

Role of Heat and Mass Transfer on Time Dependent Natural Convection Flow in a Vertical Channel with Adiabatic and Isoflux Thermal Boundary Conditions

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ABSTRACT

This study investigates transient natural-convection flow with coupled heat and mass transfer in a vertical porous channel subject to adiabatic and isoflux thermal boundary conditions. A mathematical model is formulated to describe the evolution of the velocity, temperature, and concentration fields for a two-dimensional, incompressible, laminar flow through a homogeneous and isotropic porous medium. The governing continuity, momentum, energy, and concentration equations are nondimensionalized and solved analytically using the Laplace transformation technique. The resulting solutions are inverted through the Riemann-Sum Approximation (RSA), while MATLAB PDEPE is employed to validate the analytical results. The effects of thermal and solutal buoyancy, heat generation or absorption, and wall suction/injection on the transport characteristics are examined. The results indicate that increasing thermal and solutal buoyancy enhances the fluid velocity and strengthens the associated heat and mass transfer. Higher Prandtl and Schmidt numbers reduce the temperature and concentration distributions, respectively, leading to thinner thermal and solutal boundary layers. Heat generation increases the fluid temperature and wall heat-transfer rate, whereas heat absorption produces the reverse effect. Suction suppresses the velocity, mass flow, and mass-transfer characteristics, while injection enhances them. The skin-friction coefficient increases with buoyancy forces and injection but decreases with suction. The analytical solutions obtained using the Laplace-transform and RSA procedures show close agreement with the numerical results from MATLAB PDEPE. The study demonstrates that buoyancy, thermal transport, and mass diffusion significantly influence transient natural-convection behaviour in porous channels and provides useful insights into coupled heat and mass transfer under adiabatic and isoflux boundary conditions.

Keywords: Natural convection, fluid flow, vertical channel, isothermal or isoflux boundary conditions, heat and mass transfer

1.0 INTRODUCTION

The study of natural convection developing in vertical open channels is of major importance since it is found in many applications of different scales, from cooling micro-electronic components to cool nuclear reactors, in solar systems and in Trombe walls. In the medium term the application benefits of our studies will be dedicated to the design of photovoltaic panels integrated in double-facades of buildings to exploit their thermal operating characteristics. It is necessary to investigate both the heat transfers at the rear side of heated plates (PV panels) and the chimney effect in asymmetrically heated channels (Sanvicente *et al.*, 2013). Jha and Oni (2019) investigated the role of heat generation/absorption on natural convection flow formation in a vertical annulus with prescribed time-periodic heating. Usman *et al.* (2022) analysed the effect of suction/injection on free convective flow past a vertical porous plate under the action of thermal radiation and chemical reaction when heat is supplied to the plate at constant rate. Omokhuale and Dange (2023) considered the natural convection

Couette flow in a hydrodynamic viscous fluid that is electrically conductive due to the thermal radiation effect.

Natural convection flows occur widely in industrial and environmental applications. Examples are ventilation in buildings, passive cooling in nuclear reactors and melting ice shelves. A canonical model considers fluid in a cavity subjected to differential heating via isothermal or isoflux boundary conditions on the vertical walls. The isothermal case has sidewalls subjected to sudden constant heating and cooling by the same amount. A rising/falling boundary layer is produced adjacent to the heated/cooled wall. In the cavity interior, a cavity-scale circulation and a linear thermal stratification are generated (Zhou *et al.*, 2021). Taiwo *et al.* (2020) examined the influence of isothermal and isoflux heating/cooling with heat source/sink on unsteady hydromagnetic-free convective flow of a viscous incompressible fluid in an annulus. The flow is induced by buoyancy forces due to temperature differences as a result of the isothermal heating and

constant heat flux applied on the outer surface of the inner cylinder. Isah *et al.* (2024) provided a theoretical exploration was reported to highlight the electrically viscous conducting fluid within a coaxial cylinder. Both the outer surface of the inner cylinder and the inner surface of the outer cylinder were porous, heated, and cool, respectively.

In daily life applications convection of heat-mass transfer analysis play a very important role. In the last few years natural convection flow theory highly developed and became the most rapidly established research field. A lot of phenomena's relatable to convection flow over infinite vertical plate are studied in literature. In practical life heat and mass transfer procedure appears too much, such as chemical reaction evaporation, and also in condensation. Heat and mass transfer have also industrialized applications like the buoyancy effects of diffusion of chemical reactions and thermal diffusion; this is generated by thermal conduction and mass transfer analysis. Many researchers have been working in this field and eradicate many physical problems related to heat and mass transfer and investigated that problems analytically as well as graphically (Sehra *et al.*, 2021). Olayemi *et al.* (2024) analysed a numerical investigation of the flow of an electrically conducting, incompressible, viscous fluid past a heated vertical channel with thermal radiation influence.

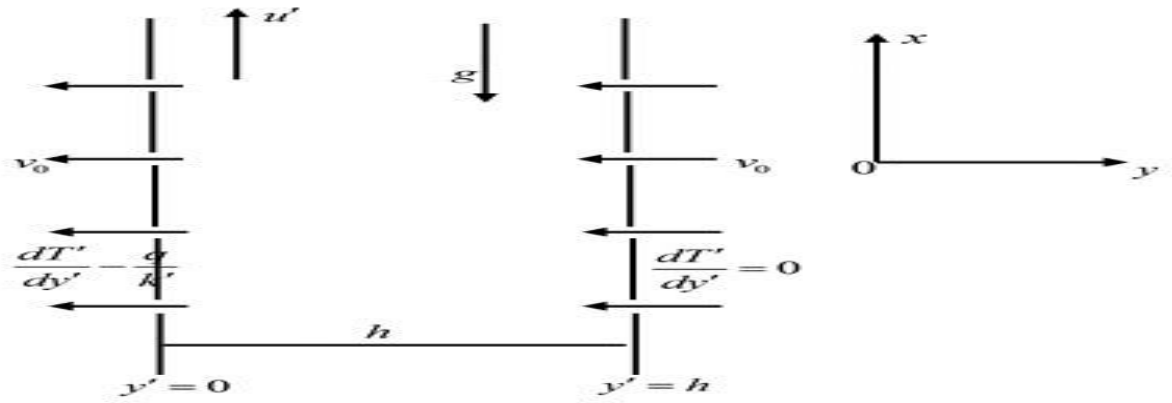
The use of isothermal or isoflux boundary conditions in channels with symmetric or asymmetric heating may yield results of acceptable accuracy in the calculation of the thermal performance of many configurations of interest. However, it is also frequent to encounter practical situations in which the effects of non-uniform heating of the channel plates must be taken into account. Furthermore, when a given global heat flux must be transferred from the plates to the fluid, and plate heating can be described by imposing boundary conditions of the Neumann type, it may be interesting to determine heat flux distributions along the wall that minimize the maximum wall temperature. This is especially so in the cooling of electronic printed circuits, for example, in which an appropriate arrangement of the dissipating components can contribute to reducing the maximum temperature reached in these components and, consequently, their risk of being damaged (Hernandez & Zamora, 2005). Aksu *et al.* (2023) explained that free convection heat transfer has increased in importance in

electronics cooling and thermal processes heating/cooling applications in the industry because it is the cheapest method. In their study, time-dependent free convection heat transfer and time-dependent buoyancy-assisted flow over the heated plate placed horizontally in a vertical channel are experimentally and theoretically for two classes of study. Chithra *et al.* (2025) presented a comprehensive analysis of conjugate heat transfer phenomena occurring within the developing region of square ducts under both isothermal and isoflux boundary conditions. Their study involved a rigorous numerical investigation, using advanced computational methods to simulate the complex heat exchange interactions between solid structures and surrounding fluid flows. Jha *et al.* (2025) reported a semi-analytical investigation of transient free convection flow of a viscous, incompressible fluid within a vertical channel subjected to third-kind thermal boundary conditions. Ji *et al.* (2026) extended their research to an unsteady behaviour of the thermal boundary layer (TBL) developing along a vertical plate subject to a constant heat flux are investigated numerically.

