

A q-Version of the Residue Theorem and Contour Integration: Theory, Computation and Python Applications

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ABSTRACT

The classical Residue Theorem is a fundamental result in complex analysis with extensive applications in evaluating contour and improper real integrals. Recent computational developments have demonstrated the effectiveness of combining residue calculus with symbolic computation for numerical verification and visualization. Motivated by these advances, this paper extends the classical residue-theoretic framework to Jackson q-calculus. A Jackson q-Residue-Type Operator and a q-contour-type integral are introduced, leading to a q-Residue Theorem and a corresponding q-Integral Evaluation Formula for the computation of q-integrals. We establish fundamental lemmas to investigate the operator's limiting behaviour and linearity, while the main theorems provide a theoretical foundation for evaluating representative q-integrals. We complement the analytical developments with computational verification in Python, using SymPy, NumPy, mpmath, and Matplotlib for symbolic computation, numerical approximation, and graphical visualisation. Illustrative examples demonstrate the approach's applicability and confirm that the derived q-formulations converge to their classical counterparts as $q \rightarrow 1^-$. The results bridge classical residue calculus, quantum calculus, and computational mathematics, providing a foundation for further investigations in q-special functions, q-difference equations, and related areas of mathematical analysis.

Keywords: Jackson q-calculus, Residue theorem, q-residue operator, q-contour integration, q-integrals, Python.

1.0 INTRODUCTION

Complex analysis remains one of the most important areas of mathematics, offering sophisticated methods for resolving issues in both the pure and applied sciences. A key result is that by studying the isolated singularities of analytic functions, the Residue Theorem provides an effective way to evaluate contour integrals and many improper real integrals. In mathematical physics, engineering, signal processing, and differential equations, where contour integration simplifies otherwise challenging integral evaluations, classical residue calculus has found extensive applications.

By creating mathematical structures that do not depend on the conventional idea of limits, quantum calculus (q-calculus) has greatly expanded classical analysis. Q-calculus is a rich framework that naturally occurs in quantum groups, special functions, orthogonal polynomials, combinatorics, and mathematical physics. It builds on the groundbreaking work of Jackson (1908, 1910) and replaces regular derivatives and integrals with their q-counterparts. In contemporary q-analysis, the Jackson q-derivative and Jackson q-integral are essential operators that offer q-deformations of numerous traditional analytical ideas.

The close connection between q-special functions, such as the q-exponential, q-trigonometric, q-gamma, and fundamental hypergeometric functions, is a

significant feature of q-calculus. The literature presents comprehensive treatments and recent developments of these functions, including Ernst (2012), Mahmudov (2020), Al-Omari (2021), and Au (2021), discussing applications to mathematical analysis, quantum mechanics, and special function theory. These functions generalise their classical counterparts, preserve many essential analytical properties, and reduce to the classical functions as $q \rightarrow 1^-$.

An essential tool for analyzing q-difference equations and q-special functions, the Jackson q-integral is a natural equivalent of the traditional Riemann integral. As a result, the q-calculus framework has been used to reformulate numerous traditional analytical methods. The development of q-integral transforms, q-differential operators, and q-special functions has advanced significantly, but extending classical theories of contour integration and residue calculus to a suitable q-setting has received little attention.

Analytic continuation, Laurent series expansions, and the calculation of residues at isolated singularities surrounded by a contour are the foundations of contour integration in classical complex analysis. Because q-difference operators replace the underlying differential and integral operators, extending these ideas to q-analysis poses theoretical and practical difficulties. Thus, creating appropriate q-analogues of contour integration methods and residue-type operators is a



logical and significant path in contemporary q -analysis.

By enabling the computational verification of theoretical derivations, recent developments in symbolic computation have also revolutionized mathematical study.

Python offers an effective environment for symbolic processing, numerical approximation, and graphical visualisation through modules like SymPy, NumPy, mpmath, and Matplotlib. In addition to increasing repeatability, integrating computational tools with q -calculus enables numerical study of Jackson q -integrals, q -special functions, and limiting behaviour as $q \rightarrow 1^-$.

In their recent work, Buwai *et al.* (2025) integrated conventional residue calculus with Python-based symbolic computing and numerical verification. While their study showed how to implement classical residue calculus using Python modules such as SymPy, mpmath, and Matplotlib, it remained limited to classical complex analysis. Inspired by these computational advances, this study extends the Python-based residue framework to Jackson q -calculus by introducing a Jackson q -Residue Operator for evaluating q -integrals.

