to choose the best course of action among several competing alternatives (Yahya, 2004).

Linear programming was developed by George B. Dantzing during the Second World War II for planning the diversified activities of the U.S. Air

Force connected with the problem of supplies to the Force and out of the necessity of solving military logistics problems. It remains one of the most used mathematical techniques in today's modern societies. From report of various surveys, it has been shown that many production companies, particularly those operating in Nigeria are yet to know fully the application of linear optimizations. Sometimes many production companies are faced with problems of how to utilize the scarce and limited available resources in order to maximize profit or minimize cost. The decision of most production managers is based on the total input used in the production and output processed. The problem of decision making based on the use of limited resource is a major factor that brought the application of linear programming model which is now considered as one of the most powerful tools which all decision makers (managers) must apply before achieving effectively the organizational objectives. (Akpan and Iwok, 2016).

Murugan and Manivel (2009) applied linear programming on textile industry with linear interactive optimizer software to determine the optimal numbers of each of three of its products to be produced to maximize profit of the organization.

Adeyemo and Otiero (2009) extended the scope of applicability of linear programming from

management sciences to environmental sciences and conducted study in South African Rand to maximize revenue where sixteen types of crops were implanted in an area of 2500ha. Kareem and Aderoba (2008) attempted to demonstrate the potency of linear programming by applying it to determine the optimal crew required for maintenance in Akure cocoa processing company in Nigeria. Kayode et al (2020) applies linear programming technique to determine the quantity of bread the University of Benin bakery company that should be produced in a day to maximize profit. Similarly, Fagoyinbo et al., (2011) used the technique of linear programming to determine the maximum profit of Geepee Nigeria Limited in the production of tanks of various types.

2.0 METHODOLOGY

2.1 Revised Simplex Method

The method to be used is the revised simplex method for standard maximization problem. The original Simplex method is a straight forward algebraic procedure. However, the way of executing the algorithm (in either algebraic or tabular form) is not the most efficient computational procedure for computers because it computes and stores many numbers that are not needed at the current iteration and that may not even become relevant for decision making at subsequent iterations. The only pieces of information relevant at each iteration are:

- (i) The coefficients of the non-basic variables.
- (ii) The coefficients of the entering basic variable in the other equations.
- (iii) The right-hand sides of the equations.

It would be very useful to have a procedure that could obtain this information efficiently without computing and storing the other coefficients. These considerations motivated the development of the revised simplex method. This method was designed to accomplish exactly the same things as the original simplex method, but in a way that is more efficient for execution on computers. Thus, it is a streamlined version of the original procedure. It computes and stores only the information that is currently needed, and it carries along the essential data in a more compact form.

2.2 The Standard Form of the Revised Simplex Method

The revised simplex method explicitly uses matrix manipulations, so it necessary to describe the problem in matrix notation. Using matrices, our standard form for the general linear programming model becomes

Maximize $z = cx$
 Subject to
 $Ax \leq b$ and $X \geq 0$,
 Where c is the row vector

$$C = [C_1, C_2, C_3 \dots C_n]$$

x , b , are the column vectors such that

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix} \quad \text{and } A \text{ is a matrix with } A = \begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ a_{m1} & a_{m2} \dots & a_{mn} \end{bmatrix}$$

To obtain the augmented form of the problem, introduce the column vector of slack variables.

$$\begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \cdot \\ \cdot \\ \cdot \\ x_{n+m} \end{bmatrix} \text{ so}$$

that the constraints become $[A \ I] \begin{pmatrix} x \\ x_s \end{pmatrix} = b$ and $x = \begin{pmatrix} x \\ x_s \end{pmatrix} \geq 0$ where I is the $m \times n$ identity matrix.

2.3 Revised Simplex Algorithm

Step 1: By considering the objective function as one of the constraints and adding the slack variable to the inequalities, the problem will be expressed in standard form.

Step 2: Using the initial revised simplex table, find an initial basic feasible solution as shown below.