The present work aims to develop a closed-form solution for generalized fractional time derivatives governing natural convection between two infinite vertical plates, with one plate isothermally heated. The governing partial differential equations are transformed into ordinary differential equations using the Laplace transform method. Closed-form solutions in the Laplace domain are obtained and inverted to the time domain using the Riemann-sum approximation. The effects of flow parameters such as buoyancy force, and Prandtl number on temperature distribution, velocity profiles, Nusselt number, and skin friction are presented both graphically and in tabular form.

2.0 MATHEMATICAL FORMULATION

The flow is assumed to be laminar because the study considers fully developed channel flow within the low-to-moderate Reynolds-number regime, where viscous effects dominate and turbulence is absent. The fluid properties are assumed constant because the temperature and concentration variations are sufficiently small for changes in the thermophysical properties to be neglected. Density variation is retained only in the buoyancy terms under the Boussinesq approximation. Thus, the assumptions are applicable to fully developed laminar flow with moderate thermal and concentration variations.

**Figure 1: Schematic Diagram of Problem One**

Under the Boussinesq approximation and the boundary-layer assumptions, the governing equations describing the flow, heat transfer, and mass transfer can be written as follows:

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_0) + g\beta_c(C' - C_0) \quad (2)$$

$$\frac{\partial T'}{\partial t'} + v_0 \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

$$\frac{\partial C'}{\partial t'} + v_0 \frac{\partial C'}{\partial y'} = D_{cm} \frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

Subject to the following initial and boundary conditions:

$$\begin{aligned} t' \leq 0 \quad u' = 0, \quad T' = T_0, C' = C_0 \quad \text{for} \quad 0 \leq y' \leq h \\ t' = 0 \quad \begin{cases} u' = u_0, & \frac{dT'}{dy'} = -\frac{q}{k}, \quad C' = C_1 \quad \text{at } y' = 0 \\ u' = 0 & \frac{dT'}{dy'} = 0, \quad C' = C_0 \quad \text{at } y' = h \end{cases} \end{aligned} \quad (5)$$

By introducing the following dimensionless quantities:

$$y = \frac{y'}{h}, \theta = \frac{(T' - T_0)k}{qh}, Gr = \frac{g\beta q h^3}{k u_0 \nu}, Gm = \frac{g\beta_c (C_1 - C_0) h^2}{\nu u_0}, t = \frac{t' \nu}{h^2}, u = \frac{u'}{u_0}, C = \frac{C' - C_0}{C_1 - C_0}, Sc = \frac{\nu}{D_{cm}}, S = \frac{v_0 h}{\nu}, Pr = \frac{\nu}{\alpha}. \quad (6)$$

Substituting Equations (1) and (6) into Equations (2)–(4) and (5), the resulting equations are transformed into the following dimensionless forms:

$$\frac{\partial u}{\partial t} + S \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC \quad (7)$$

$$Pr \left(\frac{\partial \theta}{\partial t} + S \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

$$Sc \left(\frac{\partial C}{\partial t} + S \frac{\partial C}{\partial y} \right) = \frac{\partial^2 C}{\partial y^2} \quad (9)$$

The corresponding boundary conditions for the transformed system are given by:

$$\begin{aligned} t \leq 0 \quad u = 0, \quad \theta = 0, C = 0 \quad 0 \leq y \leq 1 \\ t = 0 \quad \begin{cases} u = 1, & \frac{d\theta}{dy} = -1, \quad C = 1 \quad \text{at } y = 0 \\ u = 0 & \frac{d\theta}{dy} = 0, \quad C = 0 \quad \text{at } y = 1 \end{cases} \end{aligned} \quad (10)$$

where δ is the Casson parameter, Gr is the thermal Grashof number, G_m is the mass Grashof number, Sc is the Schmidt number, Pr is the Prandtl number, M is the magnetic parameter, Rm is the chemical reaction parameter, Ku is the permeability parameter, $-\eta$ is the suction parameter, $+\eta$ is the injection parameter, H is the heat sink parameter, H is the radiation parameter and E is the Eckert number parameter.

2.1 Solution of Role of Heat and Mass Transfer on Time Dependent Natural Convection Flow in a Vertical Channel with Adiabatic and Isoflux Thermal Boundary Conditions

Based on the aforementioned assumptions, and applying the Laplace transform technique to transform the governing partial differential equations (7) to (9) into ordinary differential equations as follows:

$$\bar{u}(y, \omega) = \int_0^\infty u(y, t) e^{-\omega t} dt, \quad \bar{\theta}(y, s) = \int_0^\infty \theta(y, t) e^{-\omega t} dt \quad \text{and} \quad \bar{C}(y, \omega) = \int_0^\infty C(y, t) e^{-\omega t} dt \quad (11)$$

where $\omega > 0$ is the Laplace transform parameter.

By applying equation (11), the governing momentum, energy and concentration equations governing flow formation and heat/mass distribution are given as:

Momentum equation

$$\frac{d^2 \bar{u}}{dy^2} + S \frac{d\bar{u}}{dy} - \omega \bar{u} + Gr \bar{\theta} + Gm \bar{C} = 0 \quad (12)$$

Energy Equation

$$\frac{d^2 \bar{\theta}}{dy^2} + SPr \frac{d\bar{\theta}}{dy} - \omega Pr \bar{\theta} = 0 \quad (13)$$

Concentration Equation

$$\frac{d^2 \bar{C}}{dy^2} + ScS \frac{d\bar{C}}{dy} - \omega Sc \bar{C} = 0 \quad (14)$$

Subject to the following dimensionless boundary conditions:

$$\begin{aligned} \bar{u} = \frac{1}{\omega}, \quad \frac{d\bar{\theta}}{dy} = -\frac{1}{\omega}, \quad \bar{C} = \frac{1}{\omega} \quad \text{at } y = 0 \\ \bar{u} = 0, \quad \frac{d\bar{\theta}}{dy} = 0, \quad \bar{C} = 0 \quad \text{at } y = 1 \end{aligned} \quad (15)$$

