

**INTERNATIONAL JOURNAL OF SCIENCE FOR GLOBAL SUSTAINABILITY****Heat Absorption Effect on Free Convection Flow Past a Vertical Porous Pipe with Mass Transfer**¹M. M. Altine, ²E. Omokhuale, ³Onwubuya I. O. and ⁴Disu A. B.¹Department of Mathematics, Federal University, Birnin Kebbi, Kebbi State.²Department of Mathematical Science, Federal University, Gusau, Zamfara State.³Department of Mathematics, Airforce Institute of Technology Kaduna, Nigeria.⁴Department of Mathematics, National Open University of Nigeria, Jabi, Abuja, Nigeria.Corresponding Author's Email altinemuhammad@gmail.com, Phone No.:+2348064819725

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ABSTRACT

This study investigates heat absorption on free convection flow past a vertical porous pipe with mass transfer. The governing equations are the continuity, momentum, energy and diffusion equations. The equations in dimensionless form were transformed into ordinary differential equations (ODEs) and solved analytically using perturbation method. Approximate solutions for the velocity, temperature and concentration were determined. Results are presented graphically showing the effects of various parameters on the fluid flow. It is observed that the velocity becomes higher when thermal Grashof number and sustentions paramètre are increased. Also, increase in magnetic parameter, Schmidt number and Prandtl number lead to fall in the velocity of the fluid. The temperature of the fluid rises also with increase in heat generation but reduces for increased values of Prandtl number. Consequently, the concentration boundary layer decreases with increasing values of Schmidt number. Furthermore, increase in suction/injection has a retarding and accelerating effect on the velocity, temperature and concentration of the fluid respectively.

Key Words: Free Convection Flow; Heat Absorption; Magnetohydrodynamics (MHD); Porous Pipe and Mass Transfer.

1.0 INTRODUCTION

Free convection flows are of great interest in a number of industrial applications such as: fiber and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns.

Magnetohydrodynamic has attracted the attention of a large number of scholars due to its diverse applications in astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc.

Several authors have presented theoretical or experimental investigations, for example in porous pipes, see Bearman (1958); where solution for the flow between porous plates with constant and equal suction at both walls was presented.

Wang (1971) extended Bearman's work for the case of the pulsating flow in a porous channel (between parallel walls) and for the corresponding problem in a porous annulus. Banks and Zaturka (1992) and Zaturka and Banks (1995) also investigated theoretically an axisymmetric flow of a viscous fluid through a porous annular pipe with constant and equal-fluxes through each pipe wall. Das *et al.* (2009)

studied mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat. Jha *et al.* (2013) examined unsteady/steady natural convection flow of reactive viscous fluid in a vertical annulus. Rao *et al.* (2013) investigated Unsteady MHD Free Convective Heat and Mass Transfer Flow Past a Semi-Infinite Vertical Permeable Moving Plate with Heat Absorption, Radiation, Chemical Reaction and Soret Effects. Machireddy (2013) analyzed chemically reactive species and radiation effects on MHD convective flow past a moving vertical cylinder. Raju *et al.* (2014) studied Heat transfer effects on a viscous dissipative fluid flow past a vertical plate in the presence of induced magnetic field. Ahmad and Jha (2015) studied computational methods of transient/steady natural convection flow of reactive viscous fluid in a vertical porous pipe. Kataria *et al.* (2019) investigated the Effects of chemical reaction and heat generation/absorption on magnetohydrodynamic (MHD) Casson fluid flow over an exponentially accelerated vertical plate embedded in porous medium with ramped wall temperature and ramped surface concentration. Thus, motivated by the above studies and applications mentioned, in this present work, we have examined heat absorption on free convection flow past a vertical porous pipe with mass transfer. The dimensionless non-linear ODEs were solved analytically using perturbation method.