Inspired by these advances, this study introduces a Jackson q -Residue Operator for evaluating q -integrals, extending the traditional residue-theoretic method to a q -calculus context. To study representative q -integrals, investigate the impact of the deformation parameter q , and confirm that the results converge to their classical counterparts as $q \rightarrow 1^-$, the proposed method combines theoretical analysis with a Python implementation. As a result, this work links quantum calculus, classical residue calculus, and contemporary computer mathematics, laying the groundwork for future advances in q -complex analysis and its applications.

2.0 MATERIALS AND METHODS

2.1 Materials

This work explores q -analogues of contour integration and residue calculus through theoretical analysis and computational testing. The theory of q -special functions and Jackson q -calculus form the foundation of the mathematical framework, while Python is used for computational verification.

The materials employed in this study include:

2.1.1 Mathematical Framework

The theoretical development is based on:

- i. Jackson q -calculus;
- ii. Jackson q -derivative and Jackson q -integral;
- iii. q -numbers and q -factorials;
- iv. q -exponential and q -trigonometric functions;
- v. Basic hypergeometric functions;
- vi. q -Residue-Type Operator;

- vii. q -Contour-Type Integral.

2.1.2 Computational Tools

The computational implementation utilizes:

- i. Python 3.x as the programming environment;
- ii. SymPy for symbolic manipulation;
- iii. NumPy for numerical computations;
- iv. mpmath for high-precision numerical Evaluation;
- v. Matplotlib for graphical visualisation of q -functions and numerical results.

2.1.3 Computational Resources

We ran numerical simulations on a standard personal computer using Python 3.x in Jupyter Notebook. The software environment enabled the symbolic computations, numerical approximations, and graphical visualisation needed to validate the theoretical results.

2.2 Methods

The methodology combines theoretical derivation and computational implementation. The theoretical component develops q -analogues of the classical residue theorem using Jackson q -calculus, while the computational component verifies the analytical results through symbolic computation and numerical approximation.

The procedure consists of the following stages.

Step 1. Construction of the q -Calculus Framework

We use the Jackson q -counterparts of classical differential and integral operators. We establish the analytical foundation by introducing basic ideas such as q -numbers, q -factorials, Jackson q -derivatives, and Jackson q -integrals.

Step 2. Development of a q -Residue-Type Operator

A q -residue-type operator is introduced to generalize the classical residue associated with isolated singularities. The proposed operator forms the foundation for evaluating q -integrals through q -analogue contour techniques.

Step 3. Development of a q -Contour-Type Integral

We formulate suitable q -contour integrals using the newly introduced q -residue operator. The relationship between the q -contour integral and the corresponding q -residue is then established theoretically.

Step 4. Analytical Evaluation of q -Integrals

Representative q -integrals involving q -rational, q -exponential, and q -trigonometric functions are evaluated analytically using the proposed q -residue formulation.

Step 5. Computational Verification

The analytical results are verified using Python through symbolic computation and numerical approximation. Numerical experiments are performed for various values of the deformation parameter q , illustrating convergence toward the corresponding classical results as $q \rightarrow 1^-$.

Step 6. Graphical Analysis

Graphical visualization is employed to illustrate:

- the influence of the deformation parameter q ;
- convergence of q -functions to their classical counterparts;
- behaviour of q -integrals;
- numerical convergence of the proposed q -residue formulation.

2.3 Fundamental Definitions

Before presenting the main theoretical results, we first define several fundamental concepts from Jackson. q -calculus.

Definition 2.1 (q-Number).

For $0 < q < 1$ and $n \in \mathbb{N}$, the q -number is defined by

$$[n]_q = \frac{1 - q^n}{1 - q}. \quad (1)$$

Definition 2.2 (q-Factorial).

The q -factorial is defined recursively by

$$[n]_q! = \prod_{k=1}^n k_q \quad (2)$$

With

$$0_q! = 1.$$

Definition 2.3 (Jackson q-Derivative).

Let f be analytic in a domain D . The Jackson q -derivative of f is defined by

$$D_q f(z) = \frac{f(z) - f(qz)}{(1-q)z}, \quad z \neq 0. \quad (3)$$

Definition 2.4 (Jackson q-Integral).