Table 1: Initial basic feasible solution

Variable in basis(B)	Solution value(b)	Basis inverse B^{-1}					Y_k
		Z	S_1	S_2	...	S_m	
z_j	0	1	0	0	...	0	$c_k - z_k$
S_1	b_1	0	1	0	...	0	y_{1k}
S_2	b_2	0	0	1	...	0	y_{2k}
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
S_m	b_m	0	0	0	0	1	y_{1k}

Step 3: To determine a variable to enter into basis (pivot) from non-basic variables, multiply the Z row in the B^{-1} by the column vector of each non-basic variable (i.e. $x_1, x_2, x_3 - x_n$). that is $c_j - z_j$.

Step 4: Select a variable to leave the basis: The least ratio of the selection column to the pivot column is used to select variables to leave the basis, just like the standard simplex technique.

Step 5: After adding the non-basic variable to the basis, conduct Gauss Jordan row operations to update the current solution.

Step 6: Repeat step 3 to step 5 until optimality is obtained when all $c_j - z_j \leq 0$

2.4 Model Assumption

Gusau sweet factory limited manufactures three types of sweet namely; Toffee, Lollipop and

Candy. The production of the three sweet requires first batch (sugar, glucose and Glycerol monostearate GMS), second batch (flavor, colorant, and condensed milk) and the third batch (fats lecithin and flour) of raw materials. Toffee, lollipop and candy requires 15kg, 20kg and 10kg of the 65kg of the first batch available. Toffee, lollipop and candy requires 30kg, 15kg and 20kg respectively of the 75kg of the second batch available. Toffee, lollipop and candy requires 2kg, 4kg and 3kg respectively of the 15kg of the third batch available. These three products are processed in the same machine. Each cartoon of toffee cost ₦900 and it is sold at ₦1300, lollipop cost ₦1000 and it is sold at ₦1500 and candy cost ₦750 and it is sold at ₦1000. How should the sweet factory allocate their production capacity to the three types of sweet so that they may make maximum profit in a day.

2.5 Data Presentation

Table 2: Quantity of raw materials available

Raw material	Product			Available material (kg)
	Toffee	Lollipop	Candy	
First batch	15	20	10	65
Second batch	30	15	20	75
Third batch	2	4	3	15
Profit (₦)	400	500	250	

Model Formulation

Let the quantity of toffee to be produce = x_1
 Let the quantity of lollipop to be produce = x_2

Let the quantity of candy to be produce = x_3
 Let Z denote the profit to be maximize

The linear programming model for the above production data is given by:

$$\text{Max } Z = 400x_1 + 500x_2 + 250x_3 \text{ (objective function)}$$

Subject to the constraints

$$15x_1 + 20x_2 + 10x_3 \leq 65$$

$$30x_1 + 15x_2 + 20x_3 \leq 75$$

$$2x_1 + 4x_2 + 3x_3 \leq 15$$

3.0 DATA ANALYSIS AND RESULTS

By introducing the slack variables, the inequalities become equality and can be written as:

$$\text{Max } z - 400x_1 - 500x_2 - 250x_3 - 0s_1 - 0s_2 - 0s_3 = 0$$

Subject to the constraints:

$$15x_1 + 20x_2 + 10x_3 + s_1 = 65$$

$$30x_1 + 15x_2 + 20x_3 + s_2 = 75$$

$$2x_1 + 4x_2 + 3x_3 + s_3 = 15$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Table 3: Initial Tableau

Variables in basis(B)	Solution value (b)	Basis invers (B ⁻¹)				Y _k	Ratio b/y _k	Add table			
		Z	s ₁	s ₂	s ₃			x ₁	x ₂	x ₃	
z _j	0	1	0	0	0	-500	—	-400	-500	-250	z-row
s ₁	65	0	1	0	0	20	3.25	15	20	10	s ₁ -row
s ₂	75	0	0	1	0	15	5	30	15	20	s ₂ -row
s ₃	15	0	0	0	1	4	3.75	2	4	3	s ₃ -row

