

Unsteady Casson Fluid Flow in a Superhydrophobic Microchannel in the Presence of Magnetic Field

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ABSTRACT

In this Study, fractional calculus is used to investigate unsteady MHD flow and heat transfer of a non-Newtonian Casson fluid in a superhydrophobic microchannel. Transient transport behavior is modeled using the Caputo–Fabrizio (CF) and Atangana–Baleanu (ABC) fractional derivatives, taking into account magnetic field effects. The results may contribute to better fluid control, less friction, increased pumping efficiency, and optimized performance in medication delivery, industrial processing, microcooling systems, and biomedical microfluidics. The Laplace transform approach is used to solve the resultant equations, and the Riemann-sum approximation (RSA) is used to get numerical solutions in the time domain. The study examines the impact of the Casson parameter, magnetic field parameter, Prandtl number, and fractional-order parameter on velocity, temperature distributions, skin friction, and Nusselt number. Key findings include that increasing the magnetic field parameter reduces fluid motion due to the Lorentz force, while the Casson parameter enhances velocity distribution. A higher Prandtl number corresponds to lower temperature and velocity distributions. The ABC formulation yields slightly higher values for velocity and temperature compared to the CF formulation, and the Nusselt number increases with both the Prandtl number and fractional-order parameter, whereas skin-friction magnitude tends to decrease with stronger magnetic fields. These results highlight the significance of magnetic-field control and fractional modeling in describing transitory Casson fluid movement.

Keywords: Atangana–Baleanu fractional derivative; Caputo–Fabrizio fractional derivative; Casson fluid; MHD; superhydrophobic microchannel; heat transfer

1.0 INTRODUCTION

Unsteady non-Newtonian fluid flows have attracted considerable research attention because of their importance in industrial processing, biomedical engineering, thermal management, coating technologies, and microfluidic systems. Non-Newtonian fluids demonstrate complex behavior unlike Newtonian fluids, featuring shear-dependent viscosity, yield stress, and memory effects that affect momentum and heat transfer. The Casson fluid model is particularly effective for yield-stress materials where deformation is reliant on applied shear stress. Thus, grasping the transient behavior of Casson fluids is crucial when flow conditions fluctuate or external forces are applied to manage fluid motion. In this context, Khan et al. (2016) examined the unsteady flow of a generalized Casson fluid caused by an oscillating infinite plate, using a Caputo time-fractional derivative. They employed the Laplace transform technique to derive exact expressions for velocity and shear stress through Wright functions, illustrating that their formulation could reproduce classical Casson and viscous-fluid models as specific instances. Sheikh et al. (2017) studied the differences between Atangana–Baleanu and Caputo–Fabrizio fractional derivatives in the context of a generalized Casson fluid under free convection with temperature and concentration gradients

and heat generation. Their analysis provided insights into the representation of transport processes in non-Newtonian fluids. Qayyum et al. (2022) later explored fractional unsteady squeezing flow in a Casson fluid using a hybrid Laplace transform–homotopy perturbation method, demonstrating that their solutions had greater accuracy compared to previous results. Nadeem et al. (2025) analyzed unsteady two-dimensional fractional Casson fluid flow, incorporating factors such as heated boundaries, oscillatory conditions, magnetic fields, porous media, and heat transfer. They utilized Fourier transforms to derive exact solutions that reflect the effects of fractional memory and electromagnetic forces. Their work highlights the relevance of fractional calculus in understanding Casson fluids, while emphasizing the importance of classical formulations for baseline comparisons. Raghunatha et al. (2023) explored Casson fluid flow in a Darcy porous medium influenced by a magnetic field and internal heat generation, applying the Bernoulli wavelet method to show how magnetic and thermal factors affect flow dynamics. Zhou et al. (2021) studied the unsteady radiative stagnation-point flow of an MHD Casson fluid over a permeable stretching surface with a non-uniform heat source, wall suction, and slip velocity, using MATLAB's `bvp4c` solver to analyze parameters affecting velocity, temperature, skin friction,

and Nusselt number. Ali et al. (2023) explored similar unsteady MHD Casson fluid flow incorporating thermal radiation and a heat sink through analytical solutions via Laplace transforms to assess velocity, temperature, concentration, and heat-transfer characteristics. Rasheed et al. (2022) extended this analysis by numerically investigating MHD Casson fluid flow over a vertical surface, factoring in thermal radiation, chemical reactions, viscous dissipation, and Hall current, linking momentum, heat, and mass transport. Anwar et al. (2021) examined MHD natural convection of a Casson fluid in porous materials under radiative heat transfer and variable wall conditions, deriving skin friction and Nusselt number expressions through Laplace transformation. Mandal and Layek (2021) explored mixed-convective MHD Casson flow over a flat surface, revealing non-monotonic velocity profiles and overshooting due to changes in stress response. Das et al. (2020) examined unsteady radiative, chemically reactive Casson fluid flow over a porous stretching sheet, using a fourth-order Runge-Kutta method. Hady et al. (2019) investigated natural convection in a dusty Casson fluid along a vertical wavy plate, finding that increased plate amplitude reduced fluid velocities and increased temperatures, highlighting the impact of fluid-particle interactions on heat transfer. Abdel-Gawad (2026) studied steady and unsteady Casson fluid flow and heat transfer over a free stretching surface, noting distinct steady-flow branches and decay of velocity components with time. Collectively, these studies indicate that factors like magnetic fields, thermal forcing, and rheological properties significantly affect Casson fluid transport, though most focus on conventional configurations rather than the interaction of unsteady Casson rheology, magnetic forces, and superhydrophobic conditions in confined microchannels.