2.0 FORMULATION

We consider the free convection flow in a finite vertical porous pipe in the presence of heat absorption. The x' -axis is taken along the pipe upward direction and the r' -axis (radial-coordinate) is taken normal to the pipe (Figure 1). A uniform magnetic field B_0 is assumed to be applied in a direction perpendicular to the pipe. The properties of the fluid are assumed to be constant. In addition, a chemically reactive species is assumed to be emitted from the vertical pipe, where it undergoes a homogeneous chemical reaction. The reaction is assumed to take place entirely in the pipe. Then, the fully developed flow under the above assumptions in a

porous pipe is governed by the following equations:

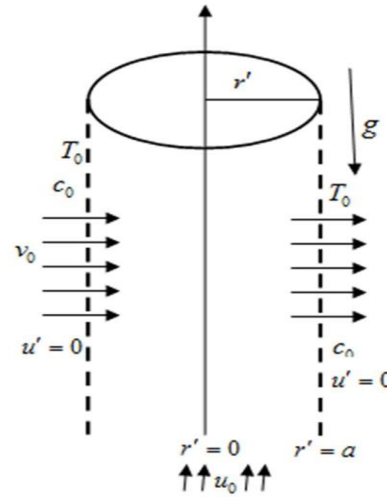


Figure 1: Physical Sketch of the problem

The equations governing the flow are:

$$\frac{v_0 b}{r'} \frac{\partial U'}{\partial r'} = v \left(\frac{\partial^2 U'}{\partial r'^2} + \frac{1}{r'} \frac{\partial U'}{\partial r'} \right) + g\beta(T' - T_0) + g\beta(C' - C_0) - \frac{\sigma B_0}{\rho} u' \quad (1)$$

$$\frac{v_0 b}{r'} \frac{\partial T'}{\partial r'} = \alpha \left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'} \frac{\partial T'}{\partial r'} \right) + \frac{Q_0}{\rho C_p} \quad (2)$$

$$\frac{v_0 b}{r'} \frac{\partial C'}{\partial r'} = D \left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'} \frac{\partial C'}{\partial r'} \right) \quad (3)$$

The boundary conditions for the problem assume the following form

$$\left. \begin{aligned} u' = 0, T' = 0 \text{ for } 0 \leq y \leq b \\ u' = 0, T' = T'_w + \varepsilon(T'_w - T_0), C' = C'_w + \varepsilon(C'_w - C_0) \text{ at } y' = 1 \\ u' = 0, T' = T_0, C' = C_0 \text{ at } y' = b \end{aligned} \right\} \quad (4)$$

on introducing the following non-dimensional quantities,

$$u = \frac{U'}{U_0}, r = \frac{r'}{bv}, \gamma = \frac{\nu_0 b}{\nu}, Sc = \frac{\nu}{D}, Pr = \frac{\nu}{\alpha}$$

$$M = \frac{\sigma B_0^2 b^2}{\rho}, S = Q_0 b^2 \nu, \theta = \frac{T' - T_0}{T'_w - T_0}, c = \frac{C' - C_0}{C'_w - C_0}$$

$$Gr = \frac{g \beta_T \nu b^2 (T'_w - T_0)}{u_0}, N = \frac{g \beta \nu b^2 (C'_w - C_0)}{u_0}$$

The non-dimensional form of the governing equations (1) – (3), reduce to:

$$\frac{\delta}{r} \frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{Mu}{r^2} + Gr\theta + Nc \quad (6)$$

$$\frac{\delta}{r} \frac{\partial \theta}{\partial r} = \frac{1}{Pr} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) + S \quad (7)$$

$$\frac{\delta}{r} \frac{\partial c}{\partial r} = \frac{1}{Sc} \left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} \right) \quad (8)$$

The corresponding boundary conditions are given by

$$\left. \begin{aligned} u = 0, \quad \theta = 1 + \varepsilon + \varepsilon^2, \quad C = 1 + \varepsilon + \varepsilon^2 \quad \text{at } r = 1 \\ u = 0, \quad \theta = 0, \quad C = 0 \quad \text{at } r = \lambda \end{aligned} \right\}$$