The Jackson q -integral from 0 to z is defined by

$$\int_0^z f(t) d_q t = (1-q)z \sum_{n=0}^{\infty} q^n f(q^n z) \quad (4)$$

Definition 2.5 (q-Laurent Expansion)

Let (f) be q -analytic in an annular domain centred at (a) . A q -Laurent expansion of (f) is represented by

$$\left[f(z) = \sum_{n=-\infty}^{\infty} c_n \Phi_n(z, a; q), \right] \quad (5)$$

where $(\Phi_n(z, a; q))$ denotes an appropriate q -basis associated with the Jackson q -difference operator and (c_n) are the q -Laurent coefficients.

Definition 2.6 (Jackson q-Residue Operator).

Let $f(z) = \frac{h(z)}{g(z)}$ be a q -meromorphic function in a neighbourhood of an isolated singularity a , where $h(a) \neq 0$, $g(a) = 0$, and $g'(a) \neq 0$. The Jackson q -Residue Operator at $z = a$ is defined by:

$$\mathcal{R}_q \{f(z); a\} = \frac{h(a)}{D_q [g(z)]|_{z=a}} \quad (6)$$

where D_q denotes the Jackson q -derivative:

$$D_q [g(z)] = \frac{g(z) - g(qz)}{(1-q)z}, \quad z \neq 0$$

Remark 2.1 (Classical Limit).

As $q \rightarrow 1^-$, the Jackson q -derivative reduces to the ordinary derivative $\lim_{q \rightarrow 1^-} D_q [g(z)] = g'(z)$.

Consequently:

$$\lim_{q \rightarrow 1^-} \mathcal{R}_q \{f(z); a\} = \frac{h(a)}{g'(a)} = \text{Res}(f, a) \quad (7)$$

which recovers the classical residue formula for simple poles.

Remark 2.2.

The Jackson q -Residue-Type Operator provides the fundamental building block for constructing q -contour-type integrals and evaluating q -integrals in a manner analogous to the classical residue theorem. It therefore establishes a theoretical bridge between residue calculus and quantum calculus.

Definition 2.7 (q-Contour-Type Integral).

Let f be a q -analytic function defined on and inside a positively oriented simple closed contour C . The q -contour-type integral of f over C is defined by:

$$\oint_C f(z) d_q z \quad (8)$$

where $d_q z$ denotes the Jackson q -differential.

If C is parameterised by $z = z(t)$ for $a \leq t \leq b$, where the q -derivatives are given by

$$d_q z = D_q [z(t)] d_q t,$$

then:

$$\begin{aligned} \oint_C f(z) d_q z &= \int_a^b f(z(t)) D_q [z(t)] d_q t \end{aligned} \quad (9)$$

where $D_q [z(t)]$ is the Jackson q -derivative of the parametrisation.

Remark 2.3

When $q \rightarrow 1^-$

$$D_q z(t) \rightarrow z'(t), \quad (10)$$

and

$$d_q(t) \rightarrow dt. \quad (11)$$

Hence,

$$\lim_{q \rightarrow 1^-} \oint_C f(z) d_q z = \oint_C f(z) dz, \quad (12)$$

showing that the q -contour-type integral reduces to the classical contour integral.

Proposition 2.1

Let f be q -analytic inside and on the contour C . Then the q -contour-type integral satisfies the following properties:

- Linearity**

$$\oint_C [\alpha f(z) + \beta g(z)] d_q z$$

$$= \alpha \oint_C f(z) d_q z$$

$$+ \beta \oint_C g(z) d_q z, \quad (13)$$

where $\alpha, \beta \in \mathbb{C}$.

ii. Orientation

If C^{-1} denotes the contour with opposite orientation, then

$$\oint_{C^{-1}} f(z) d_q z = - \oint_C f(z) d_q z. \quad (14)$$

iii. Classical Limit

As $q \rightarrow 1^-$

$$\oint_C f(z) d_q z = \oint_C f(z) dz. \quad (15)$$

3.0 MAIN RESULTS

3.1 Fundamental Lemmas

Lemma 3.1 (Classical Limiting Property)

Let f be q -analytic in a domain $D \subseteq \mathbb{C}$, and let a be an isolated q -singularity of f (Au, 2021; Kac & Cheung, 2021). Then:

$$\lim_{q \rightarrow 1^-} \mathcal{R}_q \{f(z); a\}$$

$$= \text{Res}(f, a)$$

Proof. Since $\lim_{q \rightarrow 1^-} D_q [g(z)] = g'(z)$, taking the limit of Definition 2.6 as $q \rightarrow 1^-$ yields:

$$\lim_{q \rightarrow 1^-} \mathcal{R}_q \{f(z); a\} = \lim_{q \rightarrow 1^-} \frac{h(a)}{D_q [g(z)]|_{z=a}}$$

$$= \frac{h(a)}{g'(a)}$$

$$= \text{Res}(f, a) \quad (17)$$

This completes the proof.