Superhydrophobic microchannels are effective for fluid transport, reducing resistance, and enhancing thermal performance through water-repellent surfaces created by micro- or nanoscale texturing. Enright et al. (2010) derived a general expression for the Nusselt number in heat transfer analyses within superhydrophobic Hele-Shaw microchannels, exploring their thermal behavior. Patlazhan and Vagner (2017) demonstrated that shear-dependent viscosity leads to nonlinear velocity profiles in shear-thinning fluids, with significant apparent slip lengths. Gaddam et al. (2021) emphasized the role of non-Newtonian fluid behavior on drag reduction in textured microchannels. Jha and Gwandu (2021) studied magnetohydrodynamic flow in porous superhydrophobic microchannels, discovering that magnetic parameters reduce skin friction and that suction or injection influences fluid temperature differently based on wall

configurations. Hamza et al. (2023) examined thermal radiation in hydromagnetic free convection through a vertical porous superhydrophobic microchannel, considering alternative wall-heating arrangements and suction or injection. Their findings extended the analysis of coupled magnetic, radiative, and interfacial effects in confined flows. Ojmeri and Onwubuya (2023) investigated viscous dissipation and porosity in a heated superhydrophobic microchannel, applying a perturbation technique to the coupled nonlinear equations and accounting for slip and temperature-jump conditions at one wall. Similarly, Yale et al. (2023) studied MHD free convection with viscous dissipation in a slit microchannel with a heated superhydrophobic surface and a conventional opposing wall, using perturbation analysis to determine the effects of interfacial conditions on the flow and thermal fields. Ojmeri et al. (2024) subsequently considered Arrhenius reaction kinetics and heat-source/sink effects in a hydromagnetic flow along a heated superhydrophobic microchannel, employing regular perturbation methods to analyse the nonlinear governing equations. Hamza et al. (2024) explored time-dependent mixed convection in a vertical superhydrophobic microchannel under electroosmotic flow and temperature-jump conditions, emphasizing the importance of electrokinetic effects on thermal transport. In their study Hamza et al. (2025a,b), they studied a magnetized fluid with a heat-source term in a vertical microchannel, comparing superhydrophobic and no-slip walls. They also analyzed transient fractional dusty-fluid flow in superhydrophobic microchannels, incorporating memory effects under MHD conditions, showcasing fractional modeling's relevance. Additionally, Bai et al. (2023) created an open superhydrophilic microchannel heat sink with high thermal performance and low energy needs. Cheng et al. (2025) investigated flow boiling in superhydrophobic microchannels, finding that increased voltage enhanced vapor-liquid interface oscillations and improved cooling. Mulazim et al. (2026) utilized Grey Relational Analysis to enhance heat transfer in radiative MHD flow through superhydrophobic microchannels, highlighting the significance of optimization for thermally efficient microfluidic systems. Key factors affecting microchannel flow and heat transfer include surface wettability, apparent slip, temperature jumps, magnetic forces, thermal radiation, viscous dissipation, porosity, and heat-source characteristics. However, most research focuses on Newtonian fluids, shear-thinning liquids, dusty fluids, or Casson fluids, rather than superhydrophobic microchannels.

The study investigates the unsteady flow of Casson fluids in a superhydrophobic microchannel under the influence of an external magnetic field. It focuses on how the

Casson yield stress, magnetic interactions, temporal variations, and superhydrophobic wall slip affect velocity fields, wall shear characteristics, thermal fields, and heat-transfer rates. This addresses a gap in understanding these combined effects in confined systems. The resulting analysis will provide a theoretical basis for understanding the coupled effects of non-Newtonian rheology and engineered interfacial slip in magnetically controlled microfluidic systems, with potential relevance to compact thermal-management devices and other applications involving electrically conducting yield-stress fluids.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

2.1 Physical description of the model

unsteady, laminar flow of an incompressible, electrically conducting Casson fluid in a narrow superhydrophobic microchannel influenced by a uniform transverse

magnetic field B_0 has been Considered. The fluid is situated between two infinite parallel plates separated by a distance h . The velocity field is unidirectional, expressed as $\mathbf{V} = (u(y, t), 0, 0)$, with the x – axis along the channel and the y – axis perpendicular to the plates. Where $u(y, t)$ denotes the axial velocity, y is the transverse coordinate, and t is time. The temperature distribution is represented by $T(y, t)$. Casson fluid rheology, heat diffusion, transient momentum diffusion, and magnetic resistance together govern flow characteristics. Velocity-slip boundary conditions representing superhydrophobic wall effects are essential. Instead of traditional time derivatives, the model employs Caputo–Fabrizio and Atangana–Baleanu fractional derivatives to incorporate memory-dependent transport.

