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Magnetohydrodynamic Casson Fluid Flow Past a Vertical Plate in the Presence of Thermal Conductivity and Chemical Absorption

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ABSTRACT

The present study examines effects of thermal conductivity and chemical absorption on Magnetohydrodynamic (MHD) convective boundary layer, flow of Casson fluid in a vertical plate. The governing flow equations are converted into dimensionless form using suitable non-dimensional parameters and variables and then numerically solved by explicit finite difference method (EFDM). Computations were performed to analyse the effect of thermo physical parameters of engineering interests associated with the flow quantities viz. velocity, temperature, concentration field and presented through graphs. It was found that chemical absorption parameter when increased has the tendency to enhance the fluid velocity as an opposite result is seen with higher values of Casson parameter. Comparison of present results with the previous literature was carried out to show the accuracy of the results.

Keywords: Casson fluid; EFDM; MHD; heat and mass transfer.

1.0 INTRODUCTION

Heat and mass transfer plays a relevant role in manufacturing, geothermal, geophysical, technological and engineering processes. For the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, space craft design, satellites and space vehicles. The usefulness of heat and mass transfer can also be explored in our environment in drying, filtration processes and saturation of porous materials by chemicals

Flow through a porous medium has many engineering and geophysical applications. For instance; in chemical engineering for filtration and purification process, in agricultural engineering to study the underground water resources, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs Alam *et al.* (2006).

However, when heat and mass transfer become prevalent, the fluid properties change. There is need to study the changes and interactions involved which are paramount for gaining insight of fluid variables (Uwanta, 2012). Few authors who carried out studies

on heat and mass transfer are; Raju *et al.* (2017) used finite element technique. Uwanta and Hamza (2012) investigated transient heat transfer flow in a vertical channel with porous medium and periodic suction analytically by perturbation method. Asogwa *et al.* (2013) examined heat and mass transfer over a vertical plate with periodic suction and heat sink.

Convection is of three forms which are force, mixed and free. Free convection is important amongst them due to its presence in engineering processes such as automatic control systems which consist of electronic and electrical components, constantly subjected to heating and cooled by means of free convection (Khalid *et al.*, 2015). The need to investigate the effect of radiation on unsteady free convection flow on heat and mass transfer problem

has become very important. This is because at high operating temperature, radiation effect can be quite significant. Many manufacturing, industrial and engineering processes occur at high temperature and knowledge of radiation heat transfer becomes necessary for design of reliable equipment.

The study of non-Newtonian fluids has several industrial and engineering benefits especially in extraction of crude oil from petroleum products. Also, Magnetohydrodynamic (MHD) flow of non-Newtonian fluids in porous medium are seen in polymer composite, paper and textile industries, process of petroleum, biological systems, heat storage beds and irrigation problems (Kataria and Patel, 2016). To this end, many studies have been carried out on several areas of MHD non – Newtonian fluid flows in porous medium.

The various non-Newtonian fluids are power-law fluids, micropolar fluids, viscoelastic fluids, Jeffery fluids, Rivlin-Ericksen fluids, Walter’s liquid B fluids, Casson fluids, etc. One of the most important type of non-Newtonian fluids is Casson fluid. The Casson fluid is a plastic fluid, which produces shear stress in constitutive equations (Reddy, 2016). Casson model was introduced by Casson in 1959 for the prediction of the flow behavior of pigment-oil suspensions (Casson,1959). Some studies containing the Casson fluid phenomenon are found in the works of (Ajayi *et al.*,2017; Omokhuale and Altine, 2017)

Finite difference methods is very useful in solving coupled non-linear partial differential equations whose solutions cannot be obtained analytically. Explicit Finite difference method (EFDM) is very useful in solving coupled non-linear partial differential equations whose solutions cannot be obtained analytically. Mondal *et al.* (2019) conducted a numerical exploration on inscribed MHD unsteady heat and mass transfer of Casson nanofluid flow where variable thermal conductivity, radiation and heat absorption are counterfeited using EFDM. Omokhuale and Jabaka (2019) studied magnetohydrodynamic Casson Fluid Flow over an

Infinite Vertical Plate with Chemical Reaction and Heat Generation numerically using EFDM.

In all the above studies mentioned, the situations are which the exact solutions of Casson fluid were obtained analytically was mostly by Laplace method for transient state solutions. There are few instances in which the numerical schemes were employed. The present work aims to fill the gap in the existing literatures. Therefore, motivated by the above studies and applications mentioned. We shall study Magnetohydrodynamic Casson fluid flow past a vertical plate in the presence of thermal conductivity and chemical absorption.

2.0 MATHEMATICAL FORMULATION

we consider Casson fluid over an infinite vertical plate embedded in a saturated porous medium. The flow being restricted to $y > 0$, where y is the coordinate which would be measured in the normal direction to the plate. The fluid would be assumed to be electrically conducting with a uniform magnetic field B of strength B_0 , applied in a direction perpendicular to the plate. The plate temperature is raised to T_w which is thereafter maintained constant. The rheological equation of state for the Cauchy stress tensor of Casson fluid will be given by,

$$\tau = \tau_0 + \mu \tau^*$$

$$\text{where } \tau_0 = \begin{cases} 2 \left(\mu_B + \frac{P_y}{\sqrt{2\pi}} \right) e_{ij}, & \text{when } \pi > \pi_c \\ 2 \left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \text{when } \pi < \pi_c \end{cases}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} is the $(i, j)^{th}$ component of the deformation rate.

The following assumptions were made before deriving the governing equations; incompressible flow, free convection and non-Newtonian flow, infinite vertical plate, fluid suction and magnetic field are imposed at the plate surface, the

temperature and concentration of the fluid are raised to T'_w and C'_w respectively and are higher than the ambient temperature and that of fluid. In addition, the effects of thermal conductivity, chemical absorption will be taken into account. It would also be assumed that the viscous dissipation is present.

The governing equations of the flow under the usual Boussinesq and boundary-layer approximation can be written as:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\rho \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = \mu_b \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \frac{\mu}{K_1} u' - \frac{\rho b'}{K_1} u' + g \rho \left[\beta_r (T' - T'_\infty) + \beta_m (C' - C'_\infty) \right] \tag{2}$$

$$\rho C_p \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = k_0 \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + Q(T' - T'_\infty) + v \left(\frac{\partial u'}{\partial y'} \right)^2 \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - R^* (C' - C'_\infty)^n + Q_1 (T' - T'_\infty) \tag{4}$$

The initial and boundary condition is:

$$\left. \begin{aligned} t' \leq 0, u' = 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ for all } y' \\ t' > 0, u' = u_p, T' = T'_w, C' = C'_w \text{ at } y' = 0 \\ u' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \tag{5}$$

$v_0 > 0$ is the suction parameter and $v_0 < 0$ is the injection parameter.

For the case of an optically thin gray gas the local radiant absorption is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a_R \sigma (T'^4_\infty - T'^4) \tag{6}$$

we assume that the temperature difference within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{7}$$

By using equations (6) and (7), equation (2) reduces to

$$\left[\frac{\partial T'}{\partial t'} + v \frac{\partial T'}{\partial y'} \right] = \frac{k_0}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{16a_R \sigma T'^3_\infty}{\rho C_p} (T' - T'_\infty) + \frac{Q}{\rho C_p} (T' - T'_\infty) + v \left(\frac{\partial u'}{\partial y'} \right)^2 \tag{8}$$

On introducing the following non-dimensionless quantities

$$\left. \begin{aligned} y = \frac{u_0}{v'} y', u_p = \frac{u'_p}{u_0}, t = \frac{u_0^2}{v'} t', u = \frac{u'}{u_0}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \tau^* = \frac{\tau}{\rho u^2} \end{aligned} \right\} \tag{9}$$

Using (1) and (9), equations (3) - (5) and (8) become:

$$\frac{\partial u}{\partial t} - \delta \frac{\partial u}{\partial \eta} = \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial \eta^2} - M^2 u - \frac{1}{K} u + G_T \theta + G_m \phi \tag{10}$$

$$\frac{\partial \theta}{\partial t} - \delta \frac{\partial \theta}{\partial \eta} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} - \frac{R}{Pr} \theta + \xi \theta$$

$$+Ec \left(\frac{\partial u}{\partial \eta} \right)^2 \tag{11} \quad \left(\frac{u'_{i,j} - u_{i,j}}{\Delta t} - \delta \frac{u_{i,j+1} - u_{i,j}}{\Delta \eta} \right) = \left(1 + \frac{1}{\gamma} \right) \left[\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta \eta)^2} \right]$$

$$\frac{\partial \phi}{\partial t} - \delta \frac{\partial \phi}{\partial \eta} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial \eta^2} - \gamma \phi + \lambda \theta \tag{12} \quad -M(u_{i,j}) - \frac{1}{K}(u_{i,j}) + G_T(\theta_{i,j}) + G_m(\phi_{i,j}) \tag{14}$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} u = 0, \theta = 0, \phi = 0 \text{ for all } y \leq 0 \\ u = u_p, \theta = 1, \phi = 1 \text{ at } y = 0 \\ u = 0, \rightarrow \theta = 0, \phi = 0 \text{ as } y = 1 \end{aligned} \right\} \tag{13} \quad \left(\frac{\theta'_{i,j+1} - \theta_{i,j}}{\Delta t} - \delta \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta \eta} \right) = \frac{1}{Pr} \left[\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta \eta)^2} \right] - \frac{R}{Pr}(\theta_{i,j+1} + \theta_{i,j})$$

where γ is the Casson parameter, G_T is the thermal Grashof number, G_m is the mass Grashof number, Sc is the Schmidt number, Pr is the Prandtl number, M is the magnetic parameter, γ is the chemical reaction parameter, K is the permeability parameter, δ is the suction parameter, ξ is the heat generation parameter, Ec is the Eckert number.

$$+\xi(\theta_{i,j+1} + \theta_{i,j}) + Ec \left(\frac{u_{i,j+1} - u_{i,j}}{\Delta \eta} \right)^2 \tag{15}$$

$$Sc \left(\frac{\phi'_{i,j} - \phi_{i,j}}{\Delta t} - \delta \frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta \eta} \right) = \left[\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta \eta)^2} \right] - \frac{\gamma Sc}{2}(\phi_{i,j}) + \lambda(\theta_{i,j}) \tag{16}$$

3.0 COMPUTATIONAL TECHNIQUE

To get the difference equations, the region of the flow is divided into a grid or mesh of line parallel to y axis. Where y -axis is normal to the plate. It is considered that the plate is of height $y_{max} = 150$ i.e. y varies from 1 to 150. There are $m = 150$ and $n = 300$ grid spacing in the y -directions. It is assumed that Δy is a constant mesh size along y direction and taken as $\Delta y = 0.50 (0 \leq y \leq 150)$. with the smaller time-step, $\Delta t = 0.0005$ and u', T' and C' represents the values of u, θ and C at the end of a time step respectively. Therefore, the nonlinear coupled partial differential equations (10) to (12) subject to boundary conditions (13) are solved numerically using explicit finite difference method as below:

The initial and boundary conditions becomes

$$\begin{aligned} u_{i,0} = 0, \theta_{i,0} = 0, \phi_{i,0} = 0 \text{ for all } i \text{ except } i = 0 \\ u_{i,0} = u_p, \theta_{i,0} = 1, \phi_{i,0} = 1 \\ u_{L,0} = 0, \theta_{L,0} = 1, \phi_{L,0} = 1 \end{aligned} \tag{17}$$

where L corresponds to ∞ . The suffix i corresponds to η and j is equals to t . consequently, $\Delta t = t_{j+1} - t_j$ and $\Delta y = y_{i+1} - y_i$.

The finite difference approximations of these equations were solved by using the values for $G_T = G_m = M = \delta = \lambda = 1, \gamma = 0.5, \xi = 0.5, Pr = 0.71, Sc = 0.6, S = 0.2, Ec = 0.2, \chi = 0.5, K = 0.5$, except where they are varied. A step size of $\Delta \eta = 0.01$ is used for the interval $\eta_{min} = 0$ to $\eta_{max} = 5$ for a desired accuracy and a convergence criterion of 10^{-6} is satisfied for various parameters.

4.0 RESULTS AND DISCUSSION

An explicit finite difference technique has been adopted to obtain the numerical results of the present model has been carried out for the different values of parameters embedded on the velocity, temperature and concentration within the boundary conditions. The finite difference approximations of these equations were solved by using the values for $G_T = G_m = M = \lambda = 1$, $\gamma = 0.5$, $Pr = 0.71$, $Sc = 0.6$, $S = 0.2$, $Ec = 0.2$, $\lambda = 0.5$, $K = 0.5$, except where they are varied. The interval $\eta_{min} = 0$ to $\eta_{max} = 5$ for a desired accuracy and a convergence for various parameters.

4.1 Velocity Profiles

Figures 1 to 7 depict the velocity profiles with varying parameters respectively.

Figure 1 shows the effect of Prandtl number on the velocity. It is seen that, the velocity decreases with increasing Prandtl number. Effect of Hartmann number M on the velocity is depicted in Figure 2, it is observed that, the velocity decreases with the increase in magnetic parameter. This is due to the fact that application of transverse magnetic field to an electrically conducting fluid which results in a Lorentz force (a resistive type force) similar to the drag force which leads to the deceleration of the flow. Casson parameter effect on the velocity profile is illustrated in Figure 3. It is found that, the velocity decreases with increasing Casson parameter. It is interesting to note that Casson parameter makes the velocity boundary layer thickness shorter. This occurs because of the plasticity of Casson fluid. Effect of suction parameter on the velocity is presented in Figure 4. It is observed that, the velocity is lower due to an increase in suction parameter. Figure 5 shows varied values of thermal Grashof number on the velocity, it is noticed that, the velocity rises with increasing thermal Grashof number. In Figure 6, the effect of mass Grashof number on the velocity is presented. It is seen that, the velocity increases with increase in mass Grashof number. Buoyancy force acts like a favourable pressure gradient which accelerates the fluid within the

boundary layer. The variation of radiation parameter with respect to temperature profile is shown in Figure 7. The increment in chemical absorption parameter increases the velocity of the fluid. This is due to the fact that chemical absorption of surface increases which leads velocity profile to grow up.

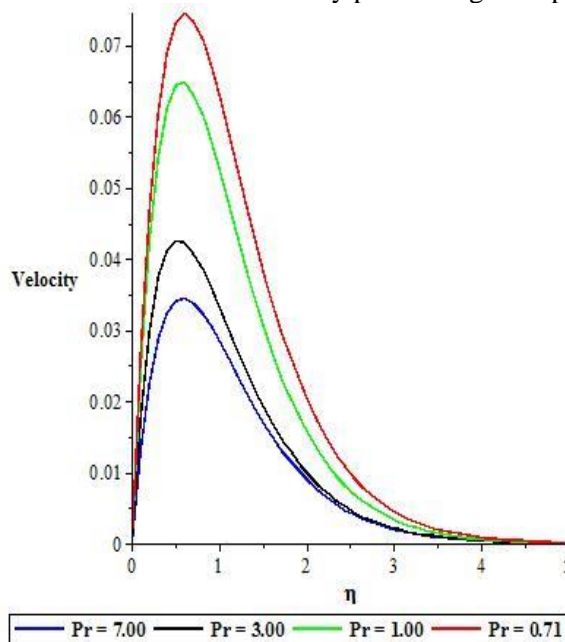


Figure 1. Effect of Prandtl number on velocity.

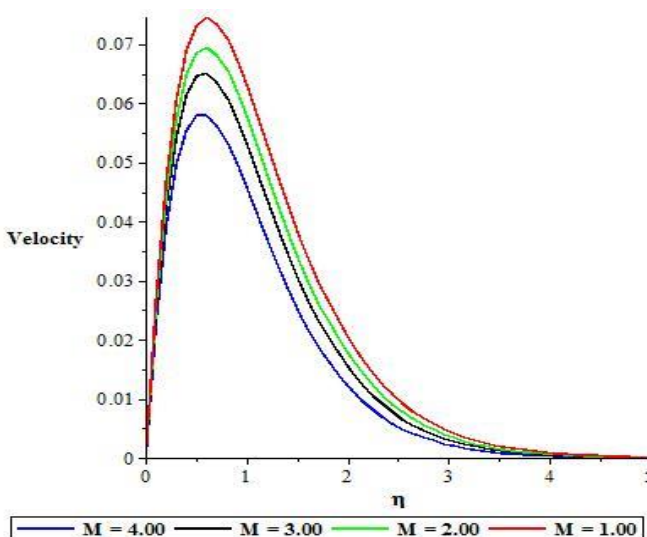


Figure 2. Effect of Hartmann number on velocity.

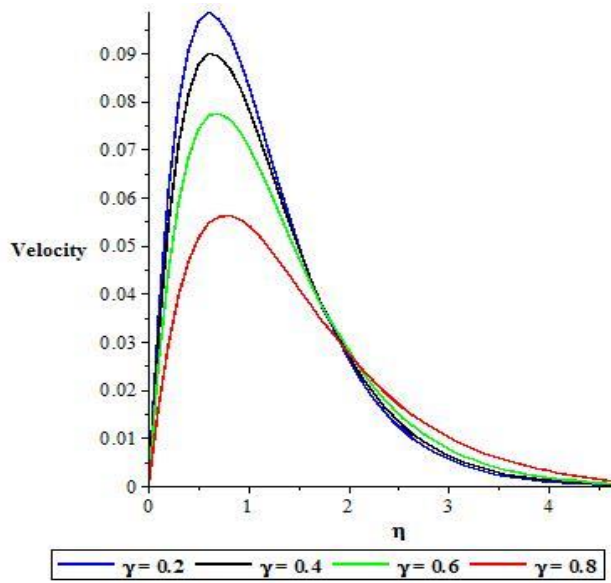


Figure 3. Effect of Casson parameter on velocity.

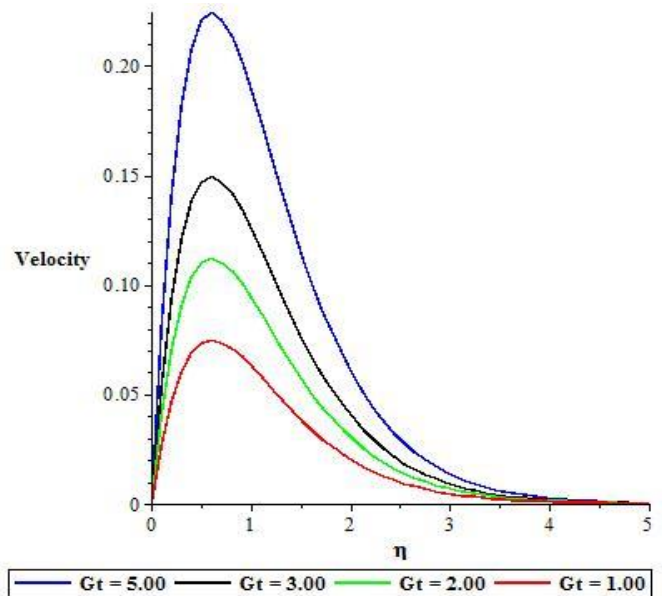


Figure 5. Effect of thermal Grashof number on velocity.

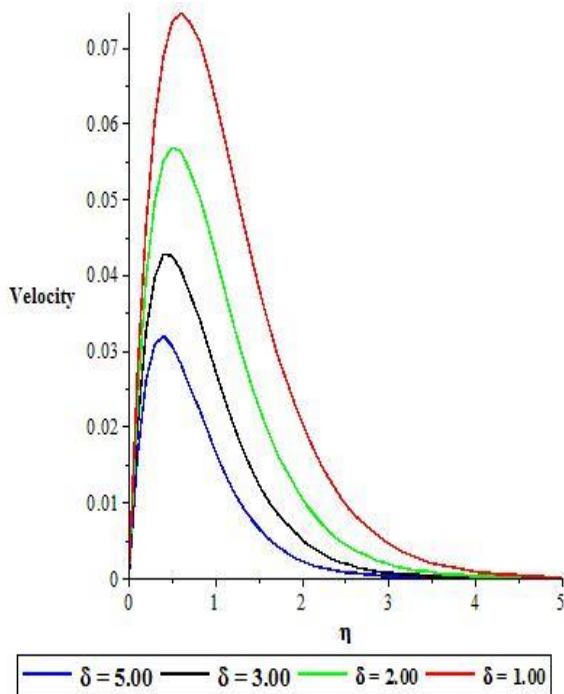


Figure 4. Effect of suction on velocity.

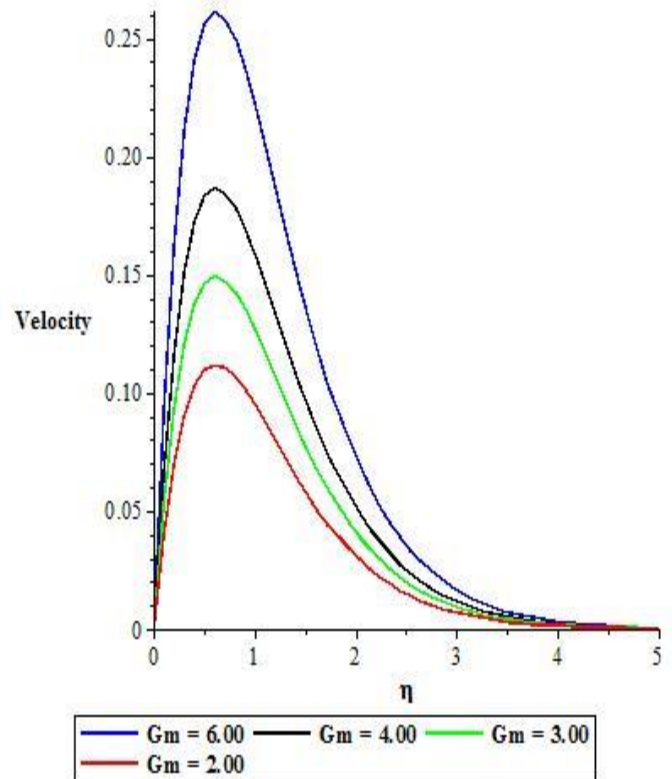


Figure 6. Effect of mass Grashof number on velocity.

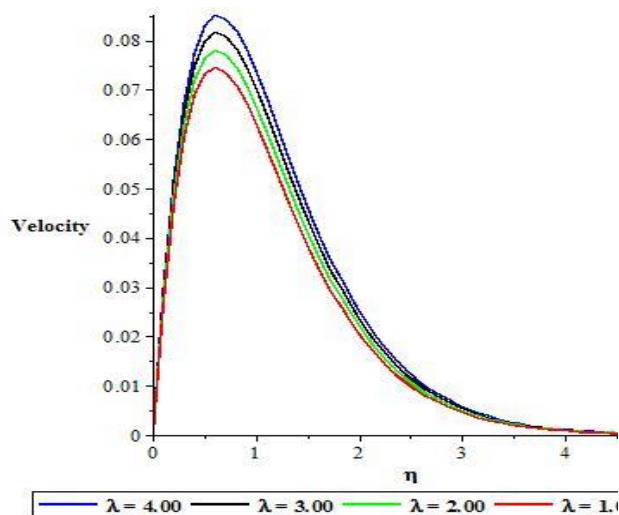


Figure 7. Effect of chemical absorption on velocity.

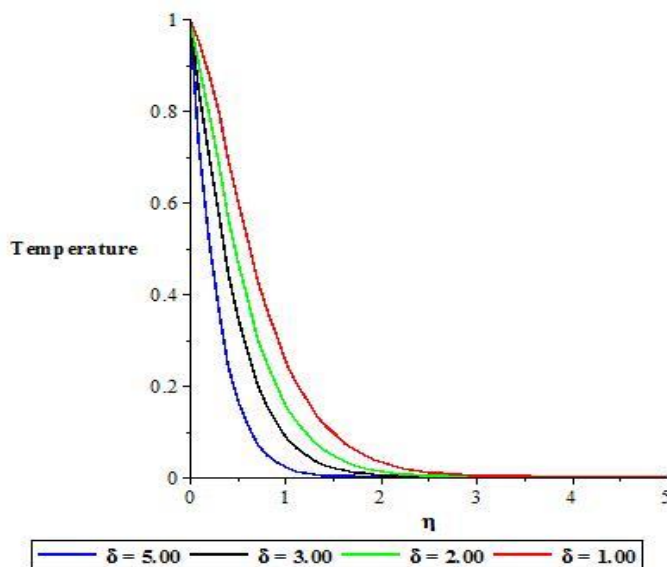


Figure 9. Effect of suction on Temperature.

4.2 Temperature profiles

Figures 8 to 9 demonstrate the temperature profiles.

In Figure 8, the influence of Prandtl number on the temperature is shown. It is seen that, the temperature decreases when the Prandtl number is increased. It is justified due to the fact that thermal conductivity of the fluid decreases with higher Prandtl number and hence retards the thermal boundary layer thickness. Figure 9 represents effect of suction parameter on temperature. It is observed that, the temperature decreases with increase in radiation.

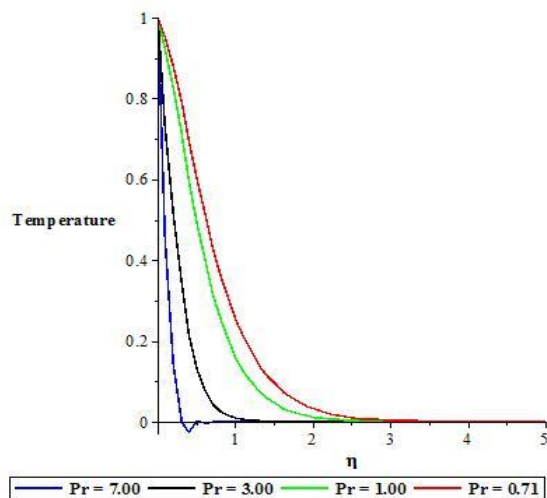


Figure 8. Effect of Prandtl number on Temperature.

4.3 Concentration profiles

Figures 10 to 11 display the concentration profiles.

Effect of Schmidt number with respect to concentration are visualized in Figure 10. As Schmidt number is the ratio of viscous diffusion rate to mass diffusion rate. Hence, the growth in Schmidt number declines the concentration of the fluid. Figure 11 shows the influence of chemical absorption parameter on the concentration. It is demonstrated that. The concentration is higher as the chemical reaction parameter is increased.

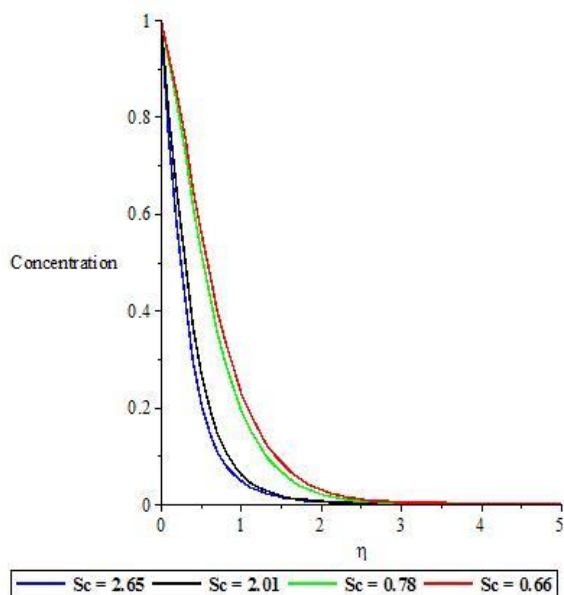


Figure 10. Effect of Schmidt number on Concentration.

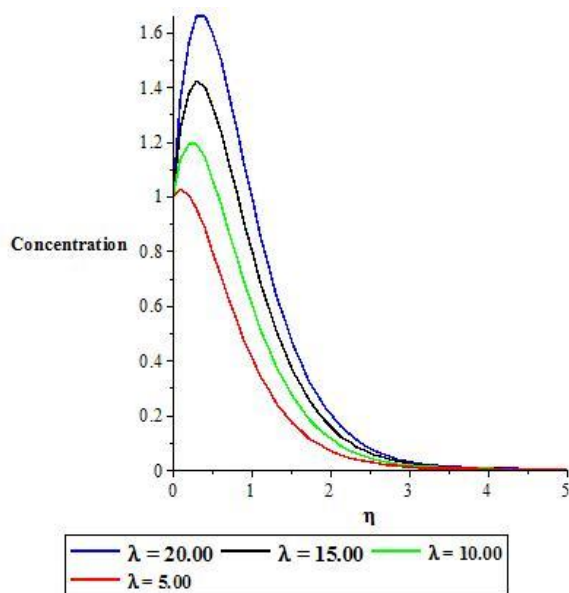


Figure 11. Effect of chemical absorption parameter on Concentration.

5.0 CONCLUSION

MHD convective boundary layer flow of a Casson fluid in a vertical plate taking into cognizance chemical absorption effect was studied. The flow

was governed by a modeled coupled nonlinear system of partial differential equations (PDEs) in dimensional form which are transformed into non-dimensional form using some non-dimensional variables. EFDM was used to approximate the velocity, temperature, concentration field. Our findings from the current study are:

- i. Velocity of the fluid increases when chemical absorption becomes significant while it decreases with increasing values of Casson parameter.
- ii. Concentration boundary layer thickness reduces by increasing the values of Schmidt number and suction.
- iii. Magnetic field retards the motion of the fluid within the boundary layer. Similarly, the velocity of the fluid rises with increasing buoyancy force..
- iv. Momentum boundary layer thickness of the fluid decreases as Prandtl number is increased.

6.0 ACKNOWLEDGEMENTS

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