Lemma 3.2 (Linearity of the Jackson q -Residue-Type Operator)

Let $f(z) = \frac{h_1(z)}{g(z)}$ and $w(z) = \frac{h_2(z)}{g(z)}$ be q -meromorphic functions sharing a simple pole at $z = a$, and let $\alpha, \beta \in \mathbb{C}$ (Al-Omari, 2021b; Mahmudov, 2020). Then:

$$\mathcal{R}_q \{\alpha f + \beta w; a\}$$

$$= \alpha \mathcal{R}_q \{f; a\}$$

$$+ \beta \mathcal{R}_q \{w; a\} \quad (18)$$

Proof. Applying Definition 2.6 to the linear

$$\text{combination } (\alpha f + \beta w)(z) = \frac{\alpha h_1(z) + \beta h_2(z)}{g(z)}.$$

$$\mathcal{R}_q \{\alpha f + \beta w; a\} = \frac{\alpha h_1(a) + \beta h_2(a)}{D_q [g(z)]|_{z=a}}$$

$$= \alpha \frac{h_1(a)}{D_q [g(z)]|_{z=a}} + \beta \frac{h_2(a)}{D_q [g(z)]|_{z=a}}$$

$$= \alpha \mathcal{R}_q \{f; a\}$$

$$+ \beta \mathcal{R}_q \{w; a\} \quad (19)$$

Hence, the Jackson q -Residue Operator is linear.

3.2 Main Theorems

Theorem 3.1 (q -Residue Theorem)

Let $f(z) = \frac{h(z)}{g(z)}$ be q -analytic on a connected domain $D \subset \mathbb{C}$, except for finitely many isolated simple q -singularities $a_1, a_2, \dots, a_k$ contained inside a positively oriented simple closed contour C . Then: (Al-Omari, 2021b; Au, 2021)

$$\oint_C f(z) d_q z = 2\pi i \sum_{j=1}^k \mathcal{R}_q \{f(z); a_j\} \quad (20)$$

where $\mathcal{R}_q \{f(z); a_k\} = \frac{h(a_k)}{D_q [g(z)]|_{z=a_k}}$ is the Jackson q -Residue Operator.

Theorem 3.2 (q -Integral Evaluation Formula)

Let $f(z)$ satisfy the hypotheses of Theorem 3.1, and suppose that the contribution of the q -contour along the semicircular arc in the upper half-plane vanishes as the radius tends to infinity (Al-Omari, 2021a; Buwai et al., 2025)(16). $a_1, a_2, \dots, a_m$ are the q -singularities lying in the upper half-plane, then the Jackson q -integral along the real axis is given by:

$$\int_{-\infty}^{\infty} f(x) d_q x = 2\pi i \sum_{j=1}^m \mathcal{R}_q \{f(z); a_j\} \quad (21)$$

where the summation extends over all q -singularities lying in the upper half-plane.

3.2 Illustrative Examples

Example 3.1

Consider the q -rational function $f(z) = \frac{1}{z^2+1}$ evaluated over a positively oriented semicircular contour C in the upper half-plane. Here, $h(z) = 1$ and $g(z) = z^2 + 1$. The isolated simple singularities occur at $z = \pm i$, of which only $a = i$ lies inside C .

Computing the Jackson q -derivative of $g(z)$ (Jackson, 1908):

$$D_q [z^2 + 1] = D_q [z^2] + D_q [1]$$

$$= (1$$

$$+ q)z \quad (22)$$

Evaluating at $z = i$:

$$[g(z)]|_{z=i} = (1 + q)i$$

Applying Definition 2.6 yields the q -residue:

$$\mathcal{R}_q \left\{ \frac{1}{z^2 + 1}; i \right\} = \frac{1}{(1 + q)i}$$

$$= -\frac{i}{1 + q} \quad (23)$$

By Theorem 3.2, the q -integral along the real axis evaluates to:

$$I_q = \int_{-\infty}^{\infty} \frac{1}{x^2 + 1} d_q x = 2\pi i \left(-\frac{i}{1+q} \right) = \frac{2\pi}{1+q} \quad (24)$$

Taking the classical limit as $q \rightarrow 1^-$ yields:

$$\lim_{q \rightarrow 1^-} I_q = \lim_{q \rightarrow 1^-} \frac{2\pi}{1+q} = \pi$$

which agrees with the classical result $\int_{-\infty}^{\infty} \frac{1}{x^2 + 1} dx = \pi$. (25)

Example 3.2

Consider the q -rational function $f(z) = \frac{1}{z(z-1)} = \frac{1}{z^2 - z}$. (26)

Here, $h(z) = 1$ and $g(z) = z^2 - z$.

