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Mathematical Modeling and Analysis of the Vertical and Horizontal Transmission Dynamics of Syphilis with Treatment Intervention

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ABSTRACT

We developed in this study a mathematical model of the transmission dynamics of Syphilis, which takes into account both vertical and horizontal transmission and the effect of treatment as a control measure. The basic reproduction number R_0^S was calculated to represent the persistence and eradication conditions of the disease. The results of the analysis show that the disease free equilibrium is locally asymptotically stable for $R_0^S < 1$, globally stable for $R_0^S < 1$ and that a stable endemic equilibrium is established for $R_0^S > 1$. The parameters whose values had a positive sensitivity index increased the transmission of Syphilis, such as contact and transmission rate, while the parameters whose values had a negative sensitivity index reduced the transmission of Syphilis, such as the treatment rate, etc. Numerical simulations also revealed that if the treatment rate is increased, the burden of Syphilis to the population is greatly reduced. This highlights the importance of treatment interventions in interrupting and maybe stopping the spread of Syphilis in human populations.

Keywords: Syphilis, Vertical transmission, Prevalence, Horizontal transmission, Incubation period.

1.0 INTRODUCTION

Syphilis is a highly contagious sexually transmitted infection (STI) that is most often transmitted during vaginal, oral, and anal sex. Syphilis may also be transmitted vertically from an infected mother to her fetus during pregnancy or at birth. Syphilis has seen a resurgence in recent years, and there has been a rise in cases in several countries (Bonyah*et al.*, 2021). It continues to be an endemic disease in many LMICs and is still a major cause of morbidity in high income countries, especially for those who practice high-risk sexual behaviors (Bonyah et al., 2021; Chico *et al*., 2022; Lasagabaster & Guerra, 2019; Gasper, 2022). For example, the prevalence of syphilis among men who have sex with men (MSM) in 77 countries was estimated to be 7.5% between 2000 and 2020, which was 0.5% among men in the general population (Milner & Zhao, 2010).

The World Health Organization (WHO) estimates that there are around 376 million new infections of sexually transmitted infections (STIs) every year, including 127 million chlamydia, 87 million gonorrhoea and 6.3 million syphilis, and 156 million trichomoniasis. Syphilis is one of these infections which can result in serious complications, especially during pregnancy, and is a significant public health problem. Infected mothers can pass the organism to their unborn children and cause congenital syphilis, miscarriage, stillbirth, premature delivery or neonatal death, or cause permanent problems in surviving infants. In 2006, it was estimated that 988,000 pregnant women were infected with syphilis, resulting in over 350,000 adverse pregnancy outcomes, such as stillbirths, congenital syphilis cases, and infant deaths (Kersh & Workowski, 2020; WHO, 2023).

Philippe Ricord described syphilis in 1838, and Treponema pallidum subspecies pallidum (Tiantian *et al*., 2023) is the bacterium responsible for the disease Known as the "great imitator" because it can mimic other conditions. Syphilis is curable with antibiotics, but is a major health problem in the world, especially in developing countries. T. pallidum subspecies pallidum is a spirochaete (spiral-shaped) bacterium which is mainly transmitted via sexual contact. Vertical transmission is however an important mode of transmission in the epidemiology of syphilis, as occurs when an infected mother passes on the bacterium to her fetus across the placenta or during delivery, either through exposure to infectious lesions or less commonly. There is a higher risk for fetal infection in the early phase of maternal syphilis because of the high number of bacteria in the body.

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Syphilis may develop into 4 clinical stages. Namely: primary, secondary, and tertiary infections. If left untreated, infection can cause fetal developmental disorders, hearing loss, neurological problems, hydrops fetalis, fetal death, and skeletal deformities in pregnant women with syphilis. Partial immunity is possible following infection in both treated and untreated people, but if not treated adequately, a person can suffer from serious neurological and cardiovascular complications, as well as adverse pregnancy outcomes like stillbirth and congenital syphilis. Therefore, syphilis is continued to be an important disease in terms of epidemiology and public health (Terlu & Terefe, 2023).Prior to the advent of effective antimicrobial treatment, syphilis was a disease with significant impacts on health. In 1900, for instance, when syphilis was not yet considered to be curable, around 15 percent of the causes of blindness were blamed on the disease. Early syphilis, at this stage, can be treated with antibiotics, such as penicillin based antibiotics and other recommended antimicrobial agents (Panel*et al*. 2016). However, there are still cases of congenital syphilis as a result of poor screening, delayed diagnosis, less access to health care services, and incomplete treatment of mothers during pregnancy.WHO estimates that millions of new syphilis infections happen each year in the world, and that these are unequaly distributed, particularly in developing countries. Although treatment and prevention are available, some countries have seen repeated syphilis epidemics since the 1970s. These outbreaks have been linked to numerous factors such as shifts in sexual behavior, the HIV epidemic of the 1990s, fluctuations in syphilis control programs, substance abuse epidemics, and demographic changes among high-risk groups (Terlu & Terefe, 2023). A key characteristic seen in all these outbreaks is the presence of a “core group” of people who are highly active with high-risk behaviour and are responsible for maintaining and magnifying the transmission of syphilis. Thus, it is important to include vertical transmission in mathematical models of syphilis to capture the dynamics of the disease and to assess the impact of intervention, especially preventing congenital syphilis.