3.0 METHOD OF SOLUTION

In order to solve the system of equations (6) – (8) subject to the boundary conditions (9), the following simple perturbation is used. The governing equations (6) – (8) are expanded in powers of ε .

$$u = u_0(r) + \varepsilon u_1(r) + \varepsilon^2 u_2(r) + \dots = \sum_{j=0}^{\infty} \varepsilon^j u_j$$

$$\theta = \theta_0(r) + \varepsilon \theta_1(r) + \varepsilon^2 \theta_2(r) + \dots = \sum_{j=0}^{\infty} \varepsilon^j \theta_j \quad (10)$$

$$c = c_0(r) + \varepsilon c_1(r) + \varepsilon^2 c_2(r) + \dots = \sum_{j=0}^{\infty} \varepsilon^j c_j$$

Substituting equation (10) into equations (6) – (8) subject to (9) the governing partial differential equation of the model reduces to the following ordinary differential equations:

$$\frac{d^2 u_0}{dr^2} + \frac{1}{r}(1-\delta) \frac{du_0}{dr} - \frac{Mu_0}{r^2} = -Gr\theta_0 - Nc_0 \quad (11)$$

$$(5) \quad \frac{d^2 u_1}{dr^2} + \frac{1}{r}(1-\delta) \frac{du_1}{dr} - \frac{Mu_1}{r^2} = -Gr\theta_1 - Nc_1 \quad (12)$$

$$\frac{d^2 u_2}{dr^2} + \frac{1}{r}(1-\delta) \frac{du_2}{dr} - \frac{Mu_2}{r^2} = -Gr\theta_2 - Nc_2 \quad (13)$$

$$\frac{d^2 \theta_0}{dr^2} + \frac{1}{r}(1-\delta Pr) \frac{d\theta_0}{dr} = -S Pr \quad (14)$$

$$\frac{d^2 \theta_1}{dr^2} + \frac{1}{r}(1-\delta Pr) \frac{d\theta_1}{dr} = 0 \quad (15)$$

$$\frac{d^2 \theta_2}{dr^2} + \frac{1}{r}(1-\delta Pr) \frac{d\theta_2}{dr} = 0 \quad (16)$$

$$\frac{d^2 c_0}{dr^2} + \frac{1}{r}(1-\delta Sc) \frac{dc_0}{dr} = 0 \quad (17)$$

$$\frac{d^2 c_1}{dr^2} + \frac{1}{r}(1-\delta Sc) \frac{dc_1}{dr} = 0 \quad (18)$$

$$(9) \quad \frac{d^2 c_2}{dr^2} + \frac{1}{r}(1-\delta Sc) \frac{dc_2}{dr} = 0 \quad (19)$$

Since equations (11) to (13) are coupled with $c_0(y), c_1(y), c_2(y), q_0(y), \theta_1(y)$ and $\theta_2(y)$ using results obtained in equations (23) - (20) the solutions of $u_0(y)$ to $u_2(y)$ using the boundary condition (9) are:

$$u_0 = B_{14} r^{m_5} + B_{15} r^{-m_6} + B_{16} + B_{17} r^{\delta Pr} + B_{18} + B_{19} r^{\delta Sc} \quad (20)$$

$$u_1 = B_{22} r^{m_5} + B_{23} r^{-m_6} + B_{24} + B_{25} r^{\delta Pr} + B_{26} + B_{27} r^{\delta Sc} \quad (21)$$

$$u_2 = B_{30} r^{m_5} + B_{31} r^{-m_6} + B_{32} + B_{33} r^{\delta Pr} + B_{34} + B_{35} r^{\delta Sc} \quad (22)$$