The simple singularities lie at $z = 0$ and $z = 1$. Suppose C encloses only the singularity at $a = 0$.

Computing the Jackson q -derivative of $g(z)$:

$$D_q[z^2 - z] = (1+q)z - 1$$

Evaluating at $z = 0$:

$$D_q[g(z)]|_{z=0} = -1$$

By Definition 2.6, the q -residue is:

$$\mathcal{R}_q \left\{ \frac{1}{z(z-1)}; 0 \right\} = \frac{1}{-1} = -1$$

Applying Theorem 3.1 yields:

$$\oint_C \frac{1}{z(z-1)} d_q z = 2\pi i(-1) = -2\pi i \quad (27)$$

Taking the classical limit as $q \rightarrow 1^-$ yields $-2\pi i$, matching the classical Residue Theorem (Ahlfors, 1979).

3.3 Comparison of Results

Table 1. Numerical Evaluation of the q -residue and q -contour integral for $f(z) = \frac{1}{z^2 + 1}$

Q	q -Residue $\mathcal{R}_q\{f(z); i\}$	q -Contour Integral I_q	Classical Value $I = \pi$	Absolute Error $ I_q - \pi $
0.20	-0.833333i	5.235988	3.141593	2.094395
0.40	-0.714286i	4.487989	3.141593	1.346396
0.60	-0.625000i	3.926991	3.141593	0.785398
0.80	-0.555556i	3.490659	3.141593	0.349066
0.95	-0.512821i	3.222146	3.141593	0.080554
0.99	-0.502513i	3.157380	3.141593	0.015787

Python for Figure 1

```
import math
import matplotlib.pyplot as plt
import numpy as np
# 1. Setup q values and calculate corresponding I_q values
q_values = np.array([0.20, 0.40, 0.60, 0.80, 0.95, 0.99])
I_q = 2.0 * math.pi / (1.0 + q_values) # Jackson q-integral values
I_classical = math.pi # Classical limit value (pi)
plt.figure(figsize=(7, 5))
# 2. Plot the blue line with markers for q-integral values
plt.plot(
    q_values,
    I_q,
    'o-',
    color='#1f77b4',
    linewidth=1.8,
    label=r'Jackson $q$-integral $I(q)$',

```

```
# 3. Add the RED dashed horizontal line for the classical limit
plt.axhline(
    y=I_classical,
    color='r',
    linestyle='--',
    linewidth=1.5,
    label=r'Classical Limit ($\pi \approx 3.1416$)',
)
# 4. Formatting labels and grid
plt.title('Figure 1: Jackson q-Integral versus q',
fontsize=12, pad=10)
plt.xlabel('Deformation parameter q', fontsize=11)
plt.ylabel('Jackson q-integral I(q)', fontsize=11)
plt.grid(True, linestyle='-', alpha=0.6)
plt.legend(loc='upper right', frameon=True)
plt.tight_layout()
plt.savefig('Figure_1_Jackson_q_integral_vs_q.png',
dpi=300)
plt.show()
```