With the above description and assumption the following model was formulate

$$\frac{\partial u}{\partial t} = \nu \left(1 + \frac{1}{\xi} \right) \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) - \frac{\sigma B_0^2 \mu}{\rho} u \quad (1)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \quad (2)$$

The boundary conditions are:

$$\left. \begin{aligned} t \leq 0 : u = 0, \quad T = 0, \quad 0 < y < L \\ t > 0 : u = \lambda' \frac{\partial u}{\partial y}, \quad T = T_0 + \gamma' \frac{\partial T}{\partial y}, \quad y = 0 \\ u = 0, \quad T = T_0, \quad y = L \end{aligned} \right\} \quad (3)$$

The appropriate dimensional quantities and parameters are:

$$\left. \begin{aligned} Y = \frac{y}{L}, \quad \lambda = \frac{\lambda'}{L}, \quad \gamma = \frac{\gamma'}{L}, \quad t = \frac{t' \nu}{L^2}, \quad U = \frac{u}{U_0}, \quad M^2 = \frac{\sigma B_0^2}{\nu \rho} \\ \theta = \frac{T - T_0}{T_h - T_0}, \quad U_0 = \frac{g\beta L^2 (T_h - T_0)}{\nu}, \quad \text{Pr} = \frac{\nu \rho C_p}{k} \end{aligned} \right\} \quad (4)$$

Then we have

$$\frac{\partial U}{\partial t} = \xi_1 \frac{\partial^2 U}{\partial Y^2} + \theta - MU \quad (5)$$

$$\text{Pr} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial Y^2} \quad (6)$$

$$\left. \begin{aligned} u = \lambda \frac{\partial U}{\partial Y}, \quad \theta = 1 + \gamma \frac{\partial \theta}{\partial Y}, \quad \text{at } Y = 0 \\ u = 0, \quad \theta = 0, \quad \text{at } Y = 1 \end{aligned} \right\} \quad (7)$$

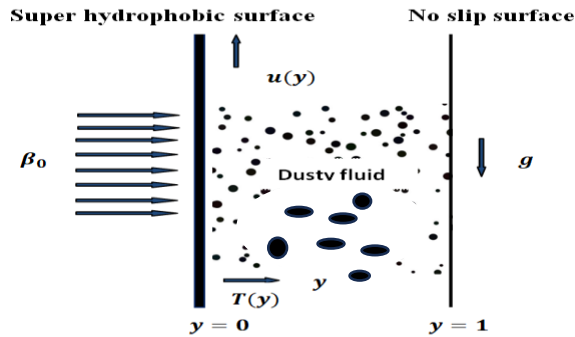


Fig.1: Geometry of the problem

where $\xi_1 = 1 + \frac{1}{\xi}$

To solve equations (5) and (6) subject to boundary conditions (7), we introduced the CF time fractional derivatives to equations (5) and (6) and solved them together using the Laplace transform method as follows:

$$\frac{\partial^\alpha U}{\partial t^\alpha} = \xi_1 \frac{\partial^2 U}{\partial Y^2} + \theta - MU \quad (8)$$

$$\text{Pr} \frac{\partial^\alpha \theta}{\partial t^\alpha} = \frac{\partial^2 \theta}{\partial Y^2} \quad (9)$$

$$\left. \begin{aligned} u &= \lambda \frac{\partial U}{\partial Y}, \quad \theta = 1 + \gamma \frac{\partial \theta}{\partial Y}, \quad \text{at } Y = 0 \\ u &= 0, \quad \theta = 0, \quad \text{at } Y = 1 \end{aligned} \right\} \quad (10)$$

2.2 Caputo-Fabrizio Fractional Derivative Approach

The Laplace transform of a function $W(y, t)$ is:

$$L[W(y, t)] = \int_0^\infty W(y, t) \cdot \exp(-st) dt, \quad \bar{W}(y, t) = \int_0^\infty W(y, t) \cdot \exp(-st) dt, \quad s > 0. \quad (11)$$

The CF model in continuous function $W(y, t)$ is

$${}^{CF}D_t^\alpha W(y, t) = \frac{1}{(1-\alpha)} \int_0^t \exp\left(-\alpha \left(\frac{t-n}{1-\alpha}\right)\right) \frac{\partial J(y, n)}{\partial n} dn, \quad \alpha \in [0, 1]. \quad (12)$$

The CF model in Laplace domain is

$$L[D_t^\alpha W(y, t)] = \frac{sL[W(y, t)] - W(y, 0)}{(1-\alpha)s + \alpha}, \quad (13)$$

Employing CF model Laplace transform

$$\bar{\theta}_{CF} = \frac{1}{SK_1} \left[K_2 \cosh(Y\sqrt{\text{Pr}C_1}) - K_3 \sinh(Y\sqrt{\text{Pr}C_1}) \right] \quad (14)$$

$$\bar{U}_{CF} = \frac{1}{S} \left[K_{10} \cosh(C_2 Y) + K_{11} \sinh(C_2 Y) + K_{12} \sinh(Y\sqrt{\text{Pr}C_1}) + K_{13} \cosh(Y\sqrt{\text{Pr}C_1}) \right] \quad (15)$$