Mathematical modelling is now a key resource for understanding the spread of infectious diseases and for assessing the effectiveness of control measures. In recent years, mathematical models have been shown to be useful tools for understanding disease dynamics, intervention strategies and public health decision making. Jalija *et al*. (2026) presented a generalized fractional Adams–Bashforth–Moulton method and provided an insight into the dynamics of typhoid fever and the fact that fractional numerical methods are reliable and efficient solutions of epidemiological models. Given the greater understanding of the interactions between multiple infections, Odiba *et al*. (2024) developed a deterministic co-infection model of COVID-19, HIV, and monkeypox, emphasizing the need for disease interactions to be taken into account when modelling transmission. Moreover, Ojonimi *et al*. (2025) introduced a fractional-order model for Hepatitis B and used the generalized Adams–Bashforth–Moulton method, showcasing the accuracy, flexibility, and promise of fractional-order models in addressing the intricate dynamics of infectious diseases. In addition, Omale *et al*.(2020) conducted sensitivity and stability analyses of the transmission dynamics of TB, which helped to identify which were the most important parameters in influencing the propagation of the disease in a real population. Likewise, Omale *et al*. (2021) built a mathematical model of TB–HIV co-infection in Kogi State, Nigeria and demonstrated that co-infection played a significant role in the dynamics of TB–HIV co-infection. Omede *et al*. (2024) discussed the vertical and horizontal transmission dynamics of HIV and Zika virus co-infection, highlighting the need to incorporate multiple modes of transmission in disease modeling and control. Amos *et al*. (2024) formulated a fractional mathematical model of the behaviour of the transmission dynamics and control of Hepatitis C virus (HCV) and illustrated that Fractional-order models are more realistic descriptions of the behaviour of the disease than classical integer order models. They identified effective control measures that need to be implemented to lower the prevalence of infections. Bolaji *et al*. (2024) developed a mathematical model to manage the transmission of monkeypox in sub-Saharan Africa and highlighted its importance in decreasing the spread of the disease through preventive approaches and health care measures. Furthermore, Bolaji *et al*. (2023) studied the effect of population density on the transmission of the Omicron variant of COVID-19 in densely populated cities in Nigeria, and found that prompt intervention strategies are essential to decrease the transmission of the Omicron variant. Philip *et al*. (2024) formulated an HIV/AIDS transmission model of fractional order with control measures, and showed that fractional modelling offers more insights into the long-term dynamics of the disease and the effectiveness of intervention measures. Likewise, Amos *et al*. (2026) developed a deterministic mathematical model of the vertical and horizontal dynamics of HIV/AIDS, and included treatment as an important control measure. Their findings showed that the higher the treatment rate, the lower the HIV/AIDS prevalence and cumulative new infections, while higher contact and vertical transmission rates are associated with increased spread of HIV/AIDS. The results highlight the need to combine the treatment interventions

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and preventive interventions for controlling the transmission of infectious diseases.

1.1 Objective of the Study

The objectives of his study are to:

1. Develop a compartmental model incorporating both vertical and horizontal transmission routes.
2. Derive the basic reproduction number R_0^S and determine its epidemiological significance.
3. Analyze the stability of disease-free and endemic equilibrium points.
4. Introduce and assess control strategies such as treatment and prevention of vertical transmission.
5. Conduct numerical simulations to evaluate the effectiveness of these control measures in reducing disease prevalence.

1.2 Novelty of the Study

The novelty of this study is the development of a mathematical model involving both horizontal and vertical transmission dynamics of syphilis with treatment as a control strategy. Compared to existing models which focus mainly on sexual transmission, the model proposed here also includes transmission from mother to child, thus giving a more complete picture of the spread of syphilis and congenital syphilis. The study also contributes by analyzing the basic reproduction number, the stability of the equilibria, sensitivity analysis and numerical simulations to identify key parameters influencing disease transmission. The findings underscore the importance of effective treatment, early diagnosis and preventive interventions to reduce syphilis transmission and control its impact on the population.