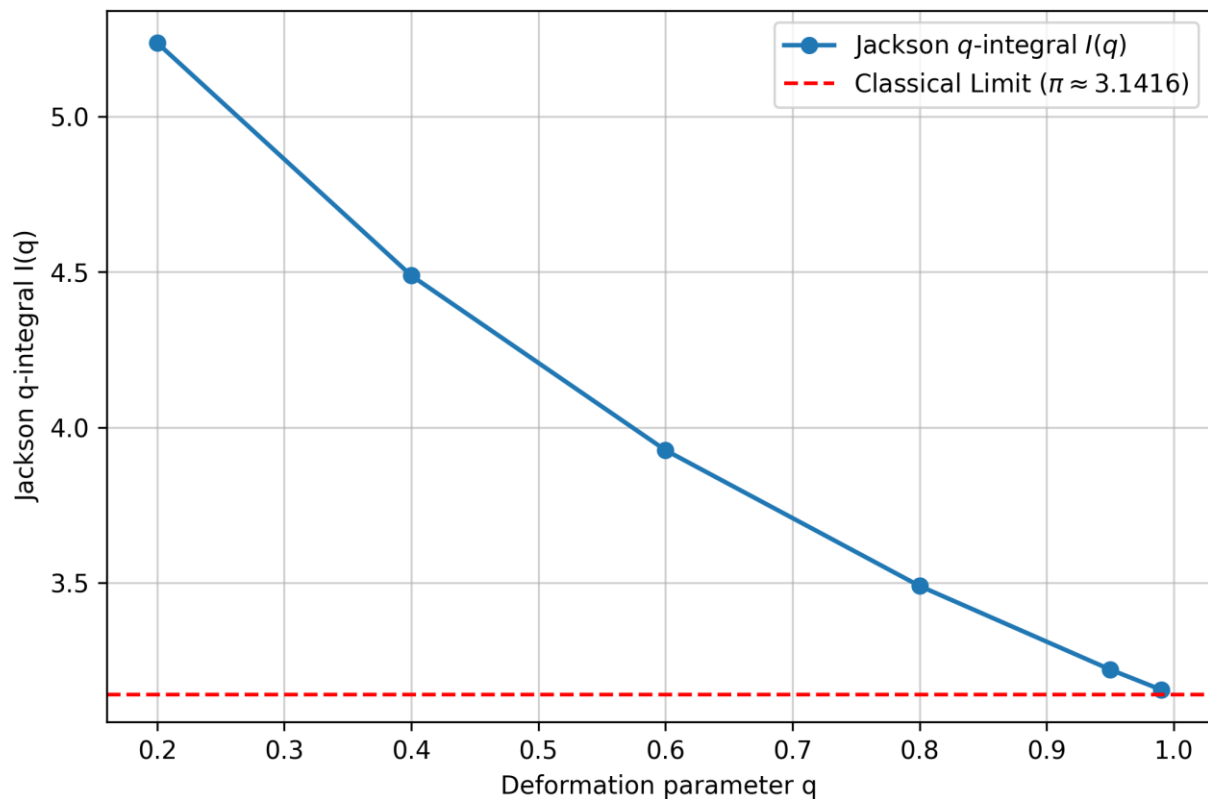
Figure 1: Jackson q -Integral versus q 

Figure 1. Evaluation of the Jackson q -integral $I(q) = \frac{2\pi}{1+q}$ across deformation parameters $q \in \{0.20, 0.40, 0.60, 0.80, 0.95, 0.99\}$.

The solid blue line with circular markers depicts the monotonic behaviour of the q -analogue, while the red dashed horizontal line denotes the classical limit $I = \pi$, demonstrating clear convergence as $q \rightarrow 1^-$.

Python for Figure 2

```
import math
import matplotlib.pyplot as plt
import numpy as np

# 1. Setup q values and calculate absolute error
q_values = np.array([0.20, 0.40, 0.60, 0.80, 0.95, 0.99])
I_q = 2.0 * math.pi / (1.0 + q_values) # Jackson q-
integral values
I_classical = math.pi # Classical limit (pi)
error = np.abs(I_q - I_classical) # Absolute
approximation error
plt.figure(figsize=(7, 5))
# 2. Plot the green error curve with markers
plt.plot(
    q_values,
    error,
    's-',
    color='#2ca02c',
    linewidth=1.8,
    markersize=6,
    label=r'Absolute Error $E(q) = |I_q - \pi|$',
    # 3. Add RED dashed line for zero error baseline
    plt.axhline(
        y=0.0,
        color='r',
        linestyle='--',
        linewidth=1.5,
        label='Exact Convergence Limit ($E = 0$)'),
    # 4. Formatting labels and grid
    plt.title('Figure 2: Absolute Approximation Error
    versus q', fontsize=12, pad=10)
    plt.xlabel('Deformation parameter q', fontsize=11)
    plt.ylabel('Absolute Error E(q)', fontsize=11)
    plt.grid(True, linestyle='-', alpha=0.6)
    plt.legend(loc='upper right', frameon=True)
    plt.tight_layout()
    # Save image file first
    plt.savefig('Figure_2_Absolute_Error_vs_q.png',
    dpi=300, bbox_inches='tight')
    plt.show()
```