The rate of heat transfer at lower plate, upper plate and Nusselt numbers are as follows:

$$q_{w0CF} = \left. \frac{d\bar{\theta}_{CF}}{dY} \right|_{y=0} = \frac{1}{SK_1} \left[-\sqrt{\text{Pr}C_1} K_3 \right] \quad (16)$$

$$q_{w1CF} = \left. \frac{d\bar{\theta}_{CF}}{dY} \right|_{y=1} = \frac{1}{SK_1} \left[K_2 \sqrt{\text{Pr } C_1} \sinh(C_2 Y) - K_3 \sqrt{\text{Pr } C_1} \cosh(C_2 Y) \right] \quad (17)$$

The skin friction at both lower and upper plates are as follows:

$$\left. \frac{d\bar{U}_{CF}}{dY} \right|_{y=0} = \frac{1}{S} \left[C_2 K_{11} + \sqrt{\text{Pr } C_1} K_{12} \right] \quad (18)$$

$$\left. \frac{d\bar{U}_{CF}}{dY} \right|_{y=1} = \frac{1}{S} \left[C_2 K_{10} \sinh(C_2) + C_2 K_{11} \cosh(C_2) + \sqrt{\text{Pr } C_1} K_{12} \cosh(\sqrt{\text{Pr } C_1}) + \sqrt{\text{Pr } C_1} K_{13} \sinh(\sqrt{\text{Pr } C_1}) \right] \quad (19)$$

2.3 Atangana Baleanu Caputo-Fabrizio Fractional Derivative Approach

The ABC model in continuous function $J(y, t)$ is

$${}^{ABC}D_t^\alpha W(y, t) = \frac{n(\alpha)}{(1-\alpha)} \int_0^t E_\alpha \exp\left(-\alpha \left(\frac{t-n}{1-\alpha}\right)^\alpha\right) \frac{\partial J(y, n)}{\partial n} dn, \quad \alpha \in [0, 1]. \quad (20)$$

The ABC model in Laplace transform domain is

$$L[D_t^\alpha J(y, t)] = \frac{s^\alpha L[J(y, t)] - s^\alpha J(y, 0)}{(1-\alpha)s^\alpha + \alpha}, \quad (21)$$

Using (21), the solution of equations (8) to (9) subject to (10) can be expressed as:

$$\bar{\theta}_{ABC} = \frac{1}{Sk_1} \left[k_2 \cosh(Y \sqrt{\text{Pr } C_{11}}) - k_3 \sinh(Y \sqrt{\text{Pr } C_{11}}) \right] \quad (22)$$

$$\bar{U}_{ABC} = \frac{1}{S} \left[k_{10} \cosh(C_{22} Y) + k_{11} \sinh(C_{22} Y) + k_{12} \sinh(Y \sqrt{\text{Pr } C_{11}}) + k_{13} \cosh(Y \sqrt{\text{Pr } C_{11}}) \right] \quad (23)$$

The rate of heat transfer at lower plate, upper plate and Nusselt numbers are as follows:

$$q_{w0ABC} = \left. \frac{d\bar{\theta}_{ABC}}{dY} \right|_{y=0} = \frac{1}{Sk_1} \left[-\sqrt{\text{Pr } C_{11}} k_3 \right] \quad (24)$$

$$q_{w1ABC} = \left. \frac{d\bar{\theta}_{ABC}}{dY} \right|_{y=1} = \frac{1}{Sk_1} \left[k_2 \sqrt{\text{Pr } C_{11}} \sinh(\sqrt{\text{Pr } C_{11}}) - k_3 \sqrt{\text{Pr } C_{11}} \cosh(\sqrt{\text{Pr } C_{11}}) \right] \quad (25)$$

The skin friction at both lower and upper plates are as follows:

$$\left. \frac{d\bar{U}_{ABC}}{dY} \right|_{y=0} = \frac{1}{S} \left[C_{22} k_{11} + \sqrt{\text{Pr } C_{11}} k_{12} \right] \quad (26)$$

$$\left. \frac{d\bar{U}_{ABC}}{dY} \right|_{y=1} = \frac{1}{S} \left[C_{22} k_{10} \sinh(C_{22}) + C_{22} k_{11} \cosh(C_{22}) + \sqrt{\text{Pr } C_{11}} k_{12} \cosh(\sqrt{\text{Pr } C_{11}}) + \sqrt{\text{Pr } C_{11}} k_{13} \sinh(\sqrt{\text{Pr } C_{11}}) \right] \quad (27)$$

The above solutions in two approaches are reported in the Laplace domain. Several challenges were encountered while inverting solutions to the time domain; this study employs a numerical scheme utilized by Jha and Apere (2010), called Riemann sum approximation (RSA). The formula is

$$L^{-1}[\bar{W}(y, t)] = W(y, t) = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} \bar{W}(y, \varepsilon) + \text{Re} \sum_{n=1}^E \bar{W} \left(y, \varepsilon + \frac{in\pi}{t} \right) (-1)^n \right]. \quad (28)$$

Where $i = \sqrt{-1}$. The detailed explanation of the formula's accuracy and its working is documented by Jha and Apere (2010).