2.2 Analytical Solution

Solving equations (12) to (14) with boundary conditions (15) using the method of undetermined coefficients, we have the closed-form solutions for the velocity, temperature and concentration respectively in the Laplace domain as:

$$\bar{\theta}(y) = A_1 e^{\lambda_1 y} + A_2 e^{\lambda_2 y}. \quad (16)$$

Applying $\frac{d\bar{\theta}}{dy} = -\frac{1}{\omega}$ at $y = 0$,

and

$$\frac{d\bar{\theta}}{dy} = 0 \text{ at } y = 1$$

$$A_1 = \frac{e^{\lambda_2}}{\lambda_1 \omega (e^{\lambda_1} - e^{\lambda_2})}, \quad A_2 = \frac{-e^{\lambda_1}}{\lambda_2 \omega (e^{\lambda_1} - e^{\lambda_2})}$$

$$\text{Where } \lambda_{1,2} = \frac{sPr \pm \sqrt{(sPr)^2 + 4\omega Pr}}{2}$$

So that

$$\bar{\theta}(y) = \frac{e^{\lambda_2 + \lambda_1 y}}{\lambda_1 \omega (e^{\lambda_1} - e^{\lambda_2})} - \frac{e^{\lambda_1 + \lambda_2 y}}{\lambda_2 \omega (e^{\lambda_1} - e^{\lambda_2})} \quad (17)$$

Similarly;

$$\bar{C}(y) = A_3 e^{\mu_1 y} + A_4 e^{\mu_2 y} \quad (18)$$

Applying $\bar{C}(0) = \frac{1}{\omega}$ at $y = 0$ and

$\bar{C} = 0$ at $y = 1$

$$A_3 = \frac{-e^{\mu_2}}{\omega (e^{\mu_1} - e^{\mu_2})}, \quad A_4 = \frac{e^{\mu_1}}{\omega (e^{\mu_1} - e^{\mu_2})}$$

$$\text{Where } \mu_{1,2} = \frac{sCs \pm \sqrt{(sCs)^2 + 4\omega Sc}}{2}$$

$$\bar{C}(y) = A_3 e^{\mu_1 y} + A_4 e^{\mu_2 y}$$

$$\bar{C}(y) = \frac{-e^{\mu_2 y} + \mu_1 y}{\omega(e^{\mu_1} - e^{\mu_2})} + \frac{e^{\mu_1 + \mu_2 y}}{\omega(e^{\mu_1} - e^{\mu_2})}$$

$$\bar{C}(y) = \frac{e^{\mu_1 + \mu_2 y}}{\omega(e^{\mu_1} - e^{\mu_2})} - \frac{e^{\mu_2 + \mu_1 y}}{\omega(e^{\mu_1} - e^{\mu_2})}$$

$$\bar{C}(y) = \frac{e^{\mu_1 + \mu_2 y} - e^{\mu_2 + \mu_1 y}}{\omega(e^{\mu_1} - e^{\mu_2})}$$

Also

$$\bar{u}(y) = A_5 e^{p_1 y} + A_6 e^{p_2 y} + d_1 e^{\lambda_1 y} + d_2 e^{\lambda_2 y} + d_3 e^{\mu_1 y} + d_4 e^{\mu_2 y}$$

$$P_{1,2} = \frac{-s \pm \sqrt{s^2 + 4\omega}}{2},$$

where

$$d_1 = \frac{-GrA_1}{\lambda_1^2 - S\lambda_1 - \omega}, d_2 = \frac{-GrA_2}{\lambda_2^2 - S\lambda_2 - \omega}, d_3 = \frac{-GmA_3}{\mu_1^2 - S\mu_1 - \omega}, d_4 = \frac{-GmA_4}{\mu_2^2 - S\mu_2 - \omega}$$

$$d_5 = d_1(e^{p_1} - e^{\lambda_1}) + d_2(e^{p_1} - e^{\lambda_2}) + d_3(e^{p_1} - e^{\mu_1}) + d_4(e^{p_1} - e^{\mu_2})$$

$$d_6 = d_1(e^{p_2} - e^{\lambda_1}) + d_2(e^{p_2} - e^{\lambda_2}) + d_3(e^{p_2} - e^{\mu_1}) + d_4(e^{p_2} - e^{\mu_2})$$

The Skin-friction/Schmit Number/Nusset-Number/Massflowrate

$$\tau = \left. \frac{d\bar{u}}{dy} \right|_{y=0,1}$$

$$\left. \frac{d\bar{u}}{dy} \right|_{y=0} = A_5 p_1 e^{p_1 y} + A_6 p_2 e^{p_2 y} + d_1 \lambda_1 e^{\lambda_1 y} + d_2 \lambda_2 e^{\lambda_2 y} + d_3 \mu_1 e^{\mu_1 y} + d_4 \mu_2 e^{\mu_2 y}$$

$$\left. \frac{d\bar{u}}{dy} \right|_{y=0} = A_5 p_1 + A_6 p_2 + d_1 \lambda_1 + d_2 \lambda_2 \mu_1 + d_3 + d_4 \mu_2$$

Similarly

$$\left. \frac{d\bar{u}}{dy} \right|_{y=1} = A_5 p_1 e^{p_1} + A_6 p_2 e^{p_2} + d_1 \lambda_1 e^{\lambda_1} + d_2 \lambda_2 e^{\lambda_2} + d_3 \mu_1 e^{\mu_1} + d_4 \mu_2 e^{\mu_2}$$

Nusset Number

$$Nu = \left. \frac{d\bar{\theta}}{dy} \right|_{y=0,1} = \lambda_1 A_1 e^{\lambda_1 y} + \lambda_2 A_2 e^{\lambda_2 y}$$

Sherwood Number

$$Sh = \left. \frac{d\bar{C}}{dy} \right|_{y=0,1} = \mu_1 A_3 e^{\mu_1 y} + \mu_2 A_4 e^{\mu_2 y}$$

Flow Rate

$$\bar{Q} = \int_0^1 \bar{u} = \frac{A_5 e^{p_1} - A_5}{p_1} + \frac{A_6 e^{p_2} - A_6}{p_2} + \frac{d_1 e^{\lambda_1} - d_1}{\lambda_1} + \frac{d_2 e^{\lambda_2} - d_2}{\lambda_2} + \frac{d_3 e^{\mu_1} - d_3}{\mu_1} + \frac{d_4 e^{\mu_2} - d_4}{\mu_2}$$

2.3 The Riemann Sum Approximation Solution of the First Problem in Time Domain

$$\theta(y, t) = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} \hat{\theta}(y, \varepsilon) + Re \sum_{n=1}^E \hat{\theta} \left(y, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], 0 \leq y \leq 1$$

$$C(y, t) = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} \hat{C}(y, \varepsilon) + Re \sum_{n=1}^E \hat{C} \left(y, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], 0 \leq y \leq 1$$

$$u(y, t) = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} \hat{U}(y, \varepsilon) + Re \sum_{n=1}^E \hat{U} \left(y, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], 0 \leq y \leq 1$$

$$\left. \frac{du}{dy} \right|_{y=0} = \tau_0 = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} \hat{\tau}_0(0, \varepsilon) + Re \sum_{n=1}^E \hat{\tau}_0 \left(0, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], y = 0$$

$$\left. \frac{du}{dy} \right|_{y=1} = \tau_1 = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} \hat{\tau}_1(1, \varepsilon) + Re \sum_{n=1}^E \hat{\tau}_1 \left(1, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], y = 1$$

$$\left. \frac{d\theta}{dy} \right|_{y=0} = Nu_0 = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} Nu_0(0, \varepsilon) + Re \sum_{n=1}^E Nu_0 \left(0, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], y = 0$$

$$\left. \frac{d\theta}{dy} \right|_{y=1} = Nu_1 = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} Nu_1(1, \varepsilon) + Re \sum_{n=1}^E Nu_1 \left(1, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], y = 1$$

$$\left. \frac{dC}{dy} \right|_{y=0} = Sh_0 = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} Sh_0(0, \varepsilon) + Re \sum_{n=1}^E Sh_0 \left(0, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], y = 0$$

$$\left. \frac{dC}{dy} \right|_{y=1} = Sh_1 = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} Sh_1(1, \varepsilon) + Re \sum_{n=1}^E Sh_1 \left(1, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right], y = 1$$

$$Q = \frac{e^{\varepsilon t}}{t} \left[\frac{1}{2} \hat{Q}(y, \varepsilon) + Re \sum_{n=1}^E \hat{Q} \left(y, \varepsilon + \frac{in\pi}{t} \right) \cdot (-1)^n \right]$$