2.0 MODEL FORMULATION

Modeling Syphilis , the total population sub-divided into six compartments according to the integer-order model; susceptible individuals S, Individuals who that are free from disease infection make up this group.; exposed individuals E ; People who have experienced infection exposure without developing contagious status; infected individuals I ; individuals on treatment T ; Population of individuals on treatment, Recovered human population R and Bacteria population B.

The susceptible human population is recruited at the rate of π, The contact probability between the susceptible humans and infected humans with Syphilis, humans on Syphilis treatment and Bacteria population are denoted as ϕ_1, ϕ_2, ϕ_3, respectively, human population die naturally at the rate of μ_h , exposed human population progresses to Syphilis infected human population at the rate of α , infected human population, and humans on Syphilis treatment die due to the disease at the rate of δ_1, δ_2 , respectively. The disease is transmitted vertically from mother to child at a rate of β . Recovered human can be again susceptible at the rate σ .

2.1 Model Assumptions

1. We assume natural death in the population.
2. We assume that human die due to Syphilis.
3. We assume horizontal transmission.

2.2 Model Flow Diagram

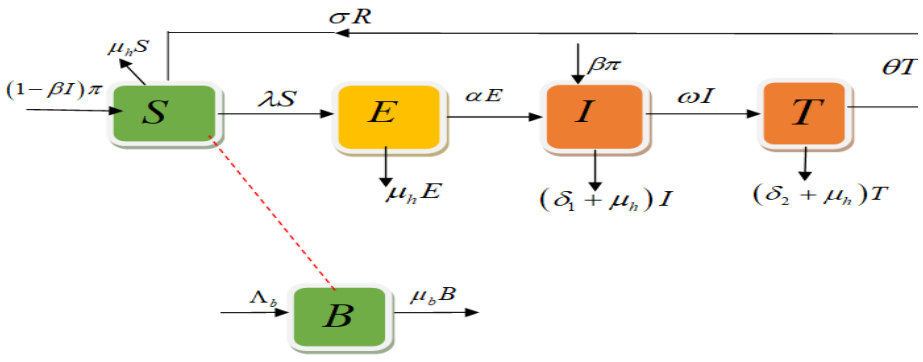

Figure.1: Syphilis Model flow Diagram

2.3 Model Equations

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$$\frac{dS}{dt} = (1-\beta I)\pi + \sigma R - \lambda S - \mu_h \mathrm{S},$$
$$\frac{dE}{dt} = \lambda S - (\alpha + \mu_h)\mathrm{E},$$
$$\frac{dI}{dt} = \alpha E + \beta\pi - (\omega + \delta_1 + \mu_h) I,$$
$$\frac{dT}{dt} = \omega I - (\theta + \delta_2 + \mu_h) T, \quad (1)$$
$$\frac{dR}{dt} = \theta T - (\sigma + \mu_h)\mathrm{R},$$
$$\frac{dB}{dt} = \Lambda_b - \mu_b \mathrm{B}$$

Where $\lambda = \frac{(\phi_1 I + \phi_2 T + \phi_3 B)}{N}$.

Table 1: Model Variables and Parameter Description

Model Variables	**Descriptions**
S	Susceptible Humans to Syphilis
E	Exposed humans to Syphilis
I	infected humans with Syphilis
T	Humans on Syphilis treatment
R	Recovered human population from Syphilis
B	Bacteria population
Model parameters	**Descriptions**
π	Recruitment rate of human population.
μ_h	Natural death rate of human population.
α	Progression rate from exposed human population to syphilis infected human population.
δ_1	Disease induced death rate of Syphilis infected human population.
δ_2	Disease induced death rate of human population on Syphilis treatment.
ω	Treatment rate of Syphilis infected human population.
ϕ_1	Contact rate between the susceptible and Syphilis infected human population.
ϕ_2	Contact rate between the susceptible and human population Syphilis treatment.
ϕ_3	Contact rate between the susceptible and Bacteria population
θ	Recovery due to treatment rate
β	Vertical transmission rate
σ	Re-infection rate
Λ_b	Recruitment rate of Bacteria population
μ_b	Natural death rate of Bacteria population

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3.0 MODEL ANALYSIS

Like all the works on mathematical epidemiology, we analyze the model in such a way that it is formulated for its basic properties viz, positivity of state variables embedded in the model, local and global stability of the disease free equilibrium of the model, sensitivity analysis for the basic reproduction numbers for the disease etc.