3.0 RESULTS AND DISCUSSION

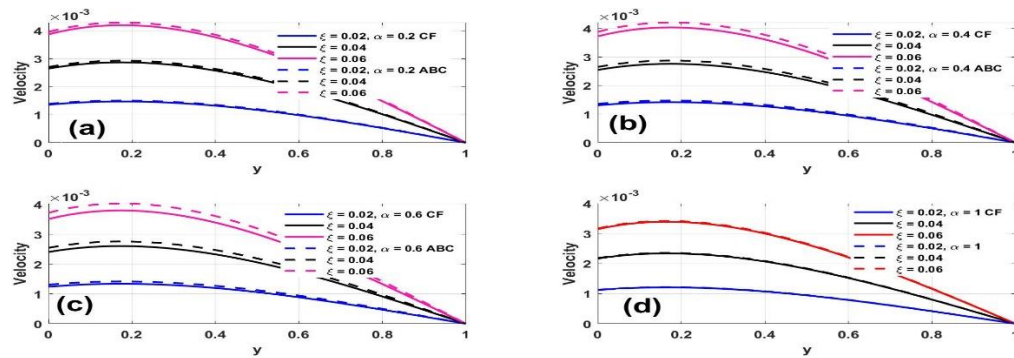


Fig. 2: Influence of Casson parameter (ξ) for varying values of fractional derivative parameter (α)

Using the Caputo–Fabrizio (CF) and Atangana–Baleanu (ABC) model, Figures 2(a–d) show how the Casson parameter (ξ) affects the velocity distribution for various values of the fractional derivative parameter (α). The findings show that velocity rises as the ξ increases, indicating an improvement in the flow velocity within the parameter range. In accordance with the specified boundary condition, the velocity profiles in each panel reach a maximum within the fluid domain before declining toward zero at $y=1$. Although the differences are still negligible, the ABC model predicts

generally slightly larger velocities than the CF model, especially around the peak area. Also, the value of velocity is decreased generally with the increase of α which implies the effect of fractional-order on momentum distribution. This restricted example demonstrates a good agreement between the two models, since the CF and ABC profiles almost coincide at $\alpha=1.0$. Therefore, the results show that the Casson parameter and the fractional derivative formulation affect the velocity field simultaneously, but the strength of these effects depends on the values of the parameters considered.

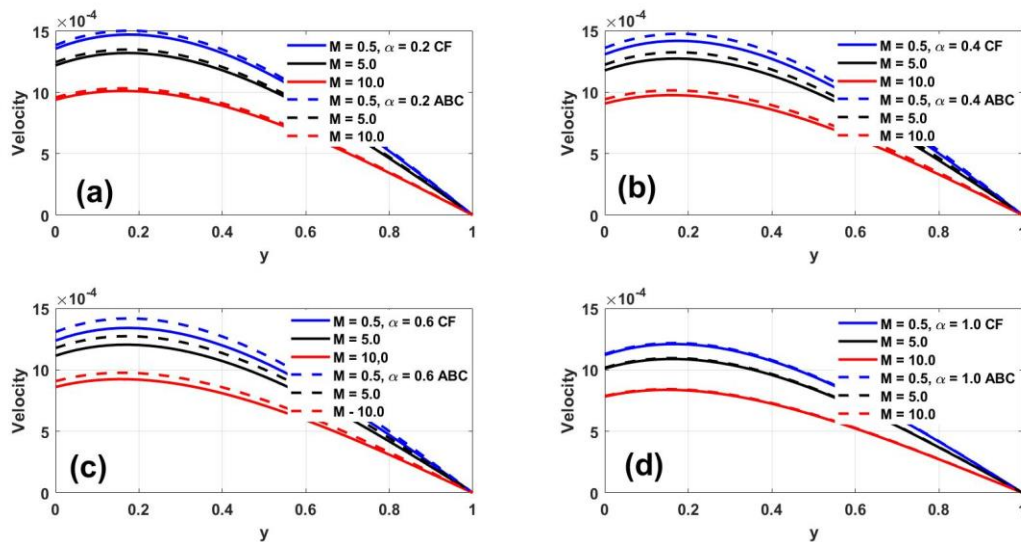


Fig. 3: Influence of Magnetic field parameter (M) on velocity for varying values of fractional derivative parameter (α)

The effects of the magnetic field parameter (M) on the velocity distribution under various values of the fractional derivative parameter for the CF and ABC models are shown in Figures 3(a–d). With the smallest magnetic field creating the highest velocity and the strongest magnetic field producing the lowest, the findings consistently demonstrate that increasing M

decreases the fluid velocity. The opposing Lorentz force produced by the interaction between the electrically conducting fluid and the magnetic field, which adds resistance to fluid motion, is physically responsible for this behavior. At $y=1$, the velocity profiles decrease toward zero while maintaining their distinctive interior maxima. Both fractional forms exhibit the same magnetic

damping behavior, but the ABC model generally predicts much larger velocities than the CF model, particularly in the vicinity of the peak region. For $\alpha = 1.0$, the differences between the two models are insignificant, indicating that the two formulations approach similar velocity estimates

at this limiting condition. These results demonstrate the additional impact of fractional order modeling on the mean hydrodynamic behavior and establish the magnetic field as an effective variable to control the fluid flow.