$c_j - z_j = -(\text{z-row column in } B^{-1})(\text{columns in add table})$

$$\text{For } x_1 \Rightarrow -[1 \ 0 \ 0 \ 0] \begin{bmatrix} -400 \\ 15 \\ 30 \\ 2 \end{bmatrix} = -[-400] = 400$$

$$\text{For } x_2 \Rightarrow -[1 \ 0 \ 0 \ 0] \begin{bmatrix} -500 \\ 20 \\ 15 \\ 4 \end{bmatrix} = -[-500] = 500$$

$$\text{For } x_3 \Rightarrow -[1 \ 0 \ 0 \ 0] \begin{bmatrix} -250 \\ 10 \\ 20 \\ 3 \end{bmatrix} = -[-250] = 250$$

Since $c_j - z_j \geq 0$. Pick the highest positive column as your pivot column. Then obtain y_k column (pivot column) by

$$Y_k = B^{-1} \times x_2 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -500 \\ 20 \\ 15 \\ 4 \end{bmatrix} = \begin{bmatrix} -500 \\ 20 \\ 15 \\ 4 \end{bmatrix}$$

Compute the ratio and determine the pivot element by

$$b/y_k = \begin{bmatrix} 0 \\ 65 \\ 75 \\ 15 \end{bmatrix} \div \begin{bmatrix} -500 \\ 20 \\ 15 \\ 4 \end{bmatrix} = \begin{bmatrix} - \\ 3.25 \\ 5 \\ 3.75 \end{bmatrix}$$

$$\text{Pivot row (new } x_2\text{-row)} = [65 \ 0 \ 1 \ 0 \ 0] \div 20 =$$

$$\begin{bmatrix} 3.25 \\ 0 \\ 0.05 \\ 0 \\ 0 \end{bmatrix}$$

Using Gauss Jordan operation

$$\text{New } z\text{-row} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - [-500] \begin{bmatrix} 3.25 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1625 \\ 1 \\ 25 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{New } s_2\text{-row} = \begin{bmatrix} 75 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - [15] \begin{bmatrix} 3.25 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 26.25 \\ 0 \\ 0.75 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{New } s_3\text{-row} = \begin{bmatrix} 15 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - [4] \begin{bmatrix} 3.25 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Table4: Tableau II

Variable in basis(B)	Solution value (b)	Basis inverse (B ⁻¹)				Y _k	Ratio b/y _k	Add table			
		Z	s ₁	s ₂	s ₃			x ₁	s ₁	x ₃	
z _j	1625	1	25	0	0	-25	—	-400	0	-250	z-row
x ₂	3.25	0	0.05	0	0	0.75	4.33	15	1	10	s ₁ -row
s ₂	26.25	0	0.75	1	0	41.25	0.6	30	0	20	s ₂ -row
s ₃	2	0	-0.2	0	1	-1	—	2	0	3	s ₃ -row

$c_j - z_j = -(z\text{-row column in } B^{-1})(\text{columns in add table})$

For $x_1 \Rightarrow -[1 \ 25 \ 0 \ 0] \begin{bmatrix} -400 \\ 15 \\ 30 \\ 2 \end{bmatrix} = -[-400 + 375] = 25$

For $s_1 \Rightarrow -[1 \ 25 \ 0 \ 0] \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = -[25] = -25$

For $x_3 \Rightarrow -[1 \ 25 \ 0 \ 0] \begin{bmatrix} -250 \\ 10 \\ 20 \\ 3 \end{bmatrix} = -[-250 + 250] = 0$

Since $c_j - z_j \geq 0$ we test for optimality. Pick the highest positive column as your pivot column.

$$Y_k = B^{-1} \times x_1 \Rightarrow \begin{bmatrix} 1 & 25 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0.75 & 1 & 0 \\ 0 & -0.2 & 0 & 1 \end{bmatrix} \begin{bmatrix} -400 \\ 15 \\ 30 \\ 2 \end{bmatrix} = \begin{bmatrix} -400 + 375 \\ 0.75 \\ 11.25 + 30 \\ -3 \end{bmatrix} = \begin{bmatrix} -25 \\ 0.75 \\ 41.25 \\ -1 \end{bmatrix}$$

$$b/y_k = \begin{bmatrix} 1625 \\ 3.25 \\ 26.25 \\ 2 \end{bmatrix} \div \begin{bmatrix} -25 \\ 0.75 \\ 41.25 \\ -1 \end{bmatrix} = \begin{bmatrix} - \\ 4.33 \\ 0.6 \\ - \end{bmatrix}$$