Table 1: Numerical comparison of values of velocity $U(y, t)$ with the semi-analytical and PDEPE solution when $Gr = 5$, $Gm = 5$, $S = 0.5$, $Pr = 0.71$, $Sc = 0.22$

$y \backslash t$	Analytical Solution				PDEPE			
	0.4	0.8	1.2	SS	0.4	0.8	1.2	SS
0.0	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	0.0000
0.2	1.1448	1.2573	1.3590	1.6125	1.1442	1.2569	1.3587	1.6121
0.4	1.0800	1.2559	1.4135	1.8065	1.0793	1.2555	1.4132	1.8062
0.6	0.8460	1.0282	1.1911	1.5974	0.8457	1.0280	1.1909	1.5971
0.8	0.4773	0.6021	0.7144	0.9946	0.4770	0.6019	0.7142	0.9942
1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

PDEPE Solution

To use MATLAB's pdepe (1-D parabolic PDE solver) the system is arranged in the standard form

$$c(y, tu, uy)u_t = \frac{\partial}{\partial y} \left(f(y, t, u, u_y) \right) + s(y, t, u, u_y),$$

With the dependent vector $u = [u, \theta, c]^T$. Choosing

$$C = \begin{bmatrix} 1 \\ Pr \\ Sc \end{bmatrix}, f = \begin{bmatrix} u_y \\ \theta_y \\ c_y \end{bmatrix}, s = \begin{bmatrix} -S u_y + Gr \theta + Gm C \\ -Pr S \theta_y \\ -Sc S c_y \end{bmatrix}$$

produces the same PDE system. The problem is then solved with $m=0$ (Cartesian geometry) using pdepe.

4.0 DISCUSSION OF FINDINGS

The consequence of time(t), Prandtl number(Pr), Suction/Injection (S), Heat source/sink parameter (H), M Hartmann number, parameter Grasshof number (Gr and Gm) on temperature and velocity profile for heat source and heat sink scenario on time-periodic and temperature-dependent cases are graphically presented in figures. Furthermore, effective parameters on heat source and sink in skin friction (τ_0, τ_1), and mass flow rate (Q) are presented graphically also. During the computation, the numerical value for ζ and Pr are as follows, otherwise stated: time t , $0.2 \leq t \leq 5.0$ and Prandtl numbers (Pr) are $Pr = 0.71$ (air) and $Pr = 7.0$ (water). The reference values for the parameter time(t), Pr , M , Gm , Gr , S , H are 0.4, 0.71, 0.2, 2.0, 2.0, 0.2, and 0.2 respectively. Computations are performed through graphical representation by assigning appropriate numerical values to the pertinent parameters with aid of the MATLAB program of version R2014a.

Table 1 shows the numerical comparison between the analytical solution of model two and a MATLAB inbuilt method (PDEPE). The result showed that there is a very good agreement between the two solutions. This validate the accuracy of the analytical solution.

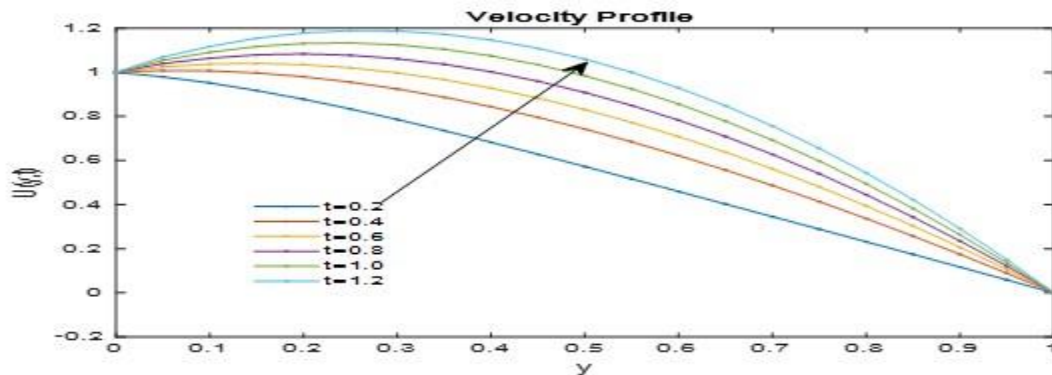
**Figure 1: Velocity profile for different time t ($Pr=0.71$, $S=0.2$, $Gr=2.0$, $Gm=2.0$)**

Figure 1 illustrates the transient variation of the velocity profile within the channel for different time intervals. It is observed that the velocity magnitude increases progressively as time advances, indicating the gradual development of the hydrodynamic boundary layer. At the initial instant, the flow velocity is relatively small due to

the dominance of viscous resistance. However, as time increases, buoyancy and inertial effects intensify, enhancing fluid motion and causing the velocity to rise until it approaches its steady-state distribution. This trend

confirms the expected behavior of time-dependent natural convection, where momentum diffusion and buoyancy forces act together to accelerate the flow as the system evolves toward equilibrium.

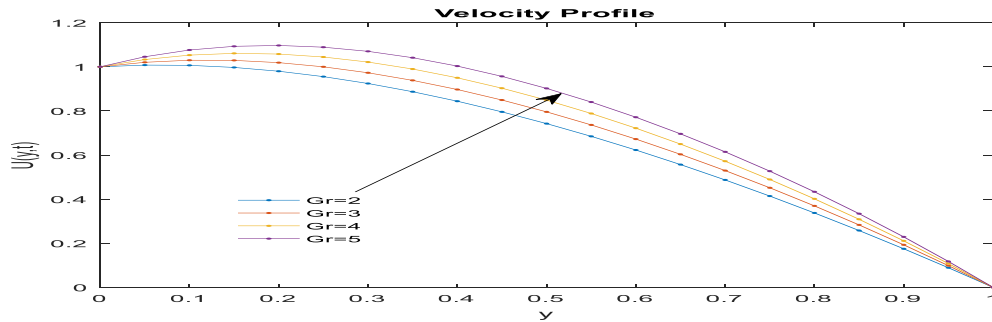


Figure 2: Velocity profile for different Gr (Pr=0.71, S=0.2, t=0.4, Gm=2.0)

Figure 2 presents the effect of the Grashof number (Gr) on the velocity distribution within the channel. It is evident that the velocity increases with an increase in Gr. This behavior arises because a larger Grashof number corresponds to a stronger buoyancy force generated by temperature differences, which enhances fluid motion in the direction of the buoyant force. As Gr increases, the buoyancy-induced flow dominates over viscous

resistance, leading to higher velocity magnitudes throughout the channel. Physically, this implies that stronger thermal or solutal gradients promote more vigorous convection currents and enhance momentum transport within the boundary layer. At lower Gr, viscous effects remain dominant, resulting in slower fluid motion and a thicker velocity boundary layer.