The values of $\theta_0(y)$ to $\theta_2(y)$ from (14) to (16) using the boundary condition (9) are:

$$\theta_0 = B_7 + B_8 r^{\delta Pr} \quad (23)$$

$$\theta_1 = B_{10} + B_{11} r^{\delta Pr} \quad (24)$$

$$u_2 = B_{12} + B_{13}r^{\delta Pr} \quad (25)$$

The values of $c_0(y)$ to $c_2(y)$ from (17) to (19) using the boundary condition (9) are:

$$c_0 = B_1 + B_2r^{\delta Sc} \quad (26)$$

$$c_0 = B_3 + B_4r^{\delta Sc} \quad (27)$$

$$c_2 = B_5 + B_6r^{\delta Sc} \quad (28)$$

Thus, the velocity field is given as:

$$u(r) = B_{14}r^{m_5} + B_{15}r^{-m_6} + B_{16} + B_{17}r^{\delta Pr} + B_{18} + B_{19}r^{\delta Sc} + (B_{22}r^{m_5} + B_{23}r^{-m_6} + B_{24} + B_{25}r^{\delta Pr} + B_{26} + B_{27}r^{\delta Sc})\epsilon + (B_{30}r^{m_5} + B_{31}r^{-m_6} + B_{32} + B_{33}r^{\delta Pr} + B_{34} + B_{35}r^{\delta Sc})\epsilon^2 \quad (29)$$

the temperature field is:

$$\theta(r) = B_7 + B_8r^{\delta Pr} + (B_{10} + B_{11}r^{\delta Pr})\epsilon + (B_{12} + B_{13}r^{\delta Pr})\epsilon^2 \quad (30)$$

The concentration field is:

$$c(r) = B_1 + B_2r^{\delta Sc} + (B_3 + B_4r^{\delta Sc})\epsilon + (B_5 + B_6r^{\delta Sc})\epsilon^2 \quad (31)$$

Using equation (29) the skin friction on the plates ($r=0$) is:

$$\tau_0 = \frac{du}{dr}\bigg|_{r=0} = m_5B_{14} - m_6B_{15} + \delta Pr B_{17} + \delta ScB_{19} + (m_5B_{22} - m_6B_{23} + \delta Pr B_{25} + \delta ScB_{27})\epsilon + (m_5B_{30} - m_6B_{31} + \delta Pr B_{33} + \delta ScB_{35})\epsilon^2$$

Using equation (30) the rate of heat transfer on the plates ($r=0$) is:

$$Nu_0 = \frac{d\theta}{dr}\bigg|_{r=1} = \delta Pr B_8 + \epsilon\delta Pr B_{11} + \epsilon^2\delta Pr B_{13} \quad (33)$$

Using equation (31) the Sherwood number on the plates ($r=0$) is obtained as:

$$Sh_0 = \frac{dc}{dr}\bigg|_{r=1} = \delta ScB_2 + \epsilon\delta ScB_4 + \epsilon^2\delta ScB_6 \quad (34)$$

4.0 RESULTS AND DISCUSSION

4.1 Velocity Profile

In this research, the effects of the Grashof number (Gr), porous medium parameter (δ), magnetic field parameter (M), Prandtl number (Pr), chemical reaction parameter (R) heat generation parameter (S) and Schmidt number (Sc) on velocity (u), temperature (θ) and concentration (c) profiles are shown graphically and discussed. Also, skin friction, the rate of heat transfer and Sherwood number are shown and discussed in a tabular form.

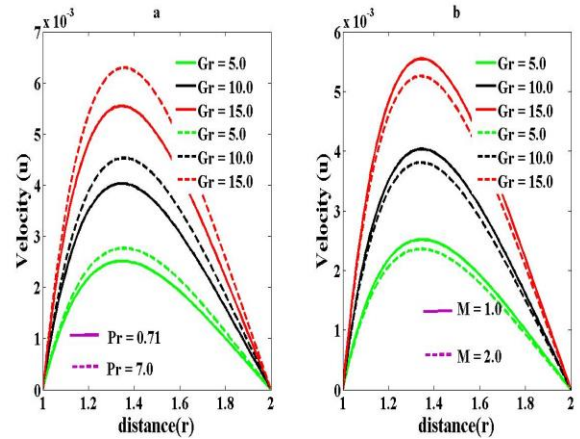


Figure 2: Effect of thermal Grashof number (Gr) on velocity (u)