Pivot row (new x_1 -row) = $[26.25 \ 0 \ 0.75 \ 1 \ 0] \div 41.25 = \begin{bmatrix} 0.064 \\ 0 \\ 0.018 \\ 0.024 \\ 0 \end{bmatrix}$

Using Gauss Jordan operation

$$\text{New z-row} = \begin{bmatrix} 1625 \\ 1 \\ 25 \\ 0 \\ 0 \end{bmatrix} - [-25] \begin{bmatrix} 0.64 \\ 0 \\ 0.018 \\ 0.024 \\ 0 \end{bmatrix} = \begin{bmatrix} 1641 \\ 1 \\ 25.45 \\ 0.6 \\ 0 \end{bmatrix}$$

$$\text{New } x_2\text{-row} = \begin{bmatrix} 3.25 \\ 0 \\ 0.05 \\ 0 \\ 0 \end{bmatrix} - [0.75] \begin{bmatrix} 0.64 \\ 0 \\ 0.018 \\ 0.024 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.77 \\ 0 \\ 0.0365 \\ -0.018 \\ 0 \end{bmatrix}$$

$$\text{New } s_3\text{-row} = \begin{bmatrix} 2 \\ 0 \\ -0.2 \\ 0 \\ 1 \end{bmatrix} - [-1] \begin{bmatrix} 0.64 \\ 0 \\ 0.018 \\ 0.024 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.64 \\ 0 \\ -0.182 \\ 0.024 \\ 1 \end{bmatrix}$$

Table 5: Tableau III

		Basis inverse (B ⁻¹)							Add table		
Variable in basis (B)	Solution value(b)	Z	s ₁	s ₂	s ₃	Y _k	Ratio b/y _k	s ₂	s ₁	x ₃	
z _j	1641	1	25.45	0.6	0			0	0	-250	z-row
x ₂	2.77	0	0.0365	-0.018	0			0	1	10	s ₁ -row
x ₁	0.064	0	0.018	0.024	0			1	0	20	s ₂ -row
s ₃	2.64	0	-0.182	0.024	1			0	0	3	s ₃ -row

$c_j - z_j = -(z\text{-row column in } B^{-1})(\text{columns in add table})$

For $s_2 \Rightarrow -[1 \ 25.45 \ 0.6 \ 0] \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = -0.6$

For $s_1 \Rightarrow -[1 \ 25.45 \ 0.6 \ 0] \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = -[25] = -25$

For $x_3 \Rightarrow -[1 \ 25.45 \ 0.6 \ 0] \begin{bmatrix} -250 \\ 10 \\ 20 \\ 3 \end{bmatrix} =$

$-[-250 + 250] = 0$

Since $c_j - z_j \leq 0$ optimality has been reached.

Thus,

$z = 400x_1 + 500x_2 + 250x_3$

$z = 400(0.064) + 500(2.77) + 250(0)$

$z = 256 + 1385$

$z = 1641$

4.0 INTERPRETATION OF RESULTS

The following result was produced after taking into account the constraints and the various machines to which it was subjected.

Table 6: Optimal solution

Variables	Value
x ₁	0.064
x ₂	2.77
x ₃	0
z	1641

Based on primary data gathered from Gusau sweet factory limited, the optimum result derived from the model indicates that two types of sweet should be produced, toffee and lollipop. Their production

quantities should be 2.77 and 0.064 units respectively as shown in table 6 (optimal solution). This will produce a maximum profit of ₦1641.

5.0 CONCLUSION

This study has analyzed the current activities of Gusau Sweet Factory Limited. Linear programming model has proven, given the raw data to determine the best production mix for the company in order to attain maximum profit. It is in the light of the above that, we recommend that the management of the company should give more attention to the production of Toffee and Lollipop rather than candy to achieve optimality.

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