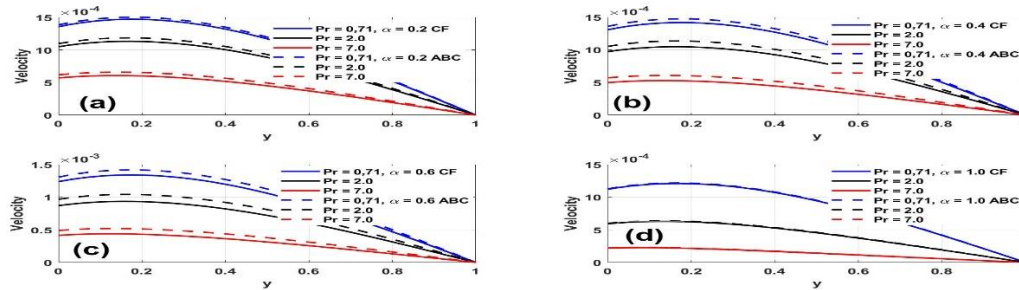


Fig. 4: Influence of Prandtl number (Pr) on velocity for varying values of fractional derivative parameter (α)

More also, Figures 4(a–d) show how the Prandtl number (Pr) affects the velocity profiles for various values of α , the findings demonstrate that raising the Prandtl number often leads in a drop in the velocity magnitude. This pattern suggests that the flow response is influenced by changes in the relative rates of thermal diffusion and momentum, as temperature-dependent buoyant forces are included in the momentum, the observed drop in velocity may be attributed to the decrease in thermal penetration caused by an increase in Pr. However, is contingent upon the particular terms that are part of the governing

momentum. Due to the choice of fractional derivative operator, the ABC model often yields somewhat higher velocity numbers than the CF model, suggesting subtle variations in the expected flow behavior. The two sets of profiles almost overlap at $\alpha = 1.0$, indicating that both models produce predictions that are similar in this limit. The findings show that the coupled thermal and momentum response of the fluid depends on the Prandtl number and fractional derivative parameter.

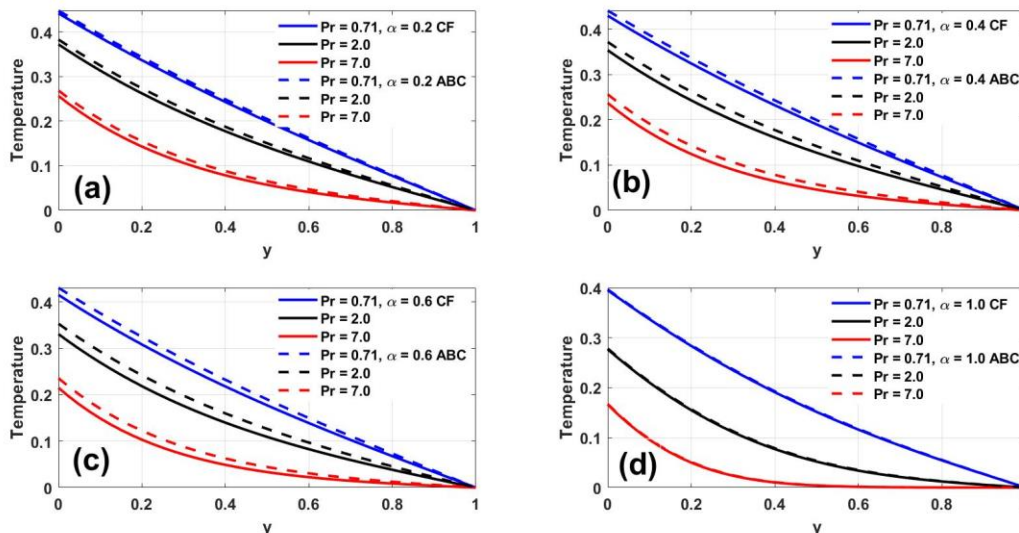


Fig. 5: Influence of Prandtl number (Pr) on temperature for varying values of fractional derivative parameter (α)

The results from Figures 5(a–d) show how the temperature distribution is affected by the Prandtl number (Pr) for various values of α . According to the specified thermal boundary condition, The findings also show that raising the Prandtl number lowers the temperature across the fluid domain. Physically, this

behavior is linked to the fact that when Pr grows, thermal diffusivity decreases in relation to momentum diffusivity, which limits heat penetration into the fluid. In general, the ABC model predicts somewhat higher temperatures than the CF model, indicating that the two fractional operators capture different aspects of the

thermal response. Additionally, changes in the α alter the temperature distribution, suggesting that the expected heat-transfer behavior is influenced by the representation of memory effects. The CF and ABC profiles almost coincide at $\alpha = 1.0$, indicating strong agreement

between the models under this limiting condition. These results demonstrate the joint significance of the fractional derivative model and the Prandtl number in characterizing temperature-field evolution and thermal transport.