3.1 Positivity of solution of Syphilis Model

Theorem 1: if the initial value S ≥ 0, $E \geq 0$, $I \geq 0$, $T \geq 0$, $R \geq 0$, $B \geq 0$. Then the solution of syphilis model is non-negative.

Proof: To show that solution of the model are positive, we first ds/dt from equation

(2)

$$S(t) \geq S(0)e^{-\frac{(\phi_1 I+\phi_2 T+\phi_3 B+\mu_h)t}{N}}.$$

$-\frac{(\phi_1 I+\phi_2 T+\phi_3 B+\mu_h)t}{N} > 0.$

Since

3.2 Invariant Region of Syphilis Model

Theorem 2

Let the total human population be $N = S_h, E_P, I_P, T_P, V_h, R_P.$

Theorem 2: The solution of the syphilis model (1) are bounded and the region $\Omega = \left\{(S,E,I,T,R) \in R^5 : N \leq \frac{\pi}{\mu_h}\right\}$ is positively invariant .

Proof.

The invariant region is proved by the visibility of the solution of the model.

The population of the model is:

$$\frac{dN}{dt} = (1-\beta I)\pi + \beta\pi - \mu_h S - \mu_h E - \mu_h I - \mu_h T - \mu_h R - \delta_1 I - \delta_2 T, \quad (3)$$

$$N(t) \leq \left(\frac{\pi}{\mu_h} + N(0) - \frac{\pi}{\mu_h}\right)e^{-ut}. \quad (4)$$

This implies that the model admits a unique solution and is invariant in the region $\Omega = \Omega_S$ where the solution is contained and the model is epidemiologically well posed and hence meaningful.

3.3 Disease Free Equilibrium Point

The disease-free equilibrium is a state in which there is no disease present in human populations.

At the disease free equilibrium point, $S^0 \neq 0, E^0 = 0, I^0 = 0, T^0 = 0, R^0 = 0, B^0 \neq 0.$

$$(S^0, E^0, I^0, T^0, R^0, B^0) = \left(\frac{\pi}{\mu_h}, 0, 0, 0, 0, \frac{\pi_b}{\mu_b}\right). \quad (5)$$

3.4 Basic Reproduction of Syphilis

The basic reproduction number is the sum of the total number of infections transmitted from one infected human to the susceptible population. The Basic Reproduction number is calculated using next generation method $R_0^S = \rho FV^{-1}$. Considering F as non-negative matrix along with other transition terms V and finding out the largest Eigen value ρ.

The largest Eigen value is:

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$$R_0^S = \frac{\pi\beta K_2 K_4 + \phi_2 \omega\alpha + \phi_1 \alpha K_4}{K_3 K_2 K_4}. \quad (6)$$

3.5 Endemic Equilibrium Point of Syphilis

The term endemic equilibrium refers to the presence of syphilis in the human population.

At endemic equilibrium point, $S^* \neq 0, E^* \neq 0, I^* \neq 0, T^* \neq 0, R^* \neq 0, B^* \neq 0.$

We were able to show that, the endemic equilibrium point of model (1) is stable.

3.6 Local Stability of Syphilis at Disease Free Equilibrium Point

We also show that, model (1) is locally asymptotically stable using the Routh-Hurwitz criterion which states that all roots of the polynomial have negative real parts if and only if $N_1 > 0, N_2 > 0, N_3 > 0, N_4 > 0, N_5 > 0, N_6 > 0$ is satisfy these conditions, then $R_0^S < 1$. Therefore, by Routh-Hurwitz criterion, the disease-free equilibrium of the Syphilis model (1) is locally asymptotically stable when $R_0^S < 1$.

Biologically, the implication of theorem 3 is when the syphilis basic reproduction number less than unity, Syphilis can be eliminated from the population if the initial sizes of the sub-populations of the Syphilis model are in the basin of attraction of ℓ_0. It is relevant to show the global asymptotic stability of the CDFE to make sure that the Syphilis elimination is independent of the initial sizes of the sub-populations.

3.7 Global Asymptotic Stability of the Disease-Free Equilibrium Point of the Syphilis Model

To study the disease-free equilibrium, we use the technique developed by Castillo-Chavez and song (2002) to investigate its global stability.

Theorem 4

The equilibrium point $\varepsilon_0 = (X^*, 0).$ *is globally asymptotically stable if* $R_0^S \leq 1$.