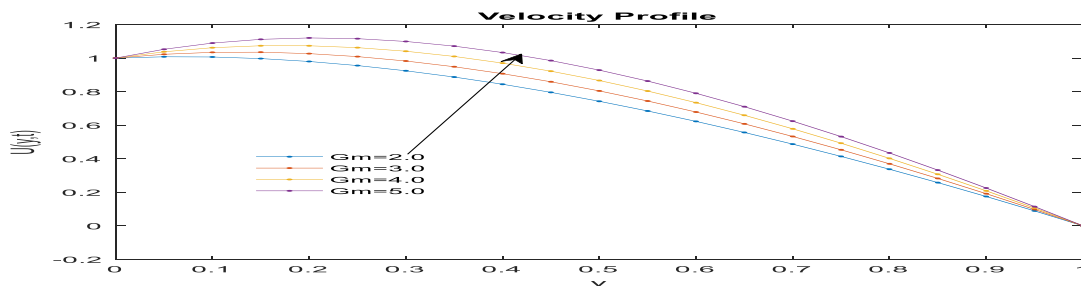


Figure 3: Velocity profile for different Gm (Pr=0.71, S=0.2, t=0.4, Gr=2.0)

Figure 3 illustrates the influence of the mass Grashof number (Gm) on the velocity profile within the channel. It is observed that the velocity increases with increasing Gm. This occurs because a larger Gm indicates a stronger solutal buoyancy force arising from concentration gradients, which enhances fluid motion in the upward direction. Physically, as the solutal buoyancy

effect becomes more dominant compared to viscous resistance, the fluid accelerates and the velocity boundary layer becomes thinner. This enhancement in velocity signifies that mass diffusion contributes significantly to the overall convective motion. For smaller values of Gm, viscous forces suppress the buoyancy effect, resulting in lower velocities and weaker convective transport.

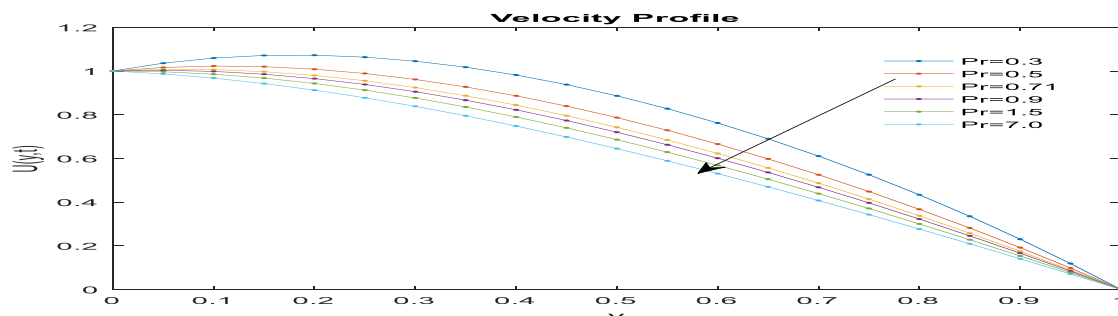


Figure 4: Velocity profile for different Pr (S=0.2, t=0.4, Gr=2.0, Gm=2.0)

Figure 4.4 depicts the influence of the Prandtl number (Pr) on the velocity profile. It is observed that the velocity decreases as Pr increases. This occurs because a higher Prandtl number corresponds to a fluid with lower thermal diffusivity, which limits the rate at which heat is transferred away from the heated surface. Consequently, the temperature gradient and the resulting buoyancy

force are reduced, leading to a weaker convective motion and lower velocity magnitudes. In contrast, fluids with smaller Pr values (higher thermal diffusivity) allow thermal energy to diffuse more readily, generating stronger buoyancy effects that enhance fluid motion and increase the velocity within the boundary layer.

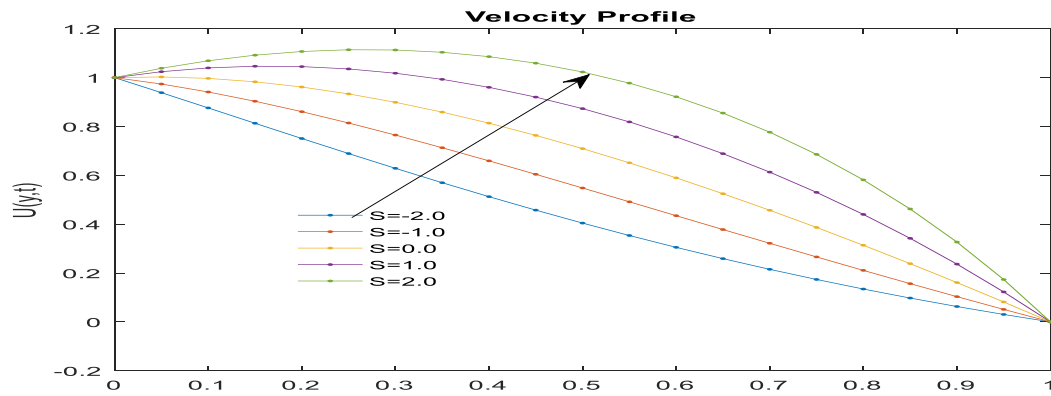


Figure 5: Velocity profile for different $\dot{S}$ ($Pr=0.71$, $t=0.4$, $Gr=2.0$, $Gm=2.0$)

Figure 5 illustrates the effect of the suction/injection parameter (S) on the velocity profile. In this study, positive values of S correspond to injection (blowing), while negative values represent suction. It is observed that the velocity increases as the positive values of S increase, whereas it decreases for negative S . Physically, injection introduces additional fluid into the boundary layer, enhancing the momentum near the wall and thickening the boundary layer, which results in higher

velocity magnitudes throughout the flow domain. In contrast, suction removes fluid from the boundary layer, suppressing fluid motion and thinning the hydrodynamic layer, thereby reducing the overall velocity. This trend confirms that wall mass transfer has a significant influence on the convective transport mechanism with injection promoting stronger flow circulation and suction stabilizing the flow by damping velocity fluctuations.