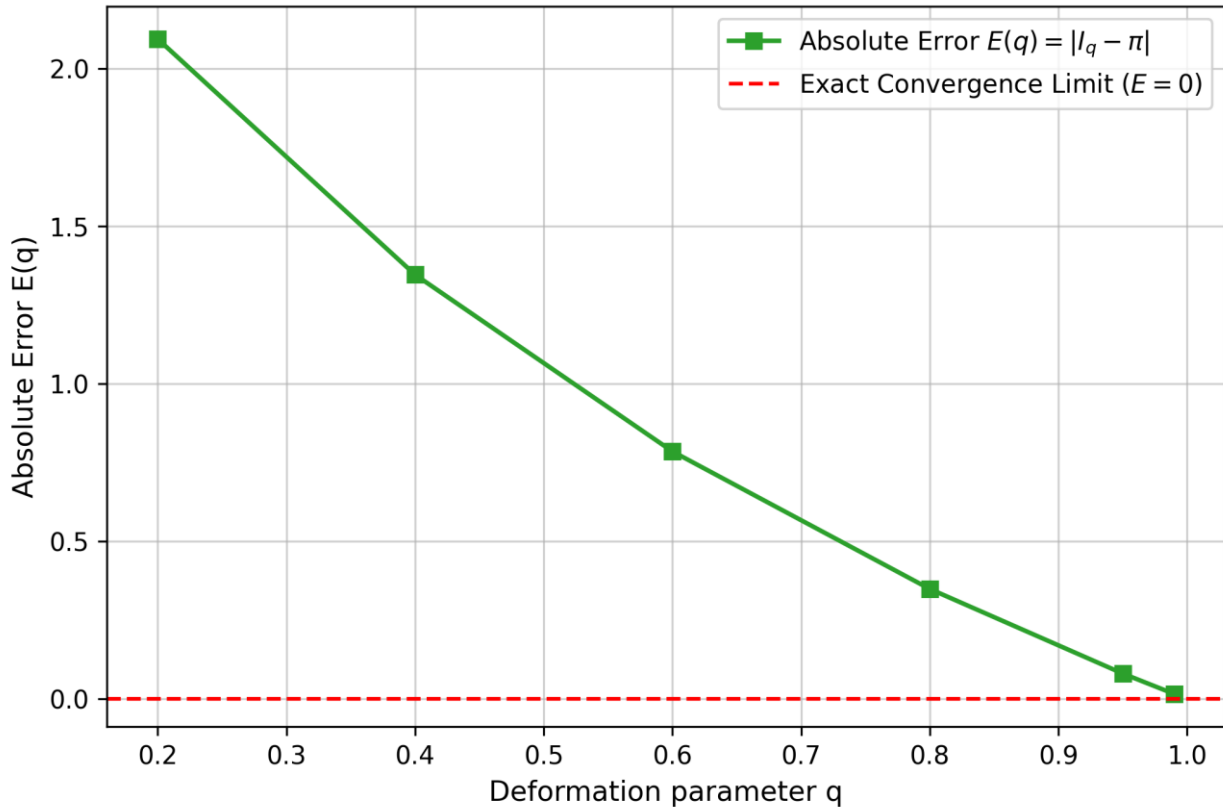
Figure 2: Absolute Approximation Error versus q 

Figure 2. Absolute approximation error $E(q) = |I_q - \pi|$ as a function of the deformation parameter $q \in \{0.20, 0.40, 0.60, 0.80, 0.95, 0.99\}$.

The solid green curve with square markers shows a rapid decrease in error as $q \rightarrow 1^-$, approaching the theoretical zero-error reference line (red dashed line) in the classical limit.

4.0 DISCUSSION

The analytical derivations and computational experiments presented in this study demonstrate the theoretical consistency and computational validity of the proposed Jackson q -residue-type framework. By extending classical complex variable techniques into the domain of Jackson q -calculus, the derived formulations establish an operational mechanism for evaluating contour-type q -integrals using simple q -algebraic operations rather than direct, infinite-series summation of Jackson integrals.