Table 1: Numerical values of skin friction for different values of Casson parameter (ξ)

ξ	α	τ_{CF_0}	τ_{CF_1}	τ_{ABCF_0}	τ_{ABCF_1}
0.2	0.2	0.0119	-0.0233	0.0115	-0.0225
0.4	0.2	0.0198	-0.0387	0.0191	-0.0372
0.6	0.2	0.0254	-0.0497	0.0245	-0.0476
0.2	0.4	0.0125	-0.0246	0.0118	-0.0231
0.4	0.4	0.0209	-0.0412	0.0197	-0.0385
0.6	0.4	0.0270	-0.0532	0.0253	-0.0494
0.2	0.6	0.0131	-0.0261	0.0123	-0.0244
0.4	0.6	0.0222	-0.0441	0.0207	-0.0407
0.6	0.6	0.0288	-0.0571	0.0267	-0.0525
0.2	1.0	0.0138	-0.0275	0.0137	-0.0274
0.4	1.0	0.0234	-0.0468	0.0234	-0.0467
0.6	1.0	0.0306	-0.0610	0.0305	-0.0609

Table 2: Numerical values of skin friction for different values of Magnetic parameter (M)

M	α	τ_{CF_0}	τ_{CF_1}	τ_{ABCF_0}	τ_{ABCF_1}
0.5	0.2	0.0119	-0.0233	0.0115	-0.0225
2.0	0.2	0.0104	-0.0201	0.0101	-0.0194
5.0	0.2	0.0063	-0.0112	0.0061	-0.0108
0.5	0.4	0.0125	-0.0246	0.0118	-0.0231
2.0	0.4	0.0109	-0.0212	0.0104	-0.0200
5.0	0.4	0.0055	-0.0118	0.0062	-0.0111
0.5	0.6	0.0131	-0.0261	0.0123	-0.0244
2.0	0.6	0.0115	-0.0225	0.0108	-0.0210
5.0	0.6	0.0069	-0.0125	0.0065	-0.0117
0.5	1.0	0.0138	-0.0275	0.0137	-0.0275
2.0	1.0	0.0120	-0.0237	0.0120	-0.0237
5.0	1.0	0.0071	-0.0131	0.0071	-0.0131

Table 3: Skin friction and Nusselt number for different values of Prandtl number (S) and fractional parameter (α) in case of suction and injection

Pr	α	(τ_{0CF})	$-(\tau_{1CF})$	(τ_{0ABC})	$-(\tau_{1ABC})$	(Nu_{0CF})	$-(Nu_{1CF})$	(Nu_{0ABC})	(Nu_{1ABC})
0.71	0.2	0.0119	0.0233	0.0115	0.0225	0.0684	0.4199	0.0705	0.4045
2.0	0.2	0.0098	0.0188	0.0092	0.0176	0.1522	0.3139	0.3547	0.3139
7.0	0.2	0.0059	0.0104	0.0053	0.0093	0.7381	0.1281	0.7398	0.1281
0.71	0.4	0.0125	0.0246	0.0118	0.0231	0.6594	0.4455	1.0417	0.4455
2.0	0.4	0.0107	0.0207	0.0097	0.0186	1.3064	0.3535	3.8380	0.3535
7.0	0.4	0.0067	0.0120	0.0058	0.0103	2.7529	0.1556	2.2529	0.1556
0.71	0.6	0.0131	0.0261	0.0123	0.0244	1.8363	0.4740	1.8363	0.4740
2.0	0.6	0.0117	0.0230	0.0106	0.0205	3.0820	0.4000	3.0820	0.4000
7.0	0.6	0.0076	0.0138	0.0066	0.0119	5.3484	0.1862	5.3884	0.1862
0.71	1.0	0.0138	0.0275	0.0137	0.0274	6.5103	0.5000	6.5103	0.5000

2.0	1.0	0.0135	0.0269	0.0134	0.0268	7.5494	0.4867	7.5494	0.4869
7.0	1.0	0.0096	0.0180	0.0095	0.0179	8.7695	0.2697	8.7695	0.2695

Table 1 shows an improved wall-shear response within the studied range, with the skin friction magnitude typically increasing with the ξ and α . Changes in the fluid's rheological properties and the ensuing velocity gradient at the wall are reflected in this behavior. Under the parameters taken into consideration, the estimated skin friction increases as α approaches unity, suggesting a greater wall-shear response in the classical-order limit. The different fractional derivative operators of the Caputo–Fabrizio (CF) and Atangana–Baleanu (ABC) models may be the cause of their differences. The direction of wall shear and the chosen sign convention are reflected in the negative skin-friction values. Although the observed trends depend on the constitutive model and boundary conditions, the results also demonstrate the joint influence of fractional-order dynamics and Casson fluid properties on the wall-shear characteristics. According to result in Table 2, the magnitude of skin friction generally decreases across the fractional derivative parameter taken into consideration when the magnetic field parameter (M) is increased. This behavior is linked to the opposite Lorentz force, which affects the wall-shear stress by changing the velocity distribution and its gradient close to the wall. On the other hand, the reported skin-friction values often increase when the α is increased, however the strength of the magnetic field and the fractional formulation determine how much of a response there is. Consistency between the two models in the classical-order limit is suggested by the close agreement between the CF and ABC results at $\alpha = 1.0$. These results show that the wall-shear properties of Casson fluid flow are concurrently influenced by fractional-order effects and magnetic field strength, with the magnetic field acting as a mechanism to control the hydrodynamic response. While Table 3 demonstrates that the Nusselt number rises with Pr and α , suggesting an improved heat-transfer rate and wall temperature gradient within the studied range. A steeper temperature gradient close to the wall may result from decreased thermal diffusivity, which is consistent with an increase in the Nusselt number with the Prandtl number. In a similar vein, when the model gets closer to the classical-order limit, the increase in α indicates an improved computed heat-transfer response. While injection may reduce heat transfer by encouraging thermal-layer thickening, wall suction may further improve heat transfer by decreasing the thermal boundary-layer thickness; However, the results shows that skin friction often increases with α but reduces with rising Pr. these highlight how the momentum and heat-

transfer properties of Casson fluid flow are influenced by thermal diffusion, fractional-order dynamics, and wall mass transfer.