From the above, it is obvious that,

Theorem 4

The equilibrium point $\varepsilon_0 = (X^*, 0).$ *is globally asymptotically stable if* $R_0^S \leq 1.$

$$\hat{G}(X,Z) = \begin{pmatrix} \phi_1 I + \phi_2 T + \phi_3 B\left(1 - \frac{S}{N}\right) \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (7)$$

From the above, it is obvious that,

$\hat{G}_1(X,Z) \geq 0$ *and* $\hat{G}_2(X,Z) = \hat{G}_3(X,Z) = \hat{G}_4(X,Z) = 0,$ which implies that $\hat{G}_1(X,Z) \geq 0.$

Consequently, since the given necessary and sufficient conditions stated above is satisfied, then the disease-free equilibrium ε_0 of Syphilis model is globally asymptotically stable when $R_0^S < 1.$

3.8 Sensitivity Analysis of Syphilis Model

Sensitivity analysis is performed to identify the parameters which favour the disease to spread among human population.

We use $S_x^{R_0} = \left(\frac{\partial R_0}{\partial x}\right)\left(\frac{x}{R_0}\right).$

$S_\beta^{R_0^S} = 0.5541,\ S_\alpha^{R_0^S} = 0.0678,\ S_\theta^{R_0^S} = -0.7092,\ S_\omega^{R_0^S} = -0.84614,\ S_{\phi_1}^{R_0^S} = 0.6321,\ S_{\phi_2}^{R_0^S} = 0.4398,$

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$$S_{\delta_1}^{R_0^S} = -0.06743,\ S_{\delta_2}^{R_0^S} = -0.2318,\ S_{\pi}^{R_0^S} = 0.65021,\ S_{\mu_h}^{R_0^S} = -0.00867.$$

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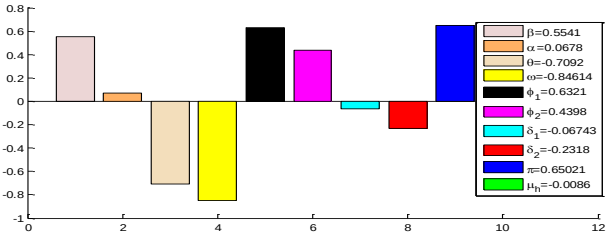


Figure 2: Sensitivity Bar chart

3.9 Interpretation of sensitivity indices of Syphilis Model

The above sensitivity indices of the basic reproduction number of the Syphilis diseases are shown in the bar chart shown in figure 2 respectively. If the value of those parameters with a positive index in the Syphilis model increases (under the assumption of keeping the values of other parameters constant), they have significant effect in accelerating the spread of disease. As such, when the value of their increases, the basic reproduction number increases. Instead, a parameter with a negative index will increase its value so that it can help reduce burdens of diseases, resulting in lower basic reproduction numbers.

4.0 NUMERICAL SIMULATION

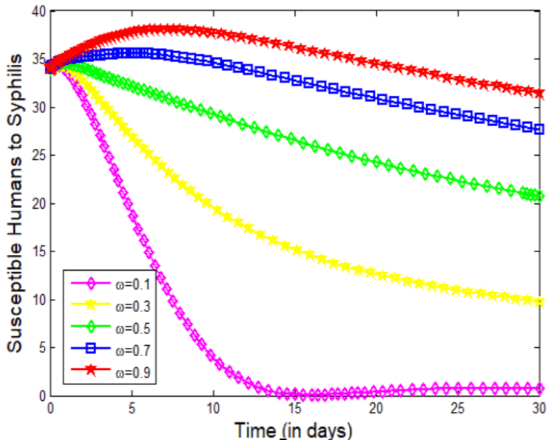


Figure 3a: Simulation of the effect of ω on susceptible humans

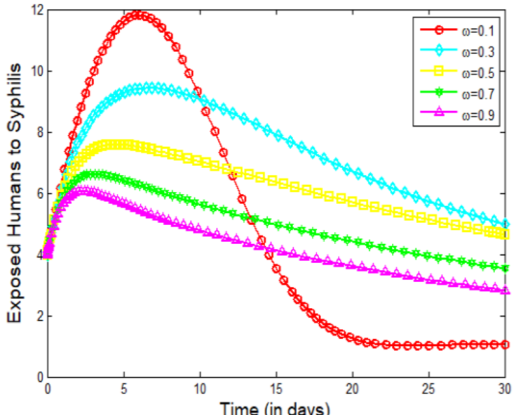
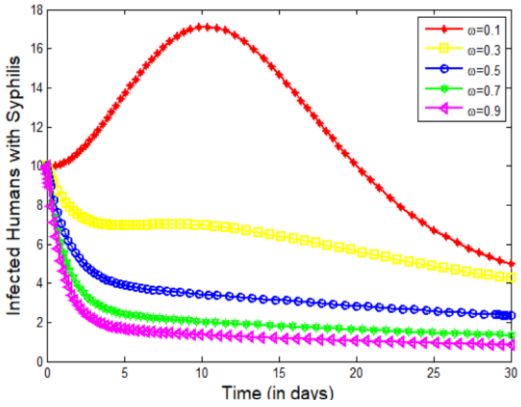