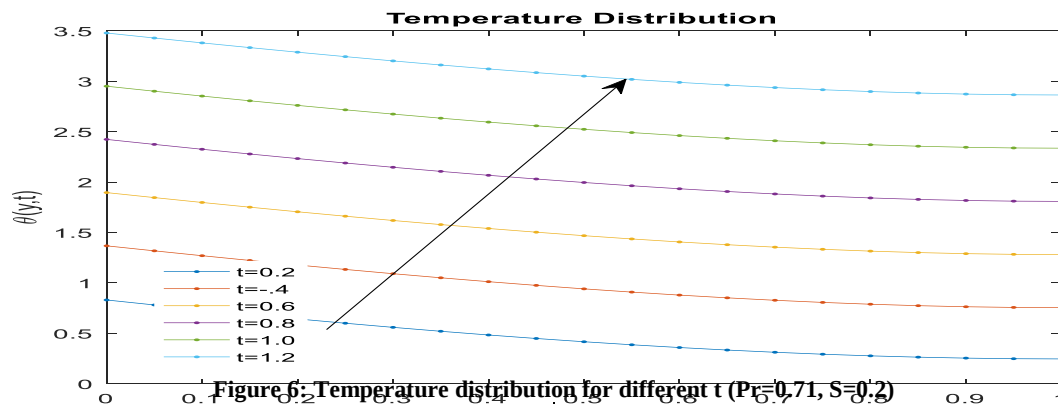


Figure 6: Temperature distribution for different t ($Pr=0.71$, $S=0.2$)

Figure 6 depicts the transient variation of the temperature distribution within the channel for different time intervals. It is observed that the temperature increases as time increases. This behavior occurs because, at the initial stage, the fluid is relatively cool, and the thermal boundary layer is thin. As time progresses, heat diffuses from the heated wall into the fluid, causing a gradual rise in temperature throughout the domain. The thickening of

the thermal boundary layer with time indicates enhanced heat penetration and energy storage in the fluid. Eventually, the temperature distribution tends toward a steady-state profile as the rate of thermal diffusion balances the convective transport. This increase in temperature with time reflects the characteristic development of transient natural convection, where the

system evolves from an initial unheated condition to a fully established thermal field.

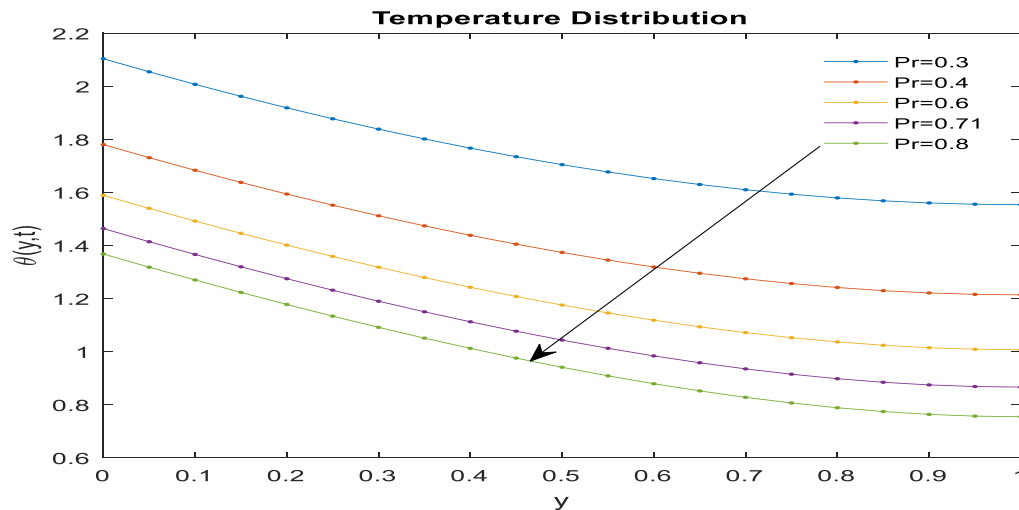


Figure 7: Temperature distribution for different Pr (t=0.4, S=0.2)

Figure 7 illustrates the influence of the Prandtl number (Pr) on the temperature distribution within the channel. It is evident that the temperature decreases as Pr increases. This occurs because fluids with higher Prandtl numbers possess lower thermal diffusivity, which reduces the rate of heat transfer from the heated wall to the surrounding fluid. As a result, the temperature within the boundary layer remains lower and the thermal boundary layer becomes thinner. Conversely, fluids with smaller Pr

values have higher thermal diffusivity, allowing heat to spread more effectively into the fluid domain and producing higher temperature profiles. Physically, this behavior implies that low-Pr fluids (such as liquid metals) experience stronger heat diffusion and more uniform temperature fields, whereas high-Pr fluids (such as oils) exhibit steep temperature gradients and reduced convective heat transfer.

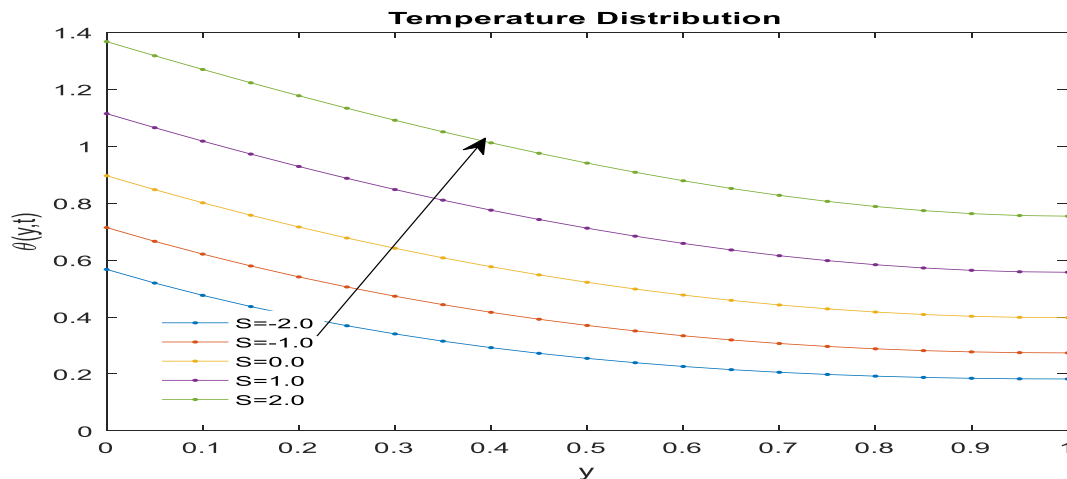


Figure 8: Temperature distribution for different S (t=0.4, Pr=0.71)

Figure 8 illustrates the effect of the suction/injection parameter (S) on the temperature distribution within the channel. It is observed that the temperature decreases with increasing suction (negative) and increases with injection (positive S). This behavior is attributed to the influence of wall mass transfer on thermal transport. Suction extracts fluid particles adjacent to the heated wall, thereby thinning the thermal boundary layer and

reducing the fluid temperature within the flow domain. In contrast, injection supplies additional fluid into the boundary layer, which enhances fluid mixing and heat accumulation, resulting in higher temperature profiles and a thicker thermal boundary layer. Physically, this implies that suction acts as a cooling mechanism that stabilizes the flow by removing heat near the wall,

whereas injection intensifies convective heating and promotes stronger thermal motion within the fluid.