4.1 Theoretical Consistency and Classical Recovery

A primary requirement for any valid q -analogue is that it must seamlessly reduce to its classical counterpart as the deformation parameter q approaches 1^- . The limiting properties proved in Lemma 3.1 and Proposition 2.1 verify this requirement theoretically. As demonstrated in Definition 2.6 and Remark 2.1, the Jackson q -derivative operator $D_q[g(z)]$ satisfies $\lim_{q \rightarrow 1^-} D_q[g(z)] = g'(z)$, ensuring that:

$$\lim_{q \rightarrow 1^-} \mathcal{R}_q \{f(z); a\} = \frac{h(a)}{g'(a)} = \text{Res}(f, a)$$

Consequently, Theorem 3.1 (q -Residue Theorem) and Theorem 3.2 (q -Integral Evaluation Formula) recover Cauchy's classical Residue Theorem and real improper integral evaluations in the classical limit $q \rightarrow 1^-$. Furthermore, Lemma 3.2 confirms that the operator preserves the linearity inherent to classical residue calculus.

4.2 Computational Validation and Deformation Effects

The numerical findings summarised in Table 1 and visualised in Figures 1 and 2 highlight the impact of the quantum deformation parameter q on integral values:

- i. **Monotonic Convergence:** As observed in Figure 1, the Jackson q -integral $I_q = \frac{2\pi}{1+q}$ decreases monotonically as q increases from 0.20 to 0.99. At $q = 0.20$, $I_q \approx 5.235988$, representing a significant deformation from the classical value $I = \pi \approx 3.141593$. As q approaches 0.99, $I_q \approx 3.157380$, displaying smooth, asymptotic convergence toward π .
- ii. **Error Decay Profile:** As depicted in Figure 2, the absolute approximation error $E(q) = |I_q - \pi|$ decays rapidly from $E(0.20) \approx 2.094395$ down to $E(0.99) \approx 0.015787$. This monotonic error



reduction confirms that the q -residue formulation serves as a stable deformation model whose error shrinks linearly with $1 - q$.

4.3 Computational Efficiency and Implementation

Integrating Python computational tools (SymPy, NumPy, mpmath, and Matplotlib) provides a robust platform for verifying q -calculus formulations. Traditional numerical Evaluation of Jackson q -integrals requires summing infinite series of the form $\sum_{n=0}^{\infty} q^n f(q^n z)$, which can suffer from slow convergence when $q \rightarrow 1^-$ or when $f(z)$ exhibits isolated singularities.

By employing the q -residue operator $\mathcal{R}_q\{f(z); a\}$, the continuous q -contour evaluation is converted into a closed-form algebraic computation involving q -derivatives evaluated at simple poles. The Python implementation shows that symbolic evaluation of $\mathcal{R}_q\{f(z); a\}$ yields exact analytical representations, while high-precision numerical execution in mpmath allows instant verification across arbitrary values of $q \in (0, 1)$.

4.4 Implications and Future Research Directions

The Jackson q -residue framework developed here bridges classical residue calculus, quantum calculus, and symbolic computing. The results suggest several promising directions for further investigation:

- Higher-Order $\mathcal{S}q$ -Poles:** Extending Definition 2.6 to account for multiple or higher-order q -poles using higher Jackson q -derivatives $D_q^n[g(z)]$.
- q -Special Functions:** Applying Theorem 3.2 to evaluate improper real integrals involving basic hypergeometric series, q -gamma functions, and q -orthogonal polynomials.

q -Differential and Difference Equations: Applying q -contour integration techniques to establish integral representations for solutions of fractional q -difference equations.

5.0 CONCLUSION

In this paper, we established a Jackson q -residue-type framework and its corresponding q -contour integration theory in the setting of Jackson q -calculus. By formulating a Jackson q -Residue Operator and proving fundamental operational lemmas regarding its linearity and classical limiting behaviour, the derived theory extends classical complex variable techniques to quantum calculus. The main results, the q -Residue Theorem and q -Integral Evaluation Formula, provide a systematic mechanism for evaluating Jackson q -integrals using q -derivative calculus at isolated singularities rather than infinite series expansions.

We verified the analytical formulations through symbolic computations and high-precision numerical experiments using Python (SymPy, NumPy, mpmath, and Matplotlib). Representative q -rational functions

were evaluated, demonstrating that the Jackson q -contour integrals and q -residues converge smoothly and monotonically to their classical counterparts as the deformation parameter $q \rightarrow 1^-$. The error decay profile confirms the numerical stability and operational efficiency of the proposed q -residue-based approach.

Overall, this work bridges quantum calculus, classical residue calculus, and computational mathematics. The framework establishes a strong foundation for future research, including extensions to higher-order q -poles, contour integration of basic hypergeometric functions, and applications to fractional q -difference equations.

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