Figure 3b: Simulation of the effect of ω on Exposed humans



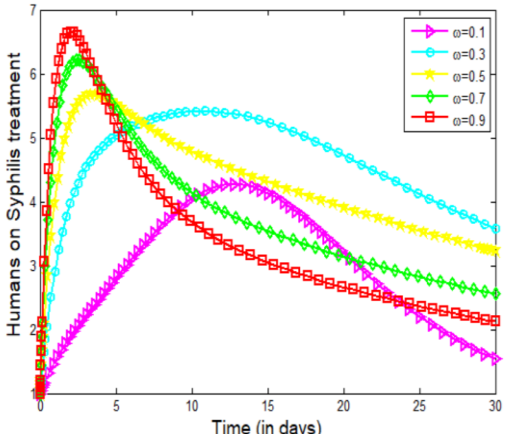


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Figure 3c: Simulation of the effect of ω on infected humans

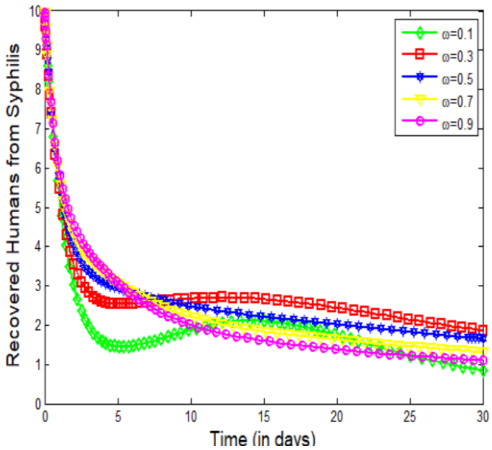


Figure 3e: Simulation of the effect of ω on Recovered humans from syphilis

Figure 3d: Simulation of the effect of ω on humans on syphilis treatment

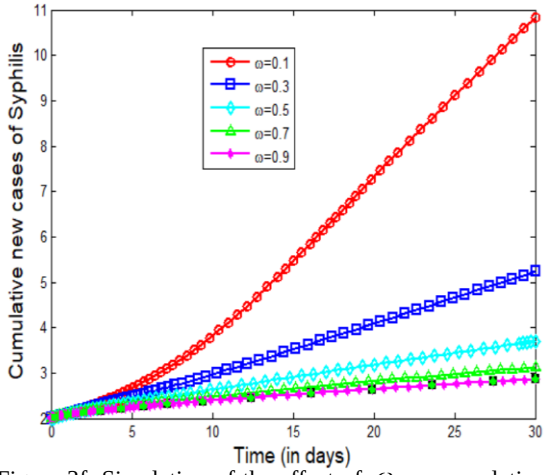


Figure 3f: Simulation of the effect of ω on cumulative new cases of syphilis