4.0 CONCLUSION

This study investigated unsteady magnetohydrodynamic (MHD) flow and heat transfer of a Casson fluid in a superhydrophobic microchannel using the Caputo–Fabrizio (CF) and Atangana–Baleanu fractional derivative in the Caputo sense (ABC) formulations. The governing equations were solved analytically in the Laplace-transform domain, while the Riemann-sum approximation was used to obtain the time-domain solutions. The effects of the Casson parameter, magnetic field strength, Prandtl number, and fractional-order parameter on velocity, temperature, skin friction, and the Nusselt number were systematically examined.

The principal conclusions of the study are as follows:

- Increasing the Casson parameter enhances fluid velocity and generally increases the magnitude of skin friction.
- Increasing magnetic field strength reduces velocity and skin-friction magnitude due to the opposing Lorentz force.
- Higher Prandtl numbers reduce velocity and temperature while increasing the Nusselt number and wall heat-transfer rate.
- The fractional-order parameter influences momentum and thermal transport, with the Nusselt number generally increasing as the order approaches unity.
- The ABC model generally predicts slightly higher velocity and temperature than the CF model, while both converge closely in the classical-order limit.
- The findings provide a theoretical basis for understanding and optimising fluid transport and heat transfer in superhydrophobic microchannels.

Therefore, this study explores fractional calculus for modeling unsteady MHD Casson fluid flow and heat transfer in superhydrophobic microchannels. The findings are intended to support the theoretical design and optimization of microfluidic systems for biomedical applications and thermal management. Future research should investigate varied wall-slip conditions, asymmetric thermal boundaries, and conduct experimental validation to assess the model's practical use.

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Constants

$$C_1 = \frac{s}{[(1-\alpha)s + \alpha]}, \quad C_{11} = \frac{s^\alpha}{[(1-\alpha)s^\alpha + \alpha]}$$

$$C_2 = \sqrt{\frac{M^2 + C_1}{\xi_1}}, \quad C_{22} = \sqrt{\frac{M^2 + C_{11}}{\xi_1}}, \quad K_1 = \gamma \sqrt{\text{Pr } C_1} \cosh(\sqrt{\text{Pr } C_1}) + \sinh(\sqrt{\text{Pr } C_1}),$$

$$k_1 = \gamma \sqrt{\text{Pr } C_{11}} \cosh(\sqrt{\text{Pr } C_{11}}) + \sinh(\sqrt{\text{Pr } C_{11}})$$

$$K_2 = K_1 - \gamma \sqrt{\text{Pr } C_1} \cosh(\sqrt{\text{Pr } C_1}),$$

$$k_2 = k_1 - \gamma \sqrt{\text{Pr } C_{11}} \cosh(\sqrt{\text{Pr } C_{11}}), \quad \cosh(\sqrt{\text{Pr } C_1}) = K_3, \quad \cosh(\sqrt{\text{Pr } C_{11}}) = k_3, \quad K_4 = \xi_1 K_1 [\text{Pr } C_1 - C_2^2],$$

$$k_4 = \xi_1 k_1 [\text{Pr } C_{11} - C_{22}^2], \quad K_5 = \frac{\cosh(\sqrt{\text{Pr } C_1})}{\cosh(C_2)}, \quad K_6 = \frac{\sinh(\sqrt{\text{Pr } C_1})}{\cosh(C_2)}, \quad K_7 = \frac{\sinh(C_2)}{\cosh(C_2)},$$

$$k_5 = \frac{\cosh(\sqrt{\text{Pr } C_{11}})}{\cosh(C_{22})}, \quad k_6 = \frac{\sinh(\sqrt{\text{Pr } C_{11}})}{\cosh(C_{22})}, \quad k_7 = \frac{\sinh(C_{22})}{\cosh(C_{22})}, \quad K_8 = \frac{K_2 K_5 - K_3 K_6 - K_2 - \lambda(\sqrt{\text{Pr } C_1}) K_3}{K_4},$$

$$k_8 = \frac{k_2 k_5 - k_3 k_6 - k_2 - \lambda \left(\sqrt{\text{Pr } C_{11}} \right) k_3}{k_4} \quad K_9 = \lambda C_2 + K_7, k_9 = \lambda C_{22} + k_7$$

$$K_{10} = \frac{\lambda C_2 K_8}{K_9} + \frac{K_2}{K_4} + \frac{\lambda \left(\sqrt{\text{Pr } C_1} \right) K_3}{K_4}, k_{10} = \frac{\lambda C_{22} k_8}{k_9} + \frac{k_2}{k_4} + \frac{\lambda \left(\sqrt{\text{Pr } C_{11}} \right) k_3}{k_4}$$

$$K_{11} = \frac{K_8}{K_9}, K_{12} = \frac{K_3}{K_4}, K_{13} = \frac{-K_2}{K_4}, k_{11} = \frac{k_8}{k_9}, k_{12} = \frac{k_3}{k_4}, k_{13} = \frac{-k_2}{k_4}$$