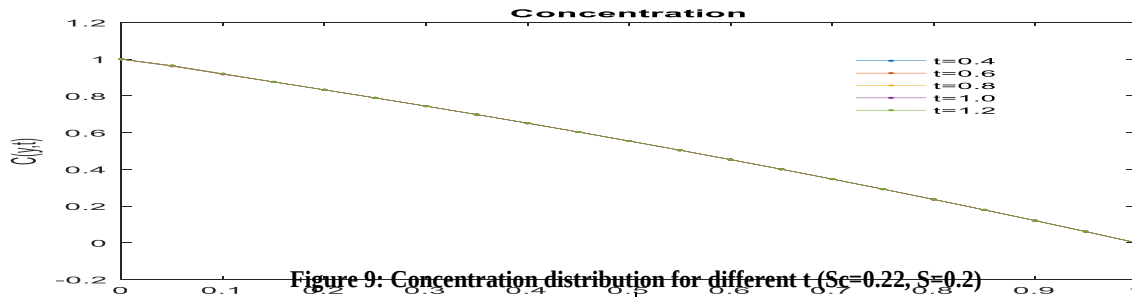


Figure 9: Concentration distribution for different t ($Sc=0.22$, $S=0.2$)

Figure 9 presents the variation of concentration distribution with time. It is observed that time has no significant effect on the concentration profile, indicating that the solutal field reaches a quasi-steady state rapidly. This behavior suggests that the rate of mass diffusion dominates the transient effects, causing the concentration gradients to stabilize within a short period after the flow begins. Consequently, the concentration distribution

remains practically unchanged as time progresses. Physically, this implies that mass transfer in the system is governed primarily by steady diffusion and convective mechanisms rather than by transient unsteadiness. Therefore, once the solute has diffused sufficiently through the boundary layer, additional time evolution contributes minimally to further concentration changes.

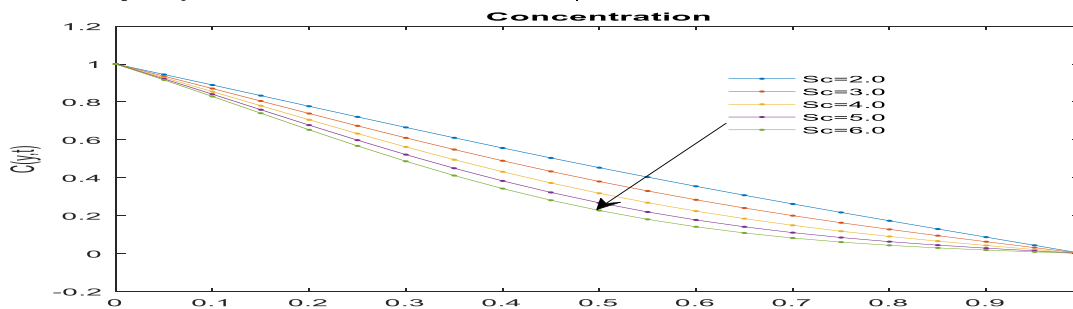


Figure 10: Concentration distribution for different Sc ($t=0.4$, $S=0.2$)

Figure 10 illustrates the influence of the Schmidt number (Sc) on the concentration distribution within the channel. It is evident that the concentration decreases as Sc increases. This behavior is due to the inverse relationship between the Schmidt number and the mass diffusivity of the fluid. A higher Sc corresponds to a lower molecular diffusivity, which restricts the rate at which solute particles spread through the fluid. Consequently, the concentration boundary layer becomes thinner, and the

overall concentration levels decrease. In contrast, fluids with lower Sc values exhibit higher mass diffusivity, allowing solute to diffuse more rapidly, which results in higher concentration profiles and a thicker concentration boundary layer. Physically, this trend signifies that the rate of mass transfer is more pronounced in low- Sc fluids, while high- Sc fluids display slower diffusion and reduced solutal mixing.

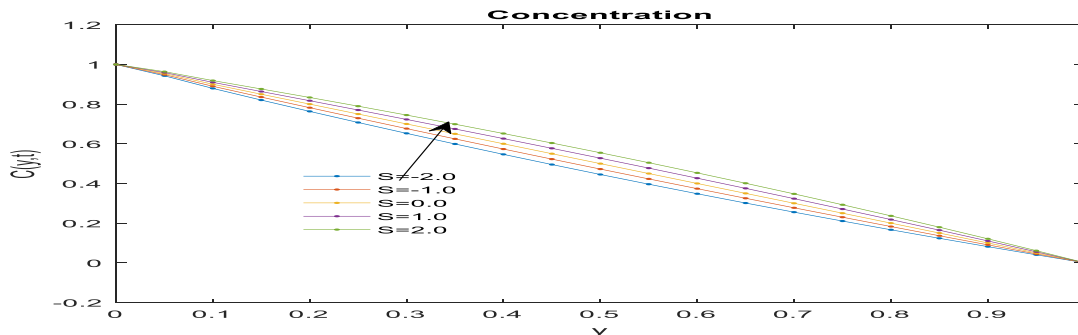


Figure 11: Concentration distribution for different S ($t=0.4$, $Sc=0.22$)

Figure 11 illustrates the influence of the suction/injection parameter (S) on the concentration distribution within the channel. It is observed that the concentration increases with increasing injection (positive S) and decreases with suction (negative S). The rise in concentration under injection conditions is attributed to the additional inflow of fluid that enriches the solute content near the wall and promotes diffusive transport across the channel. In contrast, suction removes fluid from the near-wall region,

thereby reducing the local solute concentration and thinning the concentration boundary layer. Physically, injection enhances mass transfer by introducing more solute into the fluid domain, while suction stabilizes the flow and suppresses solutal diffusion. This indicates that wall mass transfer serves as an effective control mechanism for solute distribution in convective mass transfer systems.

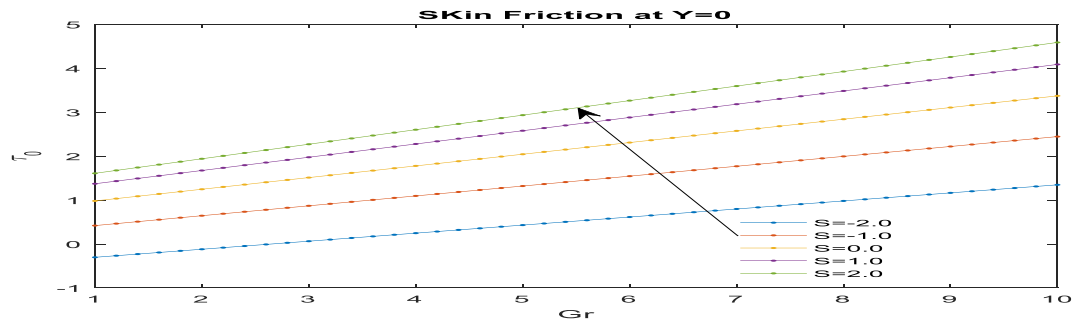


Figure 12: Skin Friction at $y=0$ for against Gr with S

Figure 4.12 presents the variation of the skin friction coefficient at the wall ($y=0$) with the Grashof number (Gr) for different values of the suction/injection parameter (S). It is observed that the skin friction increases with increasing Gr . This behavior is attributed to the enhanced buoyancy force at higher Gr , which accelerates the fluid near the wall and consequently increases the velocity gradient, resulting in greater wall shear stress. Furthermore, the figure shows that injection (positive S) leads to a further rise in skin friction, whereas

suction (negative S) causes a reduction. Injection introduces additional fluid momentum into the boundary layer, thickening it and intensifying shear at the wall, while suction removes fluid from the boundary layer, suppressing near-wall motion and lowering shear stress. Physically, this indicates that both buoyancy and wall mass transfer strongly influence the wall shear behavior. Buoyancy strengthens convective acceleration, and injection amplifies its effect, while suction stabilizes the flow by reducing momentum near the wall.