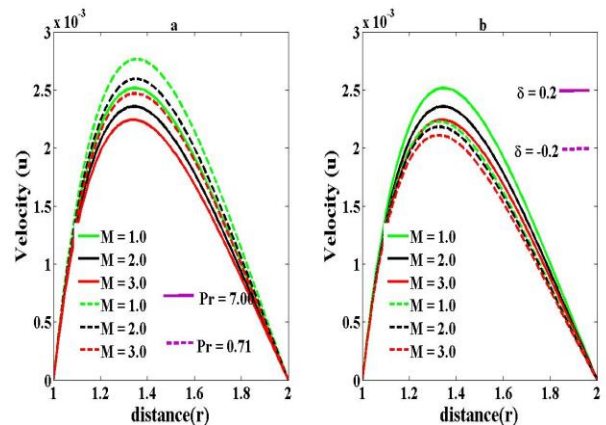


Figure 3: Effect of magnetic field parameter (M) on velocity (u)

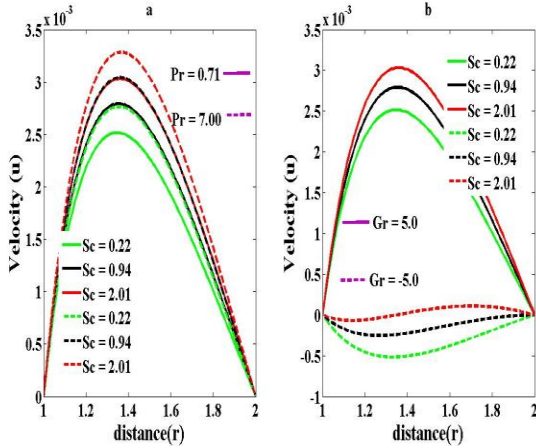


Figure 4: Effect of Schmidt number (Sc) on velocity (u)

It is observed from Figures 2 a and b that an increase in thermal Grashof number (Gr), enhances the velocity. It is seen in Figure 2a that velocity is higher in the case of water ($Pr = 7$), than in the case of air ($Pr = 0.71$). Similarly, the velocity is higher at $M = 1$ than when $M = 2$ as seen in Figure 2b. This implies that the velocity is higher at small values of magnetic field parameter. Figures 3a and b show that an increase in magnetic parameter (Pr) decreases the velocity. It is observed from Figure 3a that the velocity is higher in case of air ($Pr = 0.71$) than in the case of water ($Pr = 7$). Similarly, in Figure 3b the velocity is higher in case of injection ($\delta > 0$) than in the case of suction ($\delta < 0$). However, it is noticed that the velocity is higher in Figure 43a than in Figure 3b. Figures 4a and b depict the effect of Schmidt number (Sc) on velocity. From Figure 4a it is seen that increases in Schmidt number (Sc) increases the velocity. Also, the velocity is higher in case of water ($Pr = 7.00$) than in the case of air ($Pr = 0.71$). Moreover, in Figure 4b the velocity increases with the increase in Schmidt number for both cooling and heating of the plates ($Gr > 0$ and $Gr < 0$) while it is observed the velocity reduces drastically for heating of the plates ($Gr < 0$).