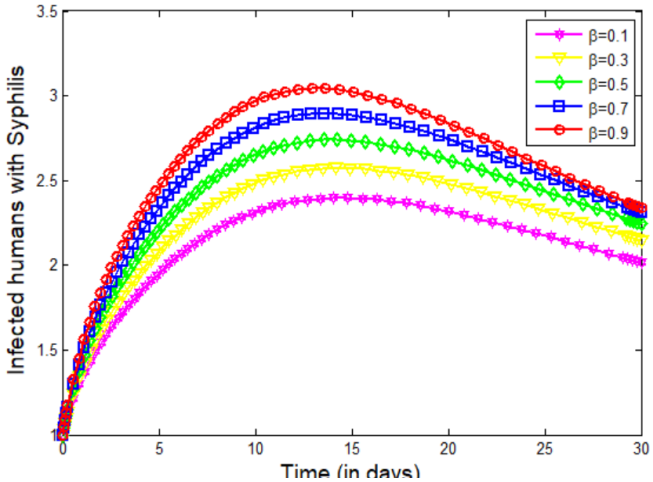


Figure 3g: Simulation of the effect of β on infected humans

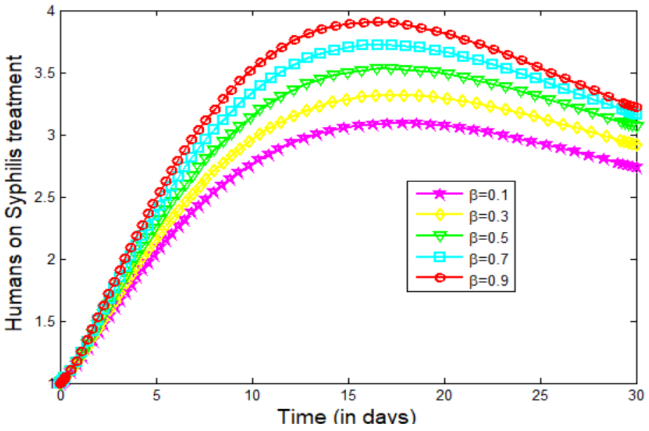
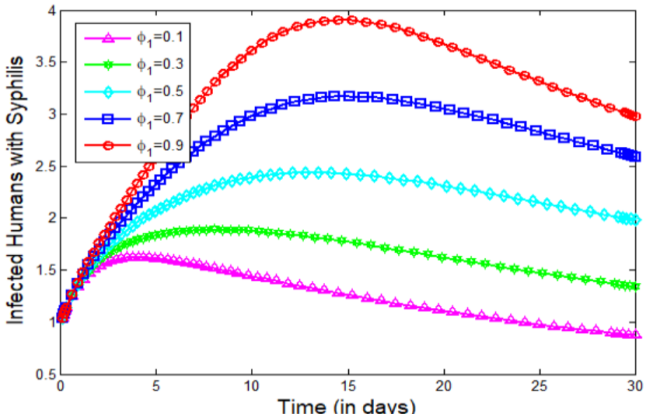


Figure 3h: Simulation of the effect of β on humans on syphilis treatment



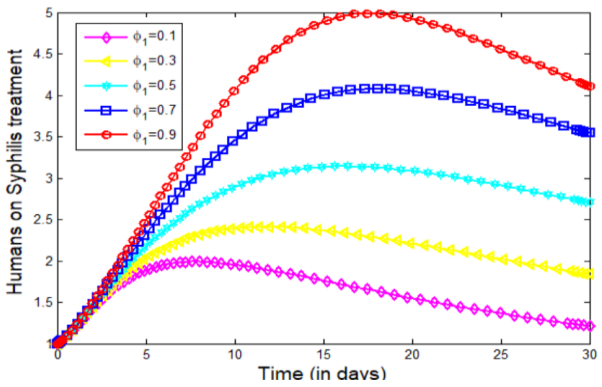


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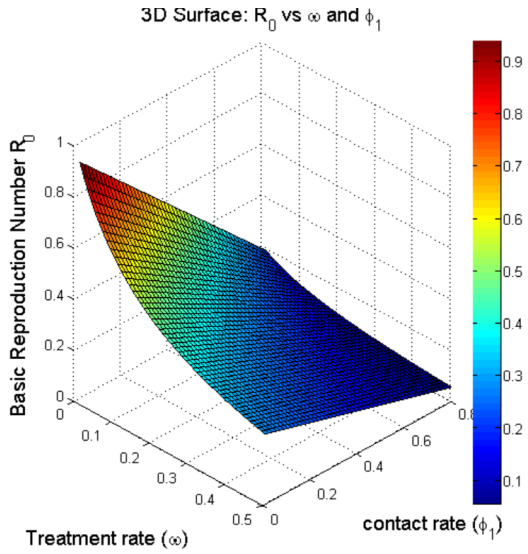
Figure 3i: Simulation of the effect of ϕ_1 on infected humans