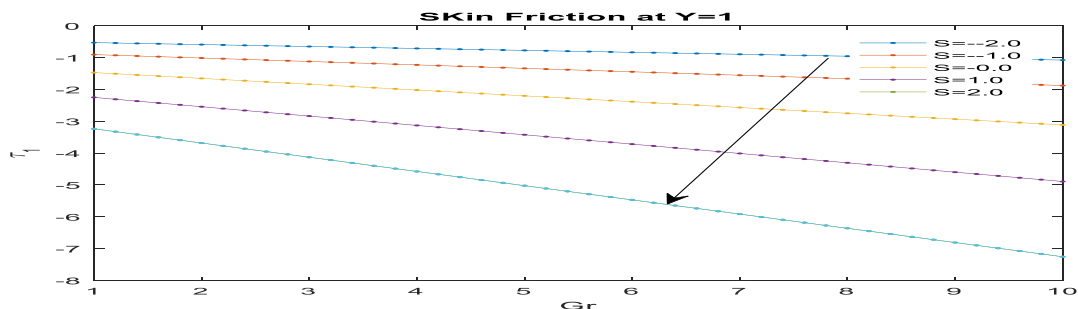


Figure 13: Skin Friction at $y=1$ for against Gr with S

Figure 13 illustrates the variation of the skin friction coefficient at the opposite wall ($y=1$) with the Grashof number (Gr) for different suction/injection conditions. In contrast to the behavior at $y=0$, the skin friction at $y=1$ increases as Gr decreases. This inverse relationship arises because, as Gr increases, buoyancy-driven upward flow strengthens near the heated wall ($y=0$) but weakens at the opposite wall, leading to reduced velocity gradients and hence lower wall shear at $y=1$. Moreover, injection (positive S) is found to reduce skin friction, while suction (negative S) enhances it. Injection tends to introduce additional fluid into the boundary layer, decreasing the

velocity gradient near the wall, whereas suction removes fluid and steepens the near-wall velocity profile, resulting in higher shear stress. Physically, this indicates that the combined effects of buoyancy and wall mass transfer manifest differently across the channel walls with enhanced convective acceleration at the heated surface and opposing shear behavior at the other wall.

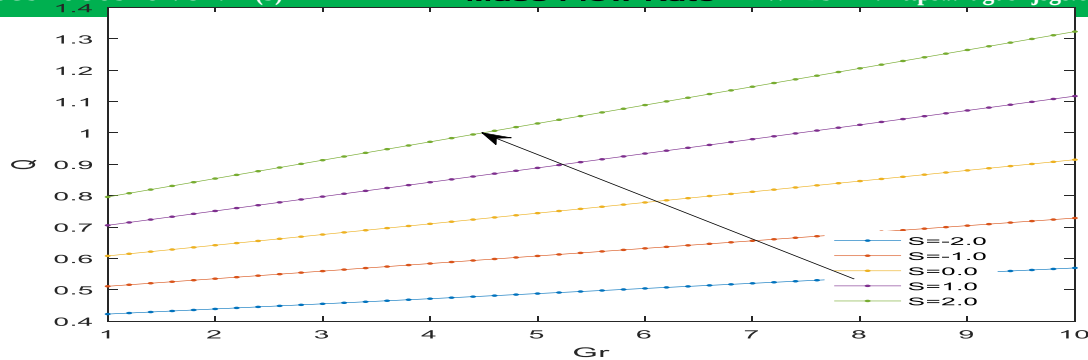
**Figure 14: Mass flow rate against Gr with S**

Figure.14 depicts the variation of the mass flow rate with the Grashof number (Gr) for different values of the suction/injection parameter (S). It is observed that the mass flow rate increases with increasing Gr. This trend is attributed to the enhancement of buoyancy forces at higher Gr, which accelerates the fluid motion and consequently promotes greater fluid transport through the channel. Additionally, the mass flow rate rises with injection (positive S) and decreases with suction (negative S). Injection introduces additional fluid and

momentum into the flow domain, thickening the boundary layer and increasing the volumetric flow, while suction extracts fluid from the near-wall region, suppressing the flow and reducing the overall mass transport. Physically, this behavior signifies that both thermal buoyancy and wall mass transfer are key mechanisms controlling the convective flow rate with buoyancy driving upward motion and injection amplifying it, whereas suction provides a stabilizing and retarding influence.

5.0 CONCLUDING REMARKS

This study examined the transient natural-convection flow associated with simultaneous heat and mass transfer in a vertical channel subject to adiabatic and isoflux thermal boundary conditions. The effects of buoyancy, wall suction and injection, magnetic field, thermal and solutal diffusion, and internal heat generation or absorption were investigated through a dimensionless mathematical model. Analytical solutions obtained using the Laplace transform method provided insight into the temporal behaviour of the velocity, temperature, and concentration fields, while the skin-friction coefficient, Nusselt number, and Sherwood number were used to assess the corresponding transport characteristics.

The results indicate that thermal and concentration buoyancy significantly influence fluid motion, with increases in the thermal and solutal Grashof numbers enhancing the flow velocity. Fluid injection generally strengthens the velocity, temperature, and concentration distributions, whereas suction produces a suppressing effect. Increasing the Prandtl and Schmidt numbers reduces the thermal and concentration fields, respectively, reflecting the development of thinner thermal and solutal boundary layers. Internal heat generation increases the fluid temperature and modifies the wall heat-transfer characteristics, while heat absorption produces the reverse response. The wall shear stress and mass-transfer rate are also strongly affected by buoyancy and wall mass transfer.

Overall, the findings demonstrate the significant interaction among fluid motion, thermal transport, and species diffusion in transient buoyancy-driven flows. The results provide useful theoretical information for controlling heat and mass transfer in engineering applications involving vertical channels, porous heat-transfer systems, cooling devices, and other transport processes where wall mass transfer and thermal boundary conditions play important roles.

RECOMENDATIONS

In view of the findings obtained from this investigation, the following areas are recommended for future research:

- Future investigations may extend the present formulation by incorporating magnetohydrodynamic (MHD), electrohydrodynamic, or electroosmotic effects. Such extensions would make the model more applicable to transport processes in microfluidic channels, electrically conducting fluids, and electromagnetic heat-transfer systems.
- Further studies could consider non-Newtonian fluids, including Jeffrey, Casson, or other rheological fluid models, as well as nanofluids. These extensions may provide a better representation of complex fluid behaviour encountered in advanced cooling technologies, industrial processes, and biomedical applications.

- iii. The analytical solutions obtained in this study could be examined using numerical techniques such as finite-difference, finite-element, or other computational methods. Numerical validation would also make it possible to investigate nonlinearities, variable fluid properties, and other complex effects that may not be readily accommodated by analytical approaches.
- iv. Experimental studies are recommended to assess the practical validity of the theoretical predictions. Controlled laboratory experiments could particularly examine the effects of wall suction and injection, heat generation or absorption, and the adopted thermal boundary conditions on fluid velocity and heat and mass transfer rates.
- v. Future research may employ optimization techniques to identify suitable combinations of governing parameters for achieving desired heat and mass transfer performance. Such investigations could contribute to the efficient design and operation of vertical channels, heat exchangers, porous cooling systems, and related thermal management devices.

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