Table 1: Skin friction, Nusselt number and Sherwood number for Heat Absorption Effect on Free Convection Flow past a Vertical Porous Pipe with Mass Transfer

Pr	Gr	N	M	Sc	δ	τ_0	Nu_0	Sh_0
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4.2 Temperature profile

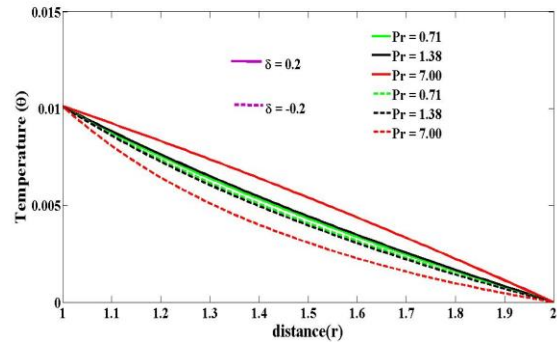


Figure 5: Effect of Prandtl number (Pr), on temperature (θ)

Figure 5 reveals that an increase in Prandtl number (Pr) increases the temperature of the fluid in the of injection ($\delta > 0$) and reversed effect is observed in the case of suction ($\delta < 0$).

4.3 Concentration profile

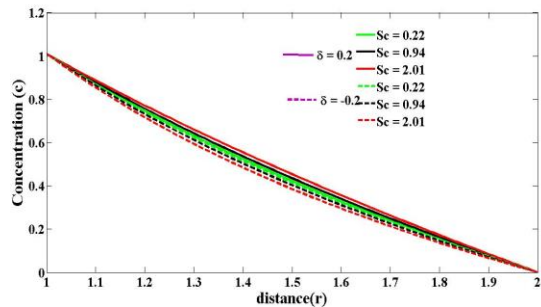


Figure 6: Effect of Schmidt number (Sc), on concentration (c)

It is noticed from Figure 6, that an increase in Schmidt number (Sc) increases the concentration of the fluid in case of injection ($\delta > 0$) while the reversed effect is seen for suction ($\delta < 0$).

0.71	5	3	1	0.22	0.2	1.8537	1.3867	1.4352
7	-	-	-	-	-	1.9654	0.8628	-
-	10	-	-	-	-	2.9788	-	-
-	-	6	-	-	-	2.5823	-	-
-	-	-	2	-	-	1.7715	-	-
-	-	-	-	0.94	-	2.0087	-	1.3644
-	-	-	-	-	0.4	1.8849	1.3185	1.4133

The comparison of variation of Skin friction (τ_0), Nusselt number (Nu_0) and Sherwood number (Sh_0) at $r = 1$ is shown in table 1. From the table it is observed that an increase in Prandtl number (Pr), thermal Grashof number (Gr), sustention parameter (N), porous parameter (δ) and Schmidt number (Sc) increases the rate of Skin friction (τ_0). While increase in the magnetic field parameter (M) decreases the rate of skin friction. Similarly, an increase in Prandtl number (Pr), Schmidt number (Sc) and porous parameter (δ) suffers the rate of heat transfer (Nu_0) and Sherwood number (Sh_0).

5.0 CONCLUSION

In this study, the governing partial differential equations in dimensionless forms of the fluid flow were transformed to ordinary differential equations and solved analytically using perturbation method. The effects of physical parameters associated with the problem were examined and their effects were shown graphically. The Skin friction, Nusselt number and Sherwood number were obtained.

From the study conducted, it is concluded that:

- i. An increase in thermal Grashof number (Gr), enhances the velocity, the velocity of fluid decreases in the case of cooling of the plate but the reversed effect is observed in the case of heating of the plate.
- ii. An increase in Prandtl number (Pr) increases the temperature of the fluid due to

injection ($\delta > 0$) and reversed effect is observed in the case of suction ($\delta < 0$). An increase in Schmidt number (Sc) decreases concentration of the fluid due to suction and contrast effect is observed due injection.

- iii. An increase in Prandtl number (Pr), thermal Grashof number (Gr), sustention parameter (N), porous parameter (δ) and Schmidt number (Sc) increases the rate of Skin friction (τ_0). While increase in the magnetic field parameter (M) decreases the rate of skin friction. Similarly, an increase in Prandtl number (Pr), Schmidt number (Sc) and porous parameter (δ) suffers the rate of heat transfer (Nu_0) and Sherwood number (Sh_0).

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