Figure 3j: Simulation of the effect of ϕ_1 on Exposed humans



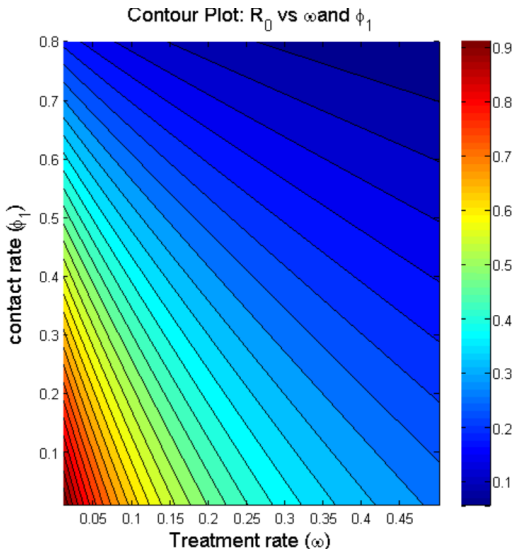


Figure.3k: Surface plot of the effect of ω and ϕ_1 on R_0^S

Figure 3l: Contour plot of the effect of ω and ϕ_1 on R_0^S

4.1 Interpretation of the Numerical Simulation

Figure (3a) depicts the simulation of the effect of the treatment rate (ω) on Syphilis in the susceptible population. It is observed that, as the treatment rate (ω) increases, the number susceptible individuals' increases. Figure (3b) shows the simulation of the effect of treatment rate (ω) on the Exposed population with Syphilis. It is observed that, as the treatment rate increases, the number of Exposed individual's decreases. Figure (3c) depicts the simulation of the effect of the treatment rate (ω) on Syphilis in the infected human population. It is observed that, as the treatment rate increases, the infected human population decreases. Figure (3d) depicts the simulation of the effect of the treatment rate (ω) on Syphilis in the treatment human population. As treatment rate goes up, so does treatment human population.

Figure (3e) depicts the simulation of the effect of the treatment rate (ω) on Syphilis in the recovered human population. As the treatment rate (ω) goes up, an increasing number of people in the human population are recovered. Figure (3f) depicts the simulation of the effect of the treatment rate (ω) on Syphilis on cumulative new cases of Syphilis. What is observed is that the more treatment rate (ω) is increased, the fewer the cumulative number of new cases of Syphilis. Figure (3g) shows the simulation of the effect of the vertical transmission rate (β) on infected humans. With the increase of the vertical transmission rate (β) as mentioned above, it has been observed that the number of infected humans increases. Figure (3h) depicts the simulation of the effect of the vertical transmission rate (β) on human population on Syphilis treatment. As vertical transmission rate (β) increases, the human population on Syphilis treatment increases, it is observed.Figure (3i) depicts the simulation of the effect of the contact rate (ϕ_1) on Syphilis in the infected human population. It is observed that, as the contact rate (ϕ_1)

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increases, the infected human population increases. Figure (3j) depicts the simulation of the effect of the contact rate (ϕ_1) on Exposed human population to Syphilis. It is observed that, as the contact rate (ϕ_1) increases, the Exposed human population to Syphilis increases. (3k) it can be observed that the basic reproduction number R_0^S reaches a peak below one (1) as the values of ω increases and ϕ_1 decreases. This means that as this level gets higher ω and lower ϕ_1, it will eventually have a lessening effect on the population of Syphilis. On the other hand, if proper actions are not taken, ϕ_1 can lead to the increase of human population infected by Syphilis. From their effect on (3l) Illustrates the contour plot of concerning . By inspecting the numerical streams in the graph, it can be seen that by changing the value of between 0 and 1, the greatest value of is 0.6 which is less than 1. This observation implies that would decrease a large outbreak of Syphilis in humans.

5.0 CONCLUSION

In this study, we carefully calibrated the mathematical model to investigate the dynamics of vertical and horizontal transmission of Syphilis with treatment as a control strategy. We found the basic reproduction number R_0^S of the Syphilis model and showed that the system is locally asymptotically stable when $R_0^S < 1$ and globally stable when $R_0^S < 1$ and maintains a stable endemic equilibrium when $R_0^S > 1$. Using sensitivity analysis we quantified the effect of some parameters on the basic reproduction number. We found that the parameter, with positive sensitivity indices enhance the spread and burden of Syphilis thereby increasing the burden of the disease. On the other hand, those parameters that have negative sensitivity indices reduced the transmission of Syphilis. In addition, numerical simulation was performed, varying the parameters involved, including infected human treatment rate ω, vertical transmission rate β and contact rate ϕ_1. Our simulations showed that the burden of Syphilis in the human population can be significantly reduced with an increase in treatment.

5.1 Limitations and Future Research Directions

There are certain limitations to this study. The model assumes homogeneous population mixing and constant epidemiological parameters, which are not necessarily representative of the epidemiological dynamics of syphilis transmission in the actual population. While vertical transmission is included, there are also other factors not considered, including stage of maternal infection, screening coverage, adherence to treatment, and access to healthcare. Further, there are no considerations for co-infections or behavioural heterogeneity. The model can be further expanded for future studies to consider these factors in predicting and controlling the transmission of syphilis.

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Declaration of competing interest

No financial conflicts or personal relationships affecting the research findings of this paper exist according to the authors.

Data availability

Values of parameters were fitted, and also parameters used are adequately cited and